



NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University,
Kerala)



DEPARTMENT OF COMPUTER SCIENCE ENGINEERING

COURSE MATERIALS



BE100 ENGINEERING MECHANICS

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- ◆ Established in: 2002
- ◆ Course offered : B.Tech in Computer Science and Engineering
M.Tech in Computer Science and Engineering

M.Tech in Cyber Security

- ◆ Approved by AICTE New Delhi and Accredited by NAAC
- ◆ Affiliated to the University of A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Producing Highly Competent, Innovative and Ethical Computer Science and Engineering Professionals to facilitate continuous technological advancement.

DEPARTMENT MISSION

1. To Impart Quality Education by creative Teaching Learning Process
2. To Promote cutting-edge Research and Development Process to solve real world problems with emerging technologies.
3. To Inculcate Entrepreneurship Skills among Students.
4. To cultivate Moral and Ethical Values in their Profession.

PROGRAMME EDUCATIONAL OBJECTIVES

- PEO1:** Graduates will be able to Work and Contribute in the domains of Computer Science and Engineering through lifelong learning.
- PEO2:** Graduates will be able to Analyse, design and development of novel Software Packages, Web Services, System Tools and Components as per needs and specifications.
- PEO3:** Graduates will be able to demonstrate their ability to adapt to a rapidly changing environment by learning and applying new technologies.
- PEO4:** Graduates will be able to adopt ethical attitudes, exhibit effective communication skills, Team work and leadership qualities.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate

consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: Ability to Formulate and Simulate Innovative Ideas to provide software solutions for Real-time Problems and to investigate for its future scope.

PSO2: Ability to learn and apply various methodologies for facilitating development of high quality System Software Tools and Efficient Web Design Models with a focus on performance optimization.

PSO3: Ability to inculcate the Knowledge for developing Codes and integrating hardware/software products in the domains of Big Data Analytics, Web Applications and Mobile Apps to create innovative career path and for the socially relevant issues.

COURSE OUTCOMES

CO1	Apply the fundamental law of mechanics and equilibrium equations for planar problems.
CO2	Classify various supports and loadings and apply the equations of equilibrium for forces system in space
CO3	Analyze and calculate the area moment of inertia of composite sections, and extend the same for mass moment of inertia of simple solids.
CO4	Combine the laws of static friction and analyze force system inclusive of frictional forces.
CO5	Compute the dynamics of rigid bodies in Cartesian and cylindrical coordinates and its applications in translation and rotation.
CO6	Analyze vibrations of single degree of freedom systems.

MAPPING OF COURSE OUTCOMES WITH PROGRAM OUTCOMES

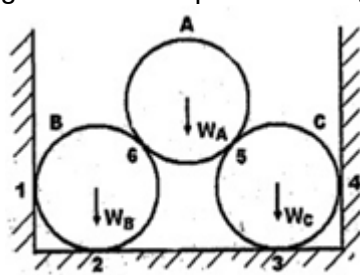
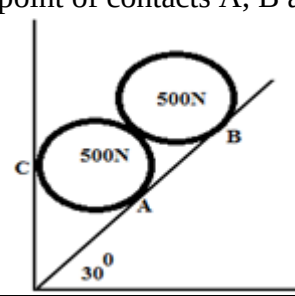
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	3	3	-	-	-	1	-	-	-	-	-	2	1	-	3
CO2	3	3	3	-	-	2	-	-	-	-	-	3	1	-	3
CO3	3	3	-	-	-	-	-	-	-	-	-	1	1	-	3
CO4	3	3	2	-	-	3	2	-	-	-	-	2	-	-	3
CO5	3	3	-	-	-	2	-	-	-	-	-	2	1	-	3
CO6	3	3	3	-	-	3	-	-	-	-	-	2	-	-	3

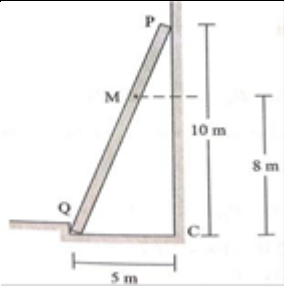
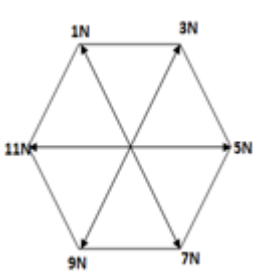
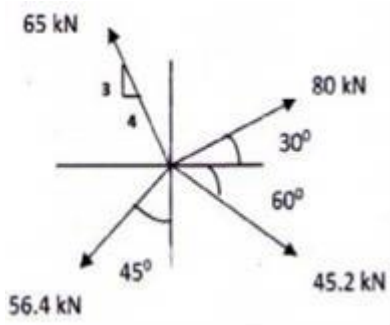
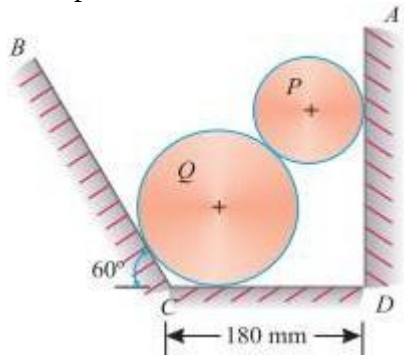
Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

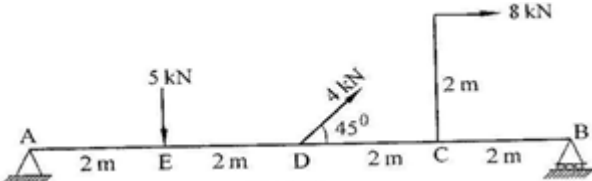
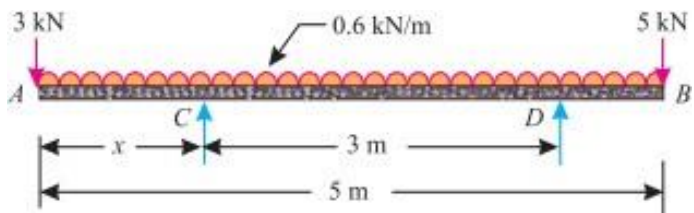
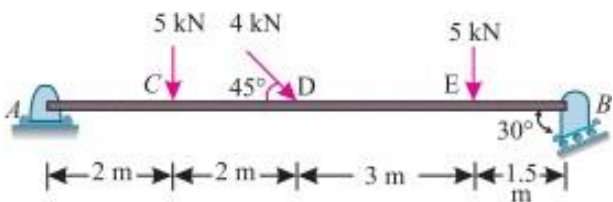
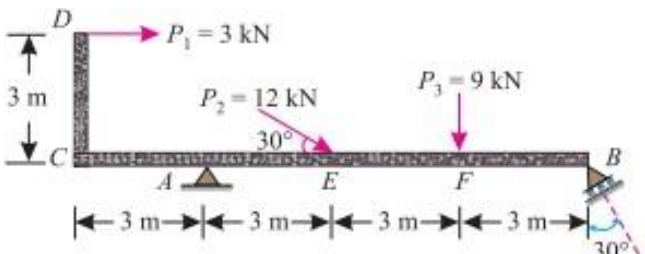
Module	Contents	Hours	Sem. Exam Marks
I	Statics: Fundamental concepts and laws of mechanics – Rigid body – Principle of transmissibility of forces	2	15%
	Coplanar force systems - Moment of a force – Principle of moments	2	
	Resultant of force and couple system	4	
	Equilibrium of rigid body – Free body diagram – Conditions of equilibrium in two dimensions – Two force and three force members.	3	
II	Types of supports – Problems involving point loads and uniformly distributed loads only.	5	15%
	Force systems in space – Degrees of freedom – Free body diagram – Equations of equilibrium – Simple resultant and Equilibrium problems.	4	
FIRST INTERNAL EXAM			
III	Properties of planar surfaces – Centroid and second moment of area (Derivations not required) - Parallel and perpendicular axis theorem – Centroid and Moment of Inertia of composite area.	3	15%
	Polar Moment of Inertia – Radius of gyration – Mass moment of inertia of cylinder and thin disc (No derivations required).	2	
	Product of inertia – Principal Moment of Inertia (conceptual level).	3	
	Theorems of Pappus and Guldinus.	1	
IV	Friction – Characteristics of dry friction – Problems involving friction of ladder, wedges and connected bodies.	6	15%
	Definition of work and virtual work – Principle of virtual work for a system of connection bodies – Problems on determinate beams only.	4	
SECOND INTERNAL EXAM			
V	Dynamics: Rectangular and Cylindrical co-ordinate system	1	20%
	Combined motion of rotation and translation – Concept of instantaneous centre – Motion of connecting rod of piston and crank of a reciprocating pump.	4	
	Rectilinear translation – Newton's second law – D'Alembert's Principle – Application to connected bodies (Problems on motion of lift only).	4	
VI	Mechanical vibrations – Free and forced vibration - Degree of freedom.	1	20%
	Simple harmonic motion – Spring-mass model – Period – Stiffness – Frequency – Simple numerical problems of single degree of freedom.	7	
END SEMESTER EXAM			

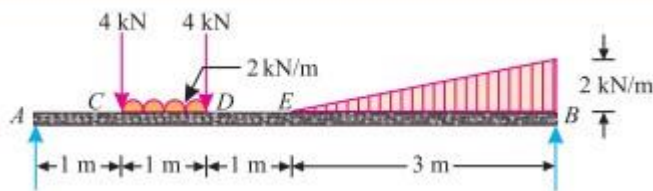
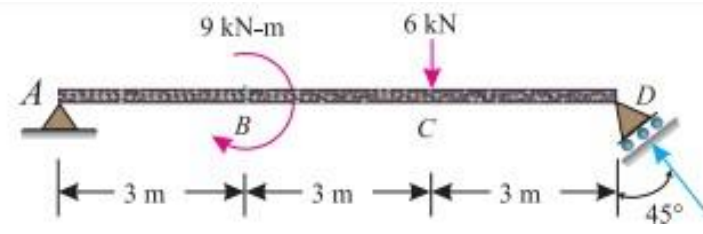
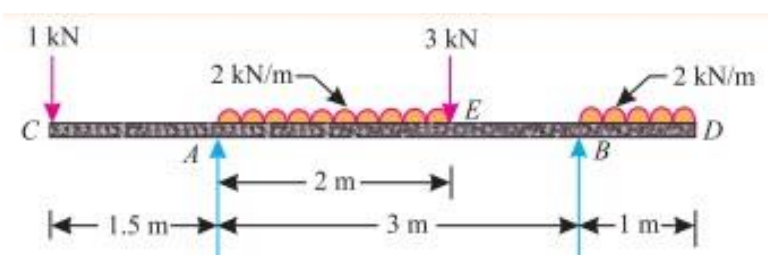
QUESTION BANK

MODULE I			
Q:N O:	QUESTIONS	CO	K L
1	State and explain Varignon's principle of moments.	CO1	K2
2	Draw the free body diagram of each sphere in the given figure. 	CO1	K3
3	The greatest and least resultants of two forces F_1 and F_2 are 17N and 3N respectively. Determine the angle between them when their resultant is 149 N.	CO1	K2
4	Two concurrent forces P & Q are acting on a plane. The maximum and minimum value of resultant is 17 N and 3 N respectively. Find the two forces	CO1	K2
5	A simply supported beam AB of span 4m is carrying point loads 10N, 6N & 4N at 1m, 2m & 3m respectively from support A. Calculate reactions at supports A and B.	CO1	K2
6	Two identical rollers, each of weight 500N are supported by an inclined plane and vertical wall as shown in figure. Assuming smooth surfaces, find the reactions at the point of contacts A, B and C. 	CO1	K2
7	A ladder of weight 30 kg is supported at wall and floor as shown in fig 1 below. A man of weight 72 kg stands on it vertically, 8 m above the floor level. There is a 100 kg force acting at top-most point of the ladder vertically. The mass distribution of the ladder is uniform. Considering all contact surfaces smooth, draw the free body diagram.	CO1	K2

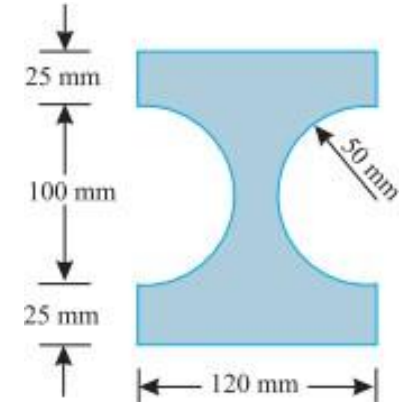
			
8	Concurrent forces of 1,3,5,7,9,11 N are applied to the center of a regular hexagon acting towards its vertices as shown in fig . Determine the magnitude and direction of the resultant. 	CO1	K2
9	Find the magnitude and direction of resultant of the forces as shown in figure. 	CO1	K2
10	Two cylinders P and Q rest in a channel as shown in Fig. The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N. If the bottom width of the box is 180 mm, with one side vertical and the other inclined at 60°, determine the pressures at all the four points of contact 		

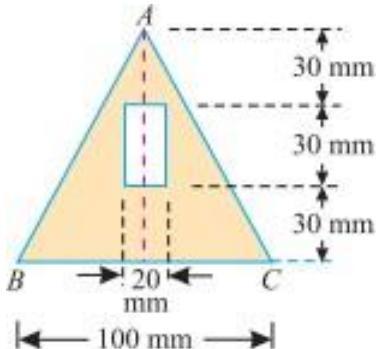
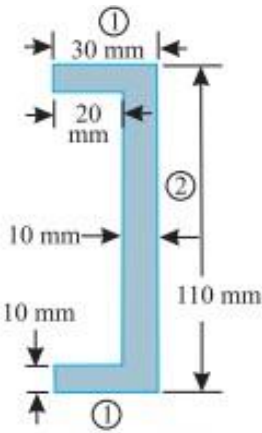
MODULE II

1	Determine the reactions at supports A and B of the beam as shown in the figure.	CO2	K2
			
2	Find the support reactions of a cantilever beam of span 6 m carrying a UDL of 6 kN/m.	CO2	K4
3	A force $2\mathbf{i}+4\mathbf{j}-3\mathbf{k}$ is applied at the point A(1,1,-2). Find the moment of the force about the point (2,-1,2)	CO2	K2
4	A force P is directed from a point A(4,1,4) metres towards a point B (-3,4,1) metres. Determine the moment of force P about x and y axis if it produces a moment of 1000 Nm about z axis	CO2	K5
5	A beam AB 5 m long, supported on two intermediate supports 3 m apart, carries a uniformly distributed load of 0.6 kN/m. The beam also carries two concentrated loads of 3 kN at left hand end A, and 5 kN at the right hand end B as shown	CO2	K5
			
6	A beam AB 8.5 m long is hinged at A and supported on rollers over a smooth surface inclined at 30° to the horizontal at B. The beam is loaded as shown. Determine graphically, or otherwise, the reactions at A and B.	CO2	K3
			
7	A beam has hinged support at A and roller support at B as shown in Fig.	CO2	K5
			

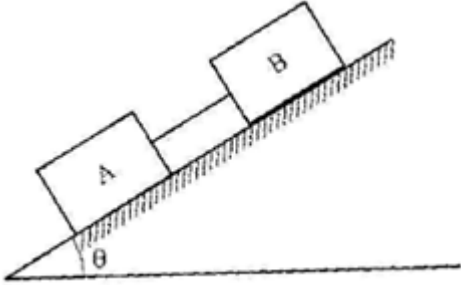
8	A simply supported beam AB of 6 m span is subjected to loading as shown in Fig. 		
9	Fig. shows as beam ABCD simply supported on a hinged support at A and at D on a roller support inclined at 45° with the vertical. Determine the horizontal and vertical components of reaction at support A. Show clearly the direction as well as the magnitude of the resultant reaction at A. 		
10	A beam AB of span 3m, overhanging on both sides is loaded as shown in Fig 		

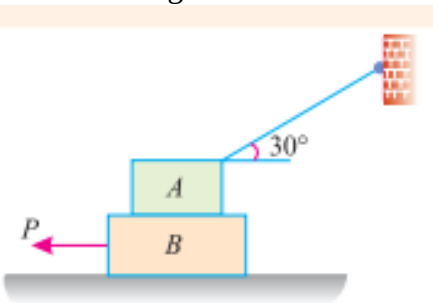
MODULE III

1	Figure shows the cross-section of a cast iron beam. 	CO3	K3
---	--	-----	----

2	Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.	CO3	K3
3	An I-section has the following dimensions in mm units :Bottom flange = 300 × 100, Top flange = 150 × 50, Web = 300 × 50. Determine mathematically the position of centre of gravity of the section.	CO3	K2
4	A rectangular hole is made in a triangular section as shown in Fig. Determine the moment of inertia of the section about X-X axis passing through its centre of gravity and the base BC. 	CO3	K3
5	State and prove parallel axis theorem. Determine moment of inertia of a built up section about X-X axis passing through centre of gravity of the section. 	CO3	K5
6	Define radius of gyration and product of inertia	CO3	K3
7	Differentiate between polar moment of inertia and product of inertia	CO3	K2
8	State the theorems of Pappus Guldinus.	CO3	K5
9	Derive the mass moment of inertia for a circular plate. (RK Bansal Pg 223)	CO3	K5
10	Derive the mass moment of inertia for a hollow circular cylinder (RK Bansal Pg 224)	CO3	K2

MODULE IV

1	A block of weight 80 N is placed on a horizontal plane where the $\mu=0.25$. Find the force that should be applied to the block at an angle of 30° to the horizontal to attain condition of limiting equilibrium.	CO4	K2
2	Distinguish between static and dynamic friction.	CO4	K1
3	Define angle of friction, coefficient of friction, cone of friction and angle of repose.	CO4	K2
4	A uniform ladder 4 m long weighs 200 N. It is placed against a wall making an angle of 60° with the floor. The coefficient of friction between the wall and the ladder is 0.25 and that between the ground and the ladder is 0.35. The ladder in addition to its own weight, has to support a man of 1000 N at the top at B. Calculate: (i) The horizontal force P to be applied to the ladder at the ground level to prevent slipping. (ii) If the force P is not applied, what should be the minimum inclination of the ladder with the horizontal, so that it does not slip with the man at the top?	CO4	K3
5	Two blocks A and B of weights 500N and 1000N are placed on an inclined plane. The blocks are connected by a string parallel to the inclined plane. The coefficient of friction between the inclined plane and block A is 0.15 and that for the block B is 0.4. Find the inclination of plane when the motion is about to take place. Also calculate the tension in the string. The block A is below the block B as shown in the figure.	CO4	K1
			
6	Find the force required to move a load of 30N up a rough inclined plane, applied parallel to the plane. The inclination of the plane is such that when the same body is kept on a perfectly smooth plane inclined at an angle, a force of 6N applied at an inclination of 30° to the plane keeps the same in equilibrium. Assume coefficient of friction between the rough plane and the load is equal to 0.3.	CO4	K2
7	Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60 N applied at an inclination of 30° to the plane, keeps the same load in equilibrium. Assume coefficient of friction	CO4	K3

	between the rough plane and the load to be equal to 0.3.		
8	An effort of 200 N is required just to move a certain body up an inclined plane of angle 15° the force acting parallel to the plane. If the angle of inclination of the plane is made 20° the effort required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body and the coefficient of friction.		
9	Two blocks A and B of weights 1 kN and 2 kN respectively are in equilibrium position as shown in Fig. 		
10	A load of 500 N is lying on an inclined plane, whose inclination with the horizontal is 30° . If the coefficient of friction between the load and the plane is 0.4, find the minimum and maximum horizontal force, which will keep the load in equilibrium.		

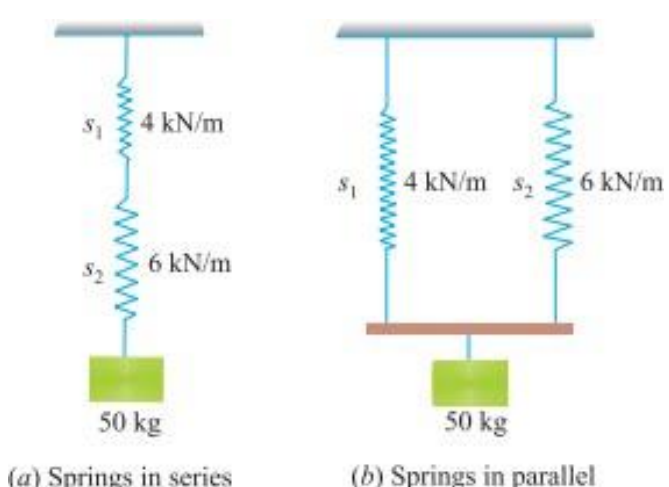
MODULE V

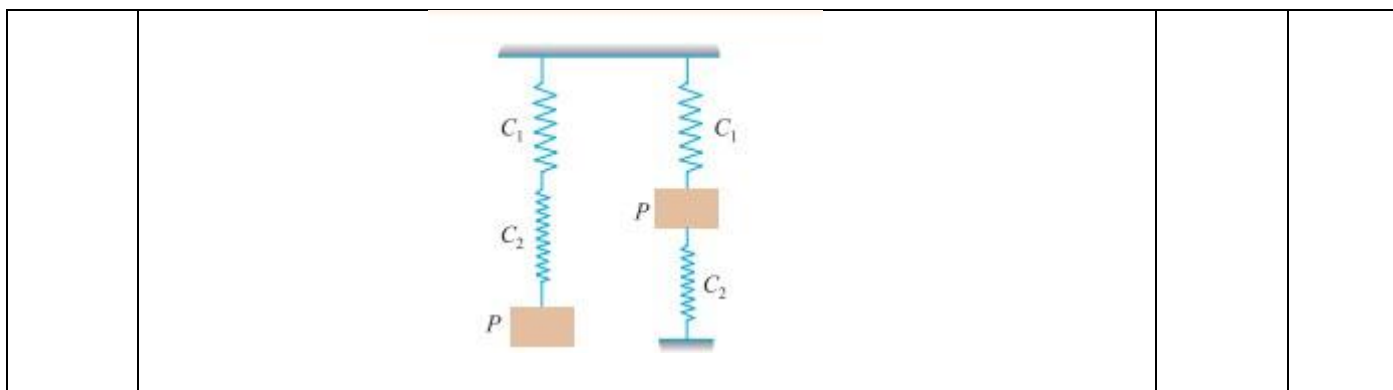
1	State and explain D'Alembert's Principle.	CO5	K2
2	A ball is thrown upward from the top of a tower, 29.4m high at a velocity of 19.6m/s. determine the time taken by the ball to reach the ground and the velocity with which it strikes the ground.	CO5	K4
3	A bus is accelerated at the rate of 0.75m/s^2 as it travels from A to B knowing that the speed of the bus was 27kmph as it passed A, determine a) time required for the bus to reach B b) the corresponding speed as it passes B. The distance between A & B is 150m.	CO5	K4 4
4	A man weighing 500 N gets into a lift. Calculate the force exerted by him on the floor of the lift when it is: i) Moving up with an acceleration of 2.5 m/s^2 and ii) Moving down with same acceleration.	CO5	K2
5	Two bodies of weight 50 N and 30 N are connected to the two ends of a light inextensible string. The string is passing over a smooth pulley. Determine the acceleration of the system and Tension in the string. Take $g = 9.80\text{ m/s}^2$.	CO5	K3
6	Two bodies of weights 20 N and 10 N are connected to the two ends of light inextensible string, passing over a smooth pulley. The weight of 20 N is placed on a smooth horizontal surface while the weight of 10 N	CO5	K2

	hanging free in air. Find the acceleration of the system and the tension in the string.		
7	Two bodies of weights 40 N and 15 N are connected to the two ends of a light inextensible string, which passes over a smooth pulley. The weight 40 N is placed on a smooth inclined plane, while the weight 15 N is hanging free in air. If the angle of the plane is 150, determine acceleration of the system and tension in the string. Take $g=9.80 \text{ m/s}^2$.	CO5	K2
8	A lift has an upward acceleration of 3.45 m/s^2 . (i) What force will a man weighing 51 kg exert on the floor of the lift. (ii) What force would he exert if the lift had a downward acceleration of 3.45 m/s^2 (iii) What upward acceleration would cause his weight to exert a force of 600 N on the floor.	CO5	K3
9	Explain the concepts of instantaneous centre with figure. Explain the following terms with respect to a Simple Harmonic Motion. 1)Amplitude 2)Time Period 3)Frequency The crank of a reciprocating engine is rotating at 210 rpm. The lengths of the crank and connecting rod are 20 cm and 100 cm respectively. Find the velocity of the piston, when crank has turned through an angle of 450 with the horizontal.		
10	A steam engine has a crank radius of length 15 cm and connecting rod of length 75 cm. The crank rotates in a clockwise direction with a constant speed of 300rpm.calculate the velocity and acceleration of the piston P at an instant when the angle $\Theta = 30^\circ$.		

MODULE VI

1	A particle, moving with simple harmonic motion, performs 10 complete oscillation per minute and its speed, is 60% of the maximum speed when it is at a distance of 8 cm from the centre of oscillation, . Find amplitude, maximum acceleration of the particle. Also find speed of the particle, when it is 6 cm far from the centre of oscillation.	CO6	K3
2	Classify different degree of freedom with proper sketches.	CO6	K2
3	Find amplitude and time period of a particle moving with simple harmonic motion, which has a velocity of 9 m/s and 4 m/s at the distance of 2 m and 3 m respectively from the centre.	CO6	K3
4	Find the velocity and acceleration after 0.3 seconds from the extreme position of a body, moving with simple harmonic motion with amplitude of 0.8 m and period of complete oscillation of 1.6 seconds.	CO6	K2
5	A body moving with simple harmonic motion, has an amplitude of 1 m and period of complete oscillation is 2 seconds. Find the velocity and acceleration of the body $\frac{2}{5}$ of a second after passing the mid-position.	CO6	K3
6	A body moving with simple harmonic, has an amplitude of 1 m and a	CO6	K2

	peroid of oscillation of 2 seconds. What will be its velocity and acceleration 0.4 seconds and after passing an extreme position .		
7	A body, moving with simple harmonic motion, has an amplitude of 1 meter and the period of complete oscillation is 2 seconds. What will be the velocity and acceleration of the body after 0.4 second from the extreme position?		
8	The weight of an empty railway wagon is 240 kN. On loading it with goods weighing 320 kN, its spring gets compressed by 80 mm. (a) Calculate its natural period of vibrations when the railway wagon is (i) empty and (ii) loaded as above. (b) It is set into natural vibrations with an amplitude of 100 mm when empty. Calculate the velocity of the railway wagon when its displacement is 40 mm from statical equilibrium position		
9	A block of mass 50 kg supported by two springs connected in series hangs from the ceiling. It can move between smooth vertical guides. The spring constants are 4 kN /m and 6 kN /m as shown in Fig. The block is pulled 40 mm down from its position of equilibrium and then released. Determine (a) period of vibrations, maximum velocity and acceleration of the block. (b) quantities in (a) above, when the block is supported by the springs connected in parallel.  (a) Springs in series (b) Springs in parallel Fig. 26.2		
10	. A weight P is attached to springs of stiffness C1 and C2 in two different cases as shown in Fig.		



APPENDIX 1

CONTENT BEYOND THE SYLLABUS

S:NO.	WEB SOURCE REFERENCES
1	Trusses- https://drive.google.com/file/d/1EjP_E_VFMUzSxBJT5OSukcUum7hFDlrw/view https://youtube/lheoBL2QaqU
2	Particle Dynamics- https://drive.google.com/file/d/1K00fUhPVE6griL6rIJ81fcrMzje5p_Sw/view https://youtube/LBMHPeJNB4E
3	Rotating frame of reference https://drive.google.com/file/d/1_KJuZv0XtdOctlbRbiboIWuLjIQENxvM/view https://youtube/P7zg_Eruudw

Engineering mechanics is that branch of science which deals with the behaviour of a body when the body is at rest or in motion. The engineering mechanics may be divided into.

* Statics



The branch of science that deals with the study of a body when the body is at rest.

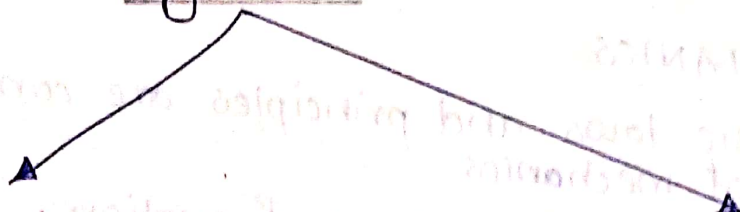
* Dynamics



The branch of science that deals with the study of a body when the body is in motion.

MODULE - 1

Dynamics



KINEMATICS



The study of a body in motion, when the forces which cause the motion are not considered.

KINETICS



The study of a body in motion when the forces which cause the motion are considered.

Statics deals with equilibrium of bodies at rest, whereas dynamics deals with the motion of bodies and the forces that cause them.

Definitions

1. Vector Quantity

- A quantity which is completely specified by magnitude and direction, is known as a vector quantity.
eg: - velocity, acceleration, force, momentum.

- A vector quantity is represented by means of a straight line with an arrow.

Length of AB represents the magnitude.



Arrow represents the direction of the vector.

2. Scalar Quantity.

A quantity which is completely specified by magnitude only, is known as a scalar quantity.
eg:- Mass, length, time.

3. Particle.

A particle is a body of infinitely small volume, or of negligible dimensions and mass of the particle is considered to be concentrated at a point.

Hence a particle is assumed to be a point and the mass of the particle is concentrated at this point.

LAWS OF MECHANICS.

The following basic laws and principles are considered to be the foundation of mechanics.

(i) Newton's first and second laws of motion.

Newton's 1st law states, "Every body continues in a state of rest or uniform motion unless it is compelled to change that state by some external force acting on it".

Newton's 2nd law states, "The net external force acting on a body in a direction is directly proportional to the rate of change of momentum in that direction".

(ii) Newton's Third law.

Newton's third law states, "To every action there is always an equal and opposite reaction".

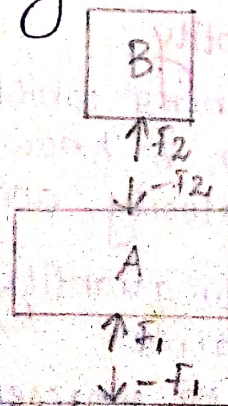
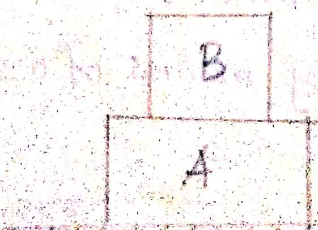


Fig. shows two bodies A and B. which are placed one above the other on a horizontal surface.

- F_1 = force exerted by horizontal surface on body A (action)
- $-F_1$ = force exerted by body A on horizontal surface (reaction)
- F_2 = force exerted by body A on body B (action)
- $-F_2$ = force exerted by body B on body A (reaction).

(iii) The gravitational law of attraction.

It states that two bodies will be attracted towards each other along their connecting line, with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Let m_1 = mass of 1st body.

m_2 = mass of 2nd body.

r = distance bet the centre of bodies.

F = force of attraction between the bodies.

A/q to the law:

$$F \propto m_1 \cdot m_2$$

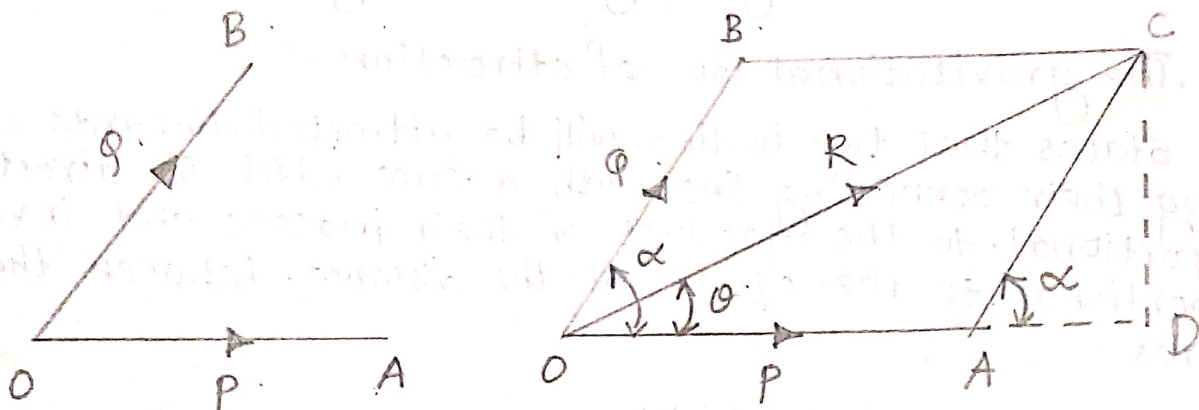
$$\propto \frac{1}{r^2}$$

$$\Rightarrow F = \frac{G m_1 m_2}{r^2}$$

G = universal constant of proportionality.
 $= 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ or $\text{m}^3/\text{kg s}^2$

(iv) The Parallelogram Law

It states that if two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.



Let two forces P and Q act at a point O as shown in fig. The force P is represented in magnitude and direction by OA & force Q is represented in magnitude and direction by OB . Let the angle between the two forces be ' α '.

The resultant of these two forces will be obtained in magnitude and direction by the diagonal of the parallelogram of which OA and OB are two adjacent sides.

Draw the parallelogram with OA and OB as adjacent sides as shown in fig. The resultant R is represented by OC in magnitude and direction.

From C draw $CD \perp OA$ produced.

$\alpha = \angle AOB$ = angle between the forces P and Q .

$$\angle DAC = \angle AOB = \alpha.$$

In parallelogram $OACB$, AC is parallel and equal to OB .
 $\therefore AC = Q$.

In ΔACD ,

(3)

$$AD = AC \cos \alpha = \phi \cos \alpha.$$

$$CD = AC \sin \alpha = \phi \sin \alpha.$$

In ΔOCD ,

$$OC^2 = OD^2 + CD^2.$$

But $OC = R$.

$$OD = p + \phi \cos \alpha.$$

$$R^2 = (p + \phi \cos \alpha)^2 + (\phi \sin \alpha)^2.$$

$$= p^2 + \phi^2 \cos^2 \alpha + 2p\phi \cos \alpha + \phi^2 \sin^2 \alpha.$$

$$= p^2 + \phi^2 (\sin^2 \alpha + \cos^2 \alpha) + 2p\phi \cos \alpha.$$

$$= p^2 + \phi^2 + 2p\phi \cos \alpha.$$

$$R = \sqrt{p^2 + \phi^2 + 2p\phi \cos \alpha}.$$

Direction of Resultant.

Let θ = angle made by resultant with OA .

Then from ΔOCD ,

$$\tan \theta = \frac{CD}{OD} = \frac{\phi \sin \alpha}{p + \phi \cos \alpha}.$$

$$\theta = \tan^{-1} \left(\frac{\phi \sin \alpha}{p + \phi \cos \alpha} \right).$$

1. Two forces of magnitude 10 N & 8 N are acting at a point. If the angle between the two forces is 60° , determine the magnitude of the resultant force.

Given:-

$$\text{Force } P = 10 \text{ N}$$

$$Q = 8 \text{ N}$$

$$\alpha = 60^\circ$$

$$\begin{aligned} R &= \sqrt{P^2 + Q^2 + 2PQ \cos \alpha} = \sqrt{10^2 + 8^2 + 2 \times 10 \times 8 \times \cos 60^\circ} \\ &= \sqrt{100 + 64 + 2 \times 10 \times 8 \times \frac{1}{2}} \\ &= \sqrt{244} = \underline{\underline{15.62 \text{ N}}} \end{aligned}$$

$$\theta = \tan^{-1} \left[\frac{Q \sin \alpha}{P + Q \cos \alpha} \right]$$

$$= \tan^{-1} \left[\frac{8 \sin 60^\circ}{10 + 8 \cos 60^\circ} \right] =$$

2. Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $20\sqrt{3} \text{ N}$, find the magnitude of each force.

$$\alpha = 60^\circ$$

$$R = 20\sqrt{3}$$

Let P be the magnitude of each force.

$$R = \sqrt{P^2 + P^2 + 2P^2 \cos \alpha}$$

$$20\sqrt{3} = \sqrt{P^2 + P^2 + 2P^2 \cos 60^\circ}$$

$$20\sqrt{3} = \sqrt{P^2 + P^2 + P^2}$$

$$20\sqrt{3} = \sqrt{3P^2}$$

$$20\sqrt{3} = \sqrt{3} P \Rightarrow \underline{\underline{P = 20 \text{ N}}}$$

3. The resultant of the two forces, when they act at an angle of 60° is 14 N . If the same forces are acting at right angles, $\textcircled{4}$ their resultant is $\sqrt{136}\text{ N}$. Determine the mag of the two forces.

Case I.

$$R_1 = 14\text{ N}$$

$$\alpha = 60^\circ$$

$$R_1 = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$14 = \sqrt{P^2 + Q^2 + 2PQ \cos 60^\circ}$$

$$14 = \sqrt{P^2 + Q^2 + PQ}$$

$$196 = P^2 + Q^2 + PQ \text{ --- } \textcircled{1}$$

Sub $\textcircled{2}$ in $\textcircled{1}$

$$196 = 136 + PQ, \Rightarrow PQ = 60 \text{ --- } \textcircled{3}$$

Multiplying by 2

$$2PQ = 120 \text{ --- } \textcircled{4}$$

Adding $\textcircled{4}$ to $\textcircled{2}$ $\textcircled{4}$

$$256 = P^2 + Q^2 + 2PQ$$

$$256 = (P+Q)^2$$

$$P+Q = 16$$

$$\therefore P = 16 - Q, \text{ sub in } \textcircled{3} (16-Q)Q = 60$$

$$16Q - Q^2 = 60, \quad Q^2 - 16Q + 60 = 0$$

$$Q = 10 \text{ \& } 6$$

$$P = 16 - 10 \text{ or } 16 - 6$$

$$= 6\text{ N or } 10\text{ N}$$

The 2 forces are 10 N & 6 N

Case II

$$R_2 = \sqrt{136}\text{ N}$$

$$\alpha = 90^\circ$$

$$R_2 = \sqrt{P^2 + Q^2 + 2PQ \cos 90^\circ}$$

$$R_2 = \sqrt{P^2 + Q^2}$$

$$R_2^2 = (P^2 + Q^2)$$

$$R_2 = 136 = P^2 + Q^2 \text{ --- } \textcircled{2}$$

LAMI'S THEOREM.

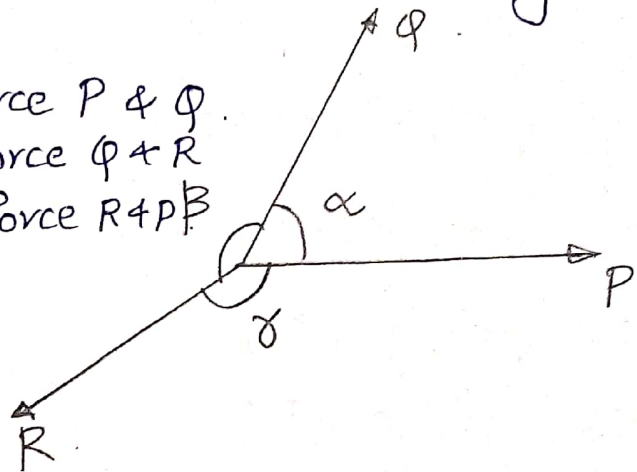
It states that, "If three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces."

Suppose the three forces P , Q and R are acting at a point O and they are in equilibrium as shown in the fig.

Let α = Angle between force P & Q .

β = Angle between force Q & R

γ = Angle between force R & P



Then according to Lami's theorem,

$P \propto \text{sine of angle between } Q \text{ and } R$

$P \propto \sin \beta$.

$$\therefore \frac{P}{\sin \beta} = \text{constant}.$$

Similarly,

$$\frac{Q}{\sin \gamma} = \text{constant}$$

$$, \quad \frac{R}{\sin \alpha} = \text{constant}.$$

$$\text{or } \frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}.$$

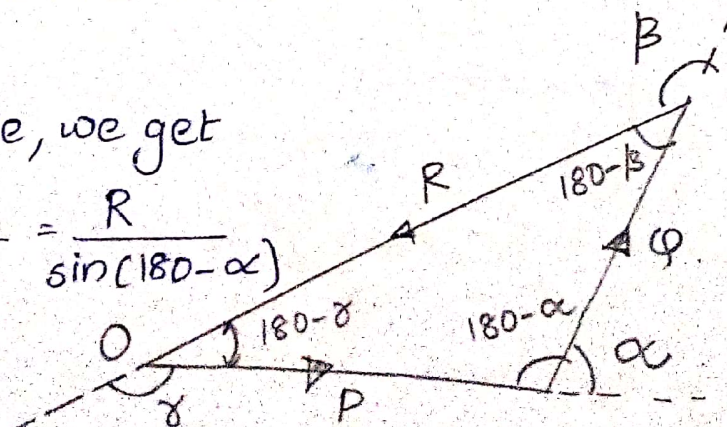
PROOF

The three forces acting on a point are in equilibrium and hence they can be represented by the three sides of a triangle taken in the same order. Now draw the force triangle as shown in fig.

Now applying sine rule, we get

$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\gamma)} = \frac{R}{\sin(180-\alpha)}$$

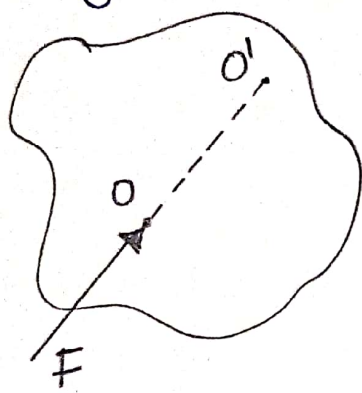
$$= \frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}$$



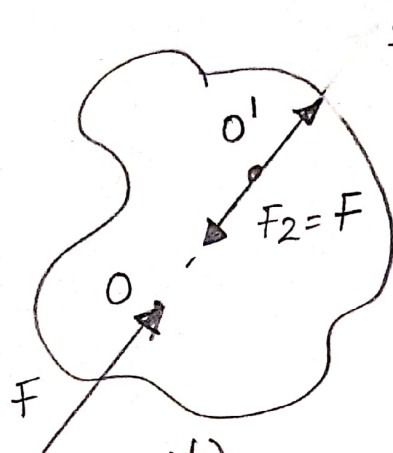
PRINCIPLE OF TRANSMISSIBILITY OF FORCES

(5)

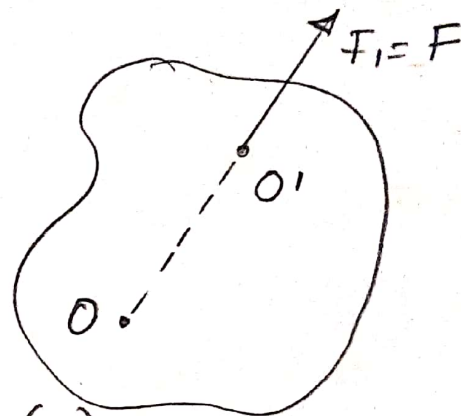
It states that if a force, acting at a point on a rigid body is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.



(a)



(b)



(c)

Consider a force F acting at point O on a rigid body. fig (1). There is another point O' in the line of action of the force F . Suppose at the point O' , 2 equal and opposite forces F_1 & F_2 are applied as in fig (b).

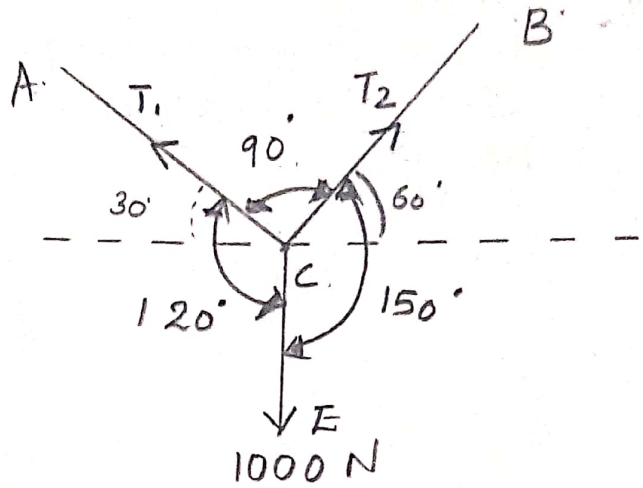
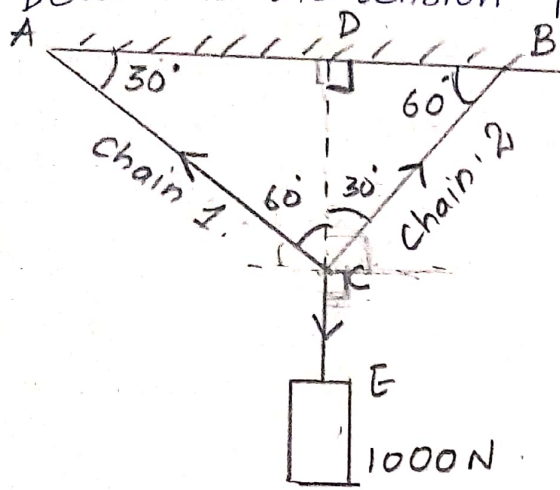
The force F and F_2 , being equal and opposite, will cancel each other, leaving a force F_1 at point O' as in fig (c). But $F_1 = F$.

The original force F acting at point O has been transferred to point O' which is along the line of action of F without changing the effect of the force on the rigid body. This proves the principle of transmissibility.

Rigid body

A body which does not deform under the action of loads or external forces is known as rigid body. The distance bet any 2 points remains the same for all the times.

1. A weight of 1000 N is supported by two chains as shown in fig. Determine the tension in each chain.



Given.

weight at C = 1000 N

$$\angle CAB = 30^\circ$$

$$\angle CBA = 60^\circ$$

$$\angle ACB = 90^\circ$$

In right angled $\triangle ACD$, $\angle ACD = 60^\circ (90^\circ - 30^\circ)$

In right angle $\triangle BCD$, $\angle BCD = 30^\circ (90^\circ - 60^\circ)$

$$\therefore \angle BCE = 180^\circ - 30^\circ = 150^\circ$$

$$\angle ACE = 180^\circ - 60^\circ = 120^\circ$$

Let T_1 be the tension in the chain 1.

T_2 be the tension in the chain 2.

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{1000}{\sin 90^\circ}$$

$$\frac{T_1}{\sin 150^\circ} = \frac{1000}{\sin 90^\circ}$$

$$T_1 = \underline{\underline{500 \text{ N}}}$$

$$\frac{T_2}{\sin 120^\circ} = \frac{1000}{\sin 90^\circ}$$

$$T_2 = \underline{\underline{866 \text{ N}}}$$

2. An electric light fixture weighing 15 N hangs from a point C, by two strings AC and BC. AC is inclined at 60° to the horizontal and BC at 45° to the vertical as shown in the fig. Using Lami's theorem determine the forces in the strings AC & BC.

Given:

Weight at C = 15 N.

$$\angle OAC = 60^\circ$$

$$\angle CBD = 45^\circ$$

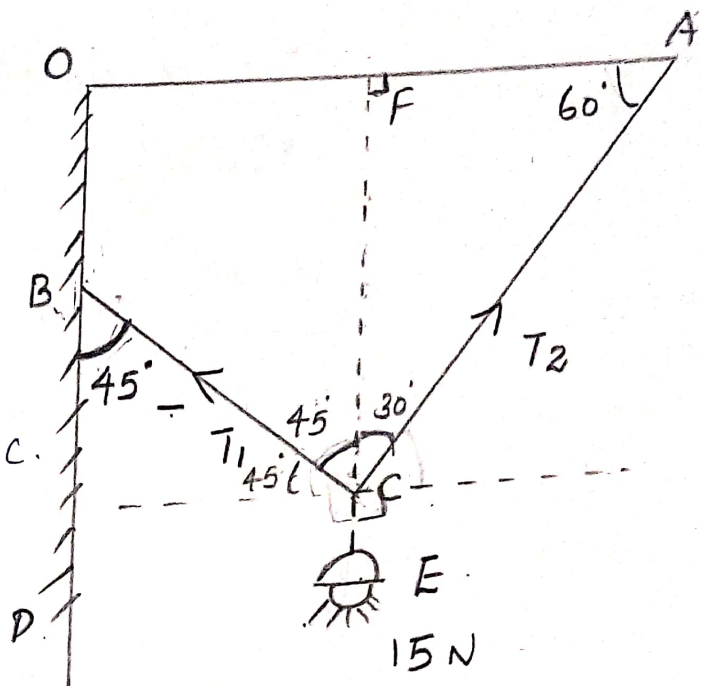
Let T_1 - force in the string BC
 T_2 - force in the string AC.

Using Lami's theorem at C.

$$\frac{15}{\sin \angle BCA} = \frac{T_1}{\sin \angle ACE} = \frac{T_2}{\sin \angle BCE}$$

$$\frac{15}{\sin 75^\circ} = \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ}$$

$$T_1 = 7.76 \text{ N}, \quad T_2 = 10.98 \text{ N}.$$



3. An electric light fixture weighing 20 N hangs from a point C, by two strings AC & BC. AC is inclined at 70° to the horizontal and BC at 30° to the vertical as shown in fig. Using Lami's theorem, determine the forces in the strings AC & BC.

$$[8.929 \text{ N}, 13.05 \text{ N}]$$

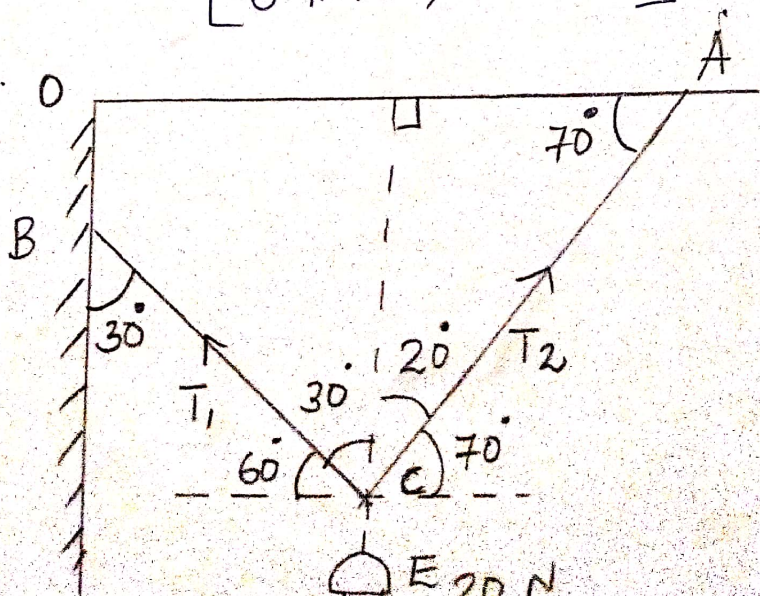
Using Lami's theorem @ C.

$$\frac{T_1}{\sin(90+70)} = \frac{T_2}{\sin(90+60)}$$

$$= \frac{20}{\sin 50^\circ}$$

$$\frac{T_1}{\sin 160^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{20}{\sin 50^\circ}$$

$$T_1 = 8.929 \text{ N}$$



$$\frac{T_2}{\sin 150^\circ} = \frac{20}{\sin 45^\circ}$$

$$T_2 = \frac{20}{\sin 45^\circ} \times \sin 150^\circ$$

$$= \underline{\underline{13.05 \text{ N}}}$$

RESOLUTION OF A NUMBER OF COPLANAR FORCES:

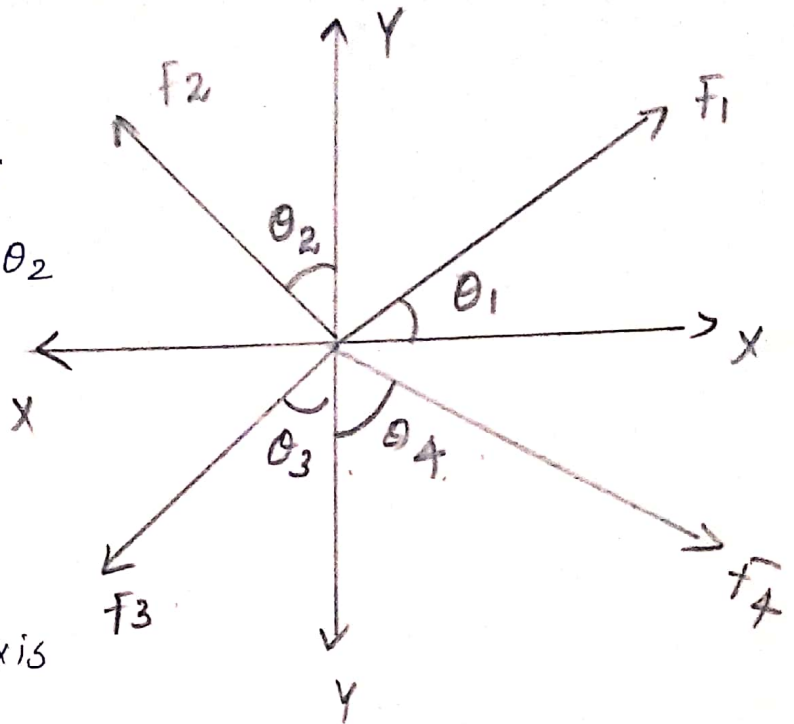
Let a number of coplanar forces, F_1, F_2, F_3, F_4 are acting at a point O .

Res Component of resultant along x-axis

$$H = F_1 \cos \theta_1 - F_2 \sin \theta_2$$

$$- F_3 \sin \theta_3$$

$$+ F_4 \sin \theta_4$$



Component of the resultant along y-axis

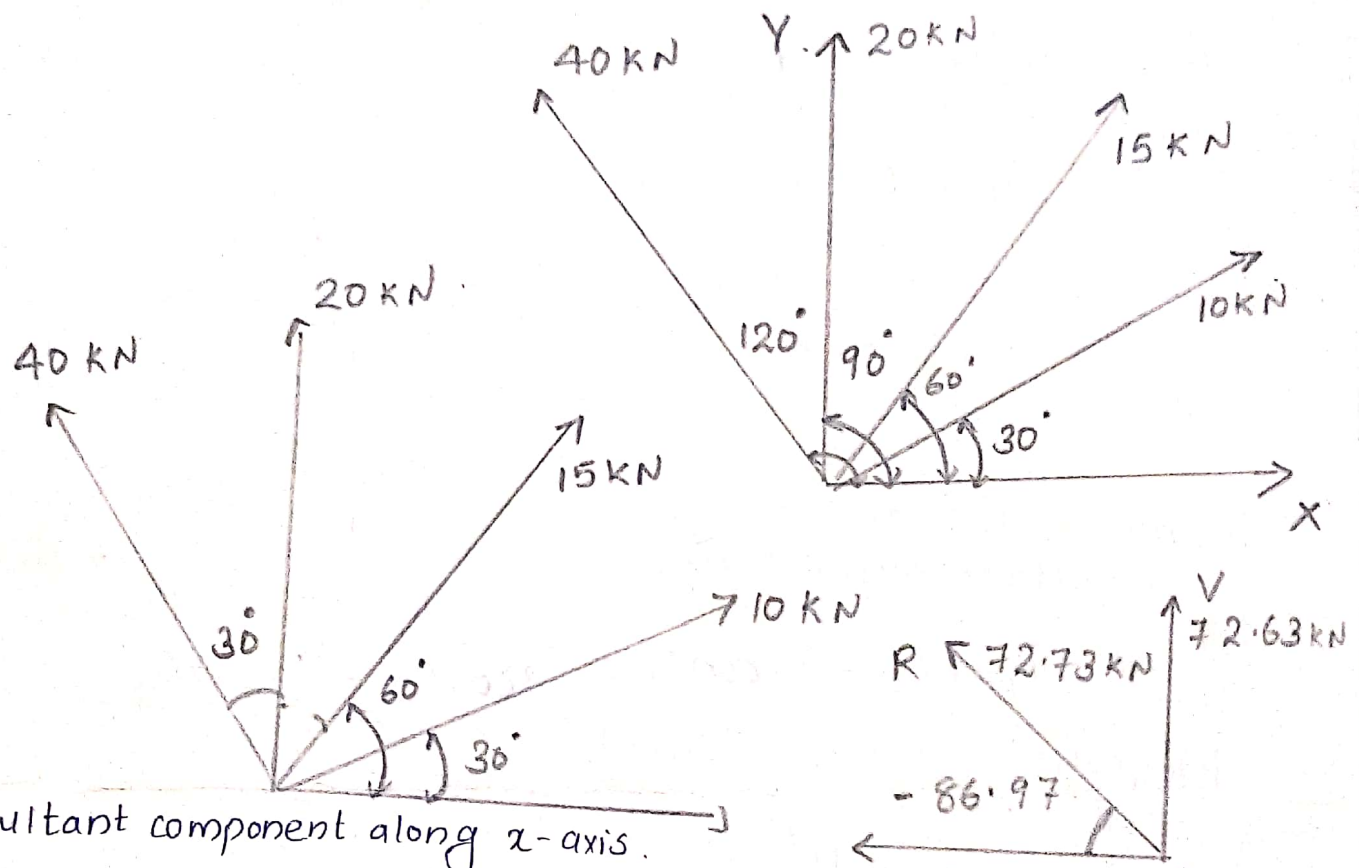
$$V = F_1 \sin \theta_1 + F_2 \cos \theta_2 - F_3 \cos \theta_3 - F_4 \cos \theta_4$$

Magnitude of the resultant.

$$R = \sqrt{V^2 + H^2}$$

$$\tan \theta = \frac{V}{H}, \quad \theta = \text{angle made by the resultant with x-axis.}$$

2. Four forces of magnitude 10 kN, 15 kN, 20 kN and 40 kN are acting at a point O as shown in fig. The angles made by 10 kN, 15 kN, 20 kN and 40 kN with x-axis are 30° , 60° , 90° and 120° resp. Find the magnitude and direction of the resultant force.



Resultant component along x-axis.

$$H = 10 \cos 30^\circ + 15 \cos 60^\circ - 40 \sin 30^\circ$$

$$= -3.84 \text{ kN}$$

Resultant component along y-axis.

$$V = 10 \sin 30^\circ + 15 \sin 60^\circ + 20 + 40 \cos 30^\circ$$

$$= 72.63 \text{ kN}$$

Magnitude of the resultant force, $R = \sqrt{H^2 + V^2}$.

$$= \sqrt{(-3.84)^2 + (72.63)^2}$$

$$= 72.73 \text{ kN}$$

Direction of the resultant force.

$$\tan \theta = \frac{V}{H} = \frac{72.63}{-3.84} = -18.91$$

$$\theta = -86.97^\circ$$

RESOLUTION OF A NUMBER OF COPLANAR FORCES (7)

Let a number of coplanar forces (forces acting in one plane are called Coplanar forces) $R_1, R_2, R_3 \dots$ are acting at a point O .

Let θ_1 = Angle made by R_1 with x-axis

θ_2 = Angle made by R_2 with x-axis.

θ_3 = Angle made by R_3 with x-axis.

R = Resultant of all forces

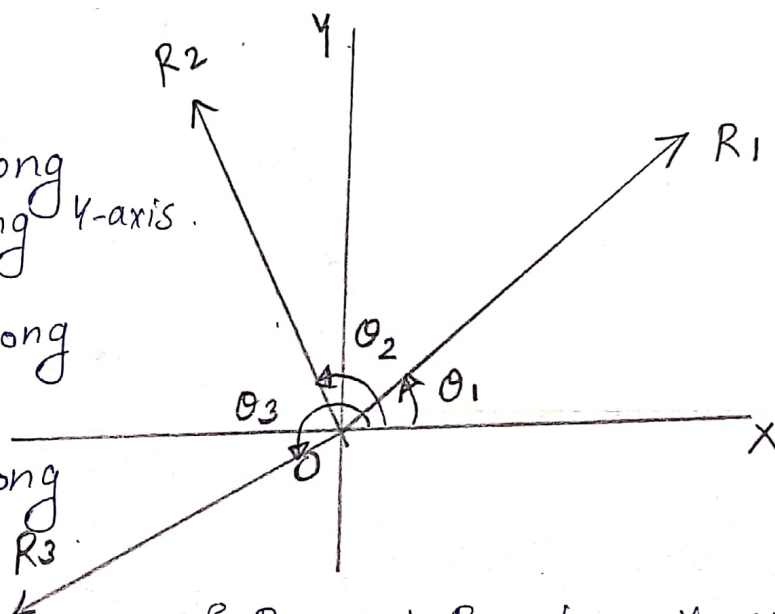
H = Component of resultant of all forces along x-axis.

V = Component of resultant of all forces along y-axis.

Each force can be resolved into 2 components, one along X-axis & other along Y-axis.

Component of R_1 along X-axis = $R_1 \cos \theta_1$

Component of R_1 along Y-axis = $R_1 \sin \theta_1$



Similarly, the components of R_2 and R_3 along X-axis & Y-axis are $(R_2 \cos \theta_2, R_2 \sin \theta_2)$ and $(R_3 \cos \theta_3, R_3 \sin \theta_3)$

Resultant component along X-axis.

= Σ Components of all forces along x-axis.

$$H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3$$

Resultant component along Y-axis.

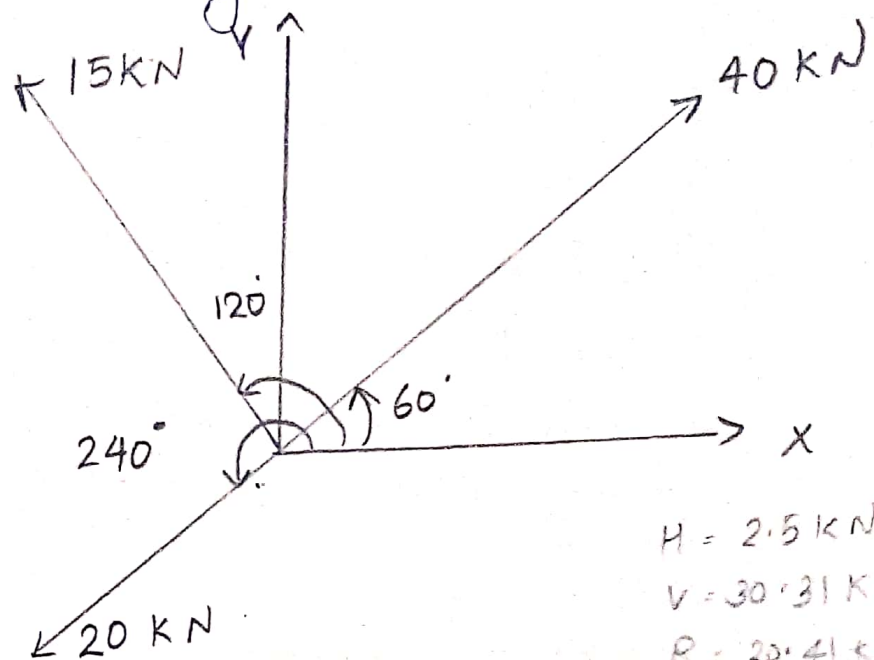
= Σ Component of all forces along y-axis.

$$V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3$$

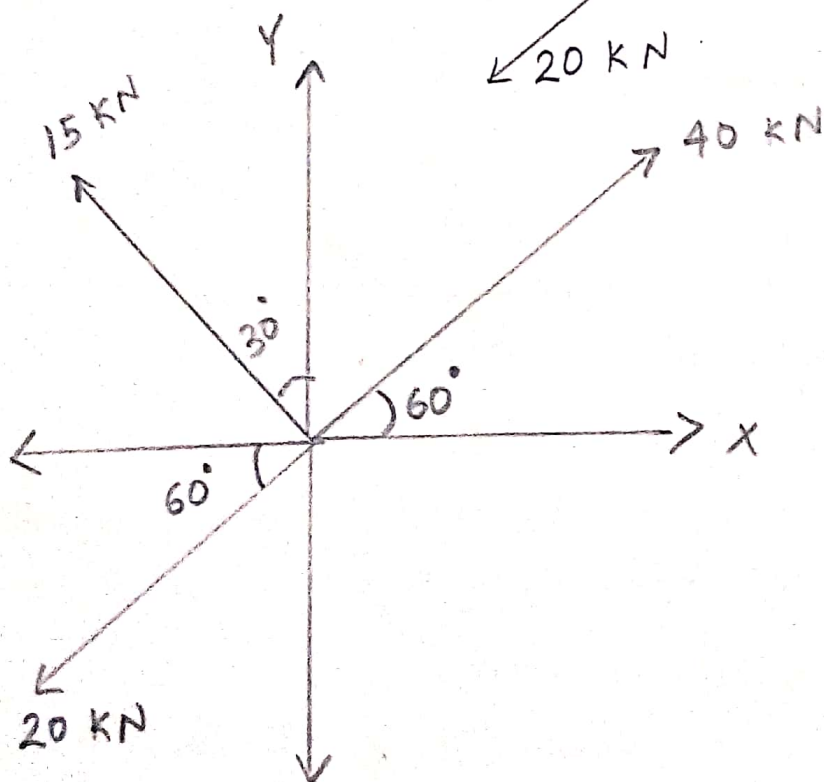
$$\text{Resultant } R = \sqrt{H^2 + V^2}$$

Angle made by R with X-axis, $\tan \theta = \frac{V}{H}$

1. Three forces of magnitude 40 kN, 15 kN & 20 kN are acting at a point O as shown in fig. The angles made by 40 kN, 15 kN, and 20 kN forces with X-axis are 60° , 120° and 240° resp. Determine the magnitude & direction of the resultant force.



$$\begin{aligned} H &= 2.5 \text{ kN} \\ V &= 30.31 \text{ kN} \\ R &= 30.41 \text{ kN} \end{aligned}$$



The direction of the resultant force.

$$\tan \theta = \frac{V}{H} = \frac{30.31}{2.5}$$

$$\theta = \tan^{-1} \left(\frac{30.31}{2.5} \right)$$

$$= 85.28^\circ$$

$$\begin{aligned} H &= 40 \cos 60^\circ - 15 \sin 30^\circ - 20 \cos 60^\circ \\ &= \boxed{2.5 \text{ kN}} \end{aligned}$$

$$\begin{aligned} V &= 40 \sin 60^\circ + 15 \cos 30^\circ - 20 \sin 60^\circ \\ &= \boxed{30.31 \text{ kN}} \end{aligned}$$

$$\begin{aligned} R &= \sqrt{H^2 + V^2} = \sqrt{2.5^2 + 30.31^2} \\ &= \boxed{30.41 \text{ kN}} \end{aligned}$$

3. The four coplanar forces are acting at a point as shown in fig. Determine the resultant in magnitude and direction.

Algebraic sum of the horizontal components.

$$H = 104 \cos 10^\circ - 156 \cos 66^\circ$$

$$- 252 \cos 3^\circ$$

$$- 228 \cos 81^\circ$$

$$= - 248.35 \text{ N}$$

Algebraic sum of the vertical components.

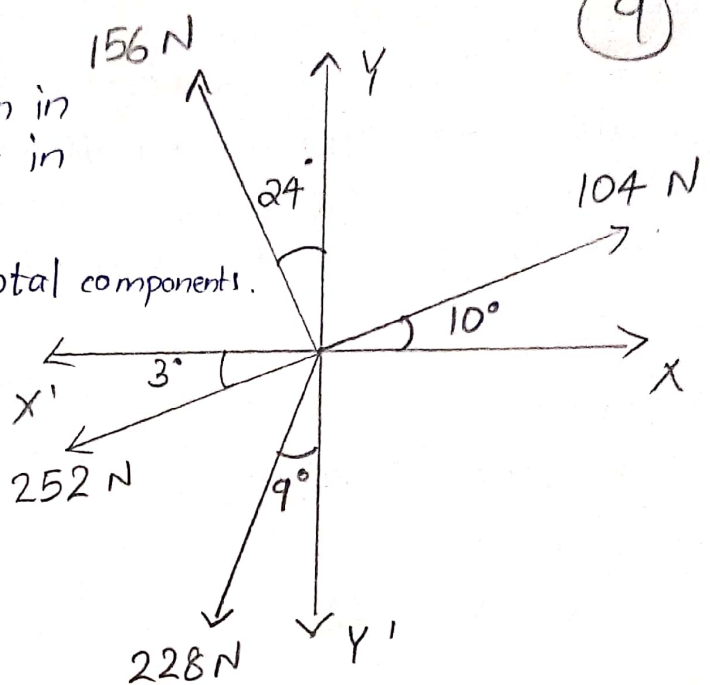
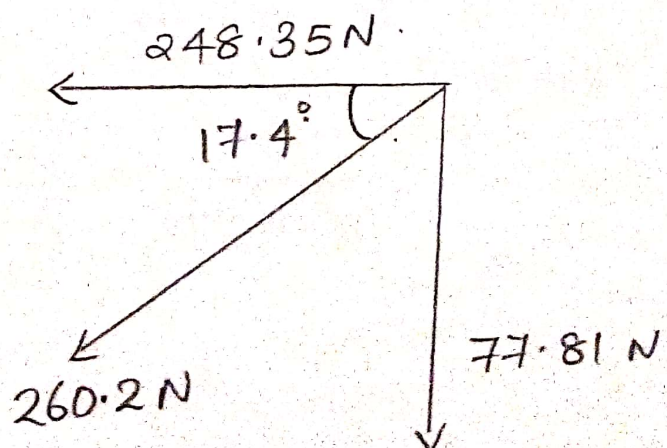
$$V = 104 \sin 10^\circ + 156 \sin 66^\circ - 252 \sin 3^\circ - 228 \sin 81^\circ$$

$$= - 77.81 \text{ N}$$

$$R = \sqrt{V^2 + H^2} = \sqrt{(-248.35)^2 + (-77.81)^2}$$

$$= 260.2 \text{ N}$$

$$\tan \theta = \frac{V}{H} = \frac{77.81}{248.35} \Rightarrow \theta = 17.4^\circ$$



4. The resultant of 4 forces, which are acting at a point O as shown in fig. is along Y-axis. The magnitude of forces are 10kN, 20kN and 40kN resp. The angles made by 10kN, 20kN, and 40kN with X-axis are 30° , 90° and 120° resp. Find the mag. and direction of force F if resultant is 72 kN

Given:-

Resultant is along Y-axis.

$$\Sigma H = 0$$

$$\Sigma V = R = 72 \text{ kN}$$

$$\text{But } \Sigma H = 10 \cos 30^\circ + F \cos \theta + 40 \cos 120^\circ$$

$$0 = -11.34 + F \cos \theta$$

$$F \cos \theta = 11.34 \quad \text{--- (1)}$$

$$72 = 10 \sin 30^\circ + F \sin \theta + 20 + 40 \sin 120^\circ$$

$$72 = 59.64 + F \sin \theta$$

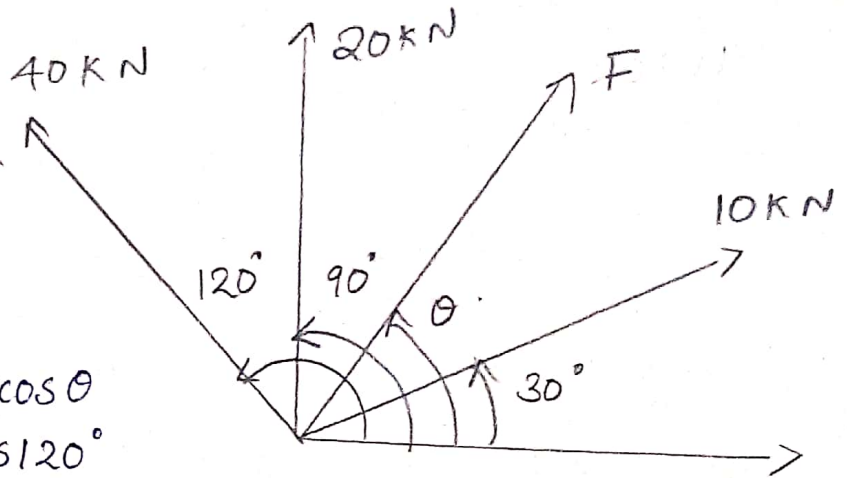
$$F \sin \theta = 12.36 \quad \text{--- (2)}$$

$$\frac{F \sin \theta}{F \cos \theta} = \frac{12.36}{11.34}, \quad \tan \theta = 1.089$$

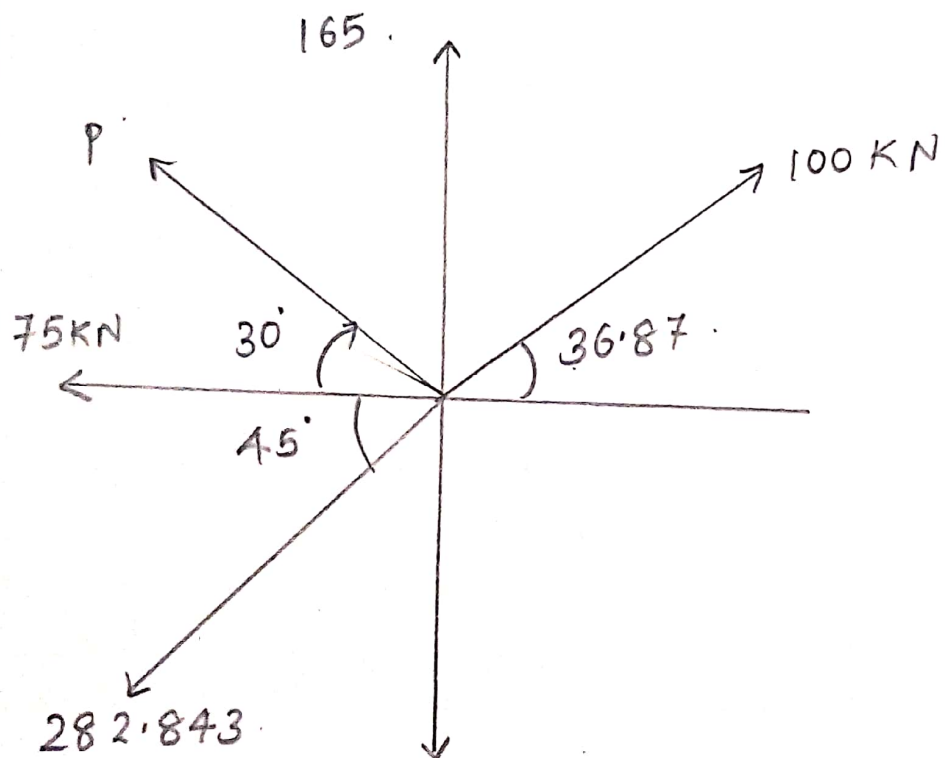
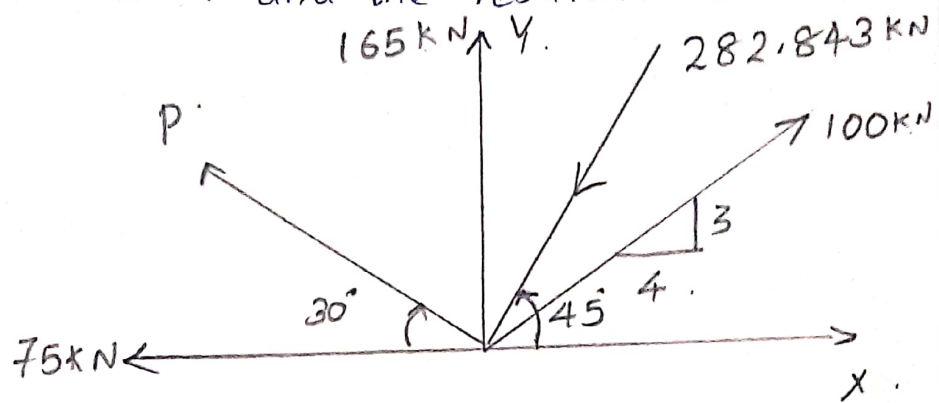
$$\theta = 47.46^\circ$$

Sub in (1)

$$F = \frac{11.34}{\cos 47.46} = \underline{\underline{16.77 \text{ N}}}$$



- 5 If 5 forces act on a particle as shown in fig¹⁰ and the algebraic sum of horizontal components of all these forces is -324.904 kN . calculate the magnitude of 'P' and the resultant of all the forces.



$$\theta = \tan^{-1} \frac{3}{4} = 36.87^\circ$$

$$\Sigma H = 100 \cos 36.87^\circ - P \cos 30^\circ - 75 - 282.843 \cos 45^\circ = -324.904$$

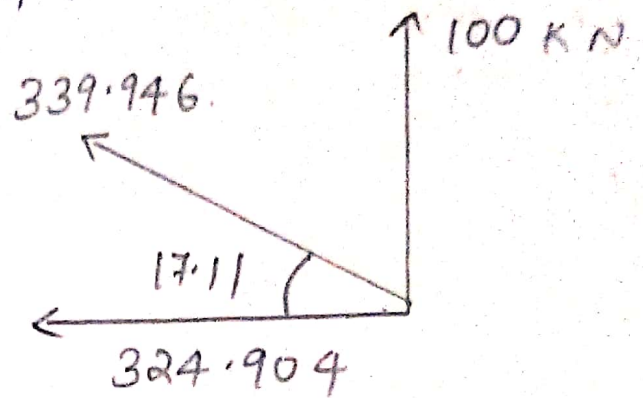
$$129.903 = P \cos 30^\circ$$

$$P = 149.99 \text{ kN}$$

$$\Sigma V = 100 \sin 36.87^\circ + 165 + 150 \sin 30^\circ - 282.843 \sin 45^\circ = 100 \text{ kN}$$

$$R = \sqrt{\Sigma V^2 + \Sigma H^2} = \sqrt{100^2 + 324.904^2} = 339.945 \text{ kN}$$

$$\theta = \tan^{-1} \frac{E_V}{E_H} = \tan^{-1} \left(\frac{100}{324.904} \right) = 17.10$$

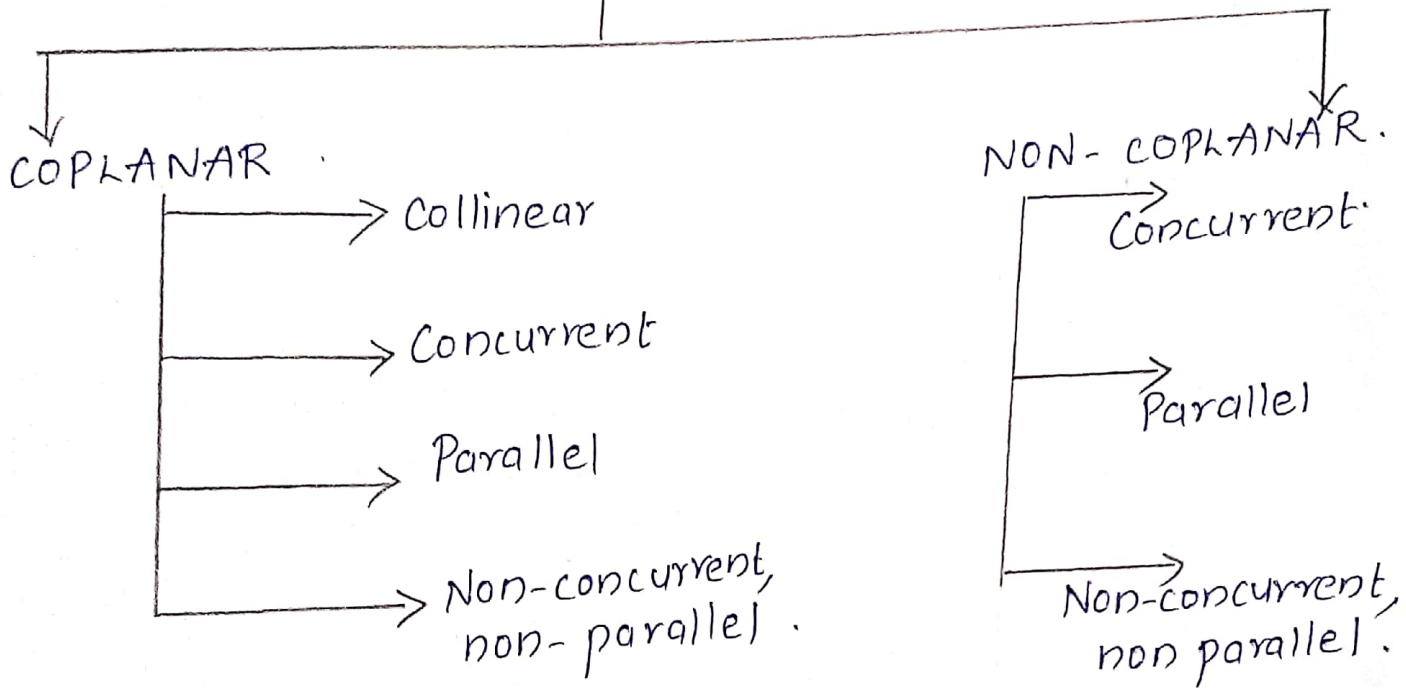


FORCE SYSTEM.

(10)

When several force acts on a body, they are called a FORCE SYSTEM.

Classification of force system.



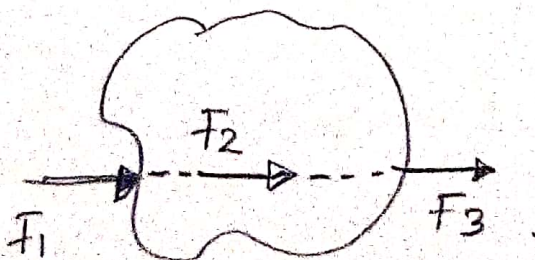
COPLANAR FORCE SYSTEM $\rightarrow$ If ~~the~~ all the forces in a system lie in the same plane.

NON-COPLANAR FORCE SYSTEM $\rightarrow$ All the forces in a system lie in different plane.

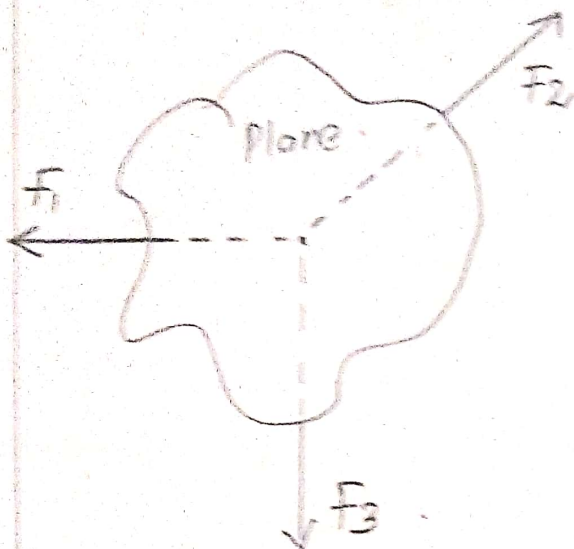
Collinear force system $\rightarrow$ Forces which are having same lines of action.

Concurrent force system $\rightarrow$ Forces which intersect at a common point.

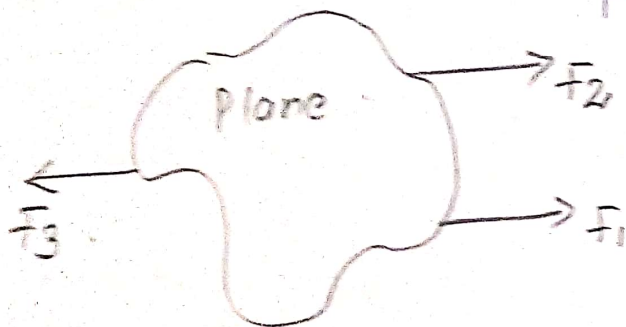
Coplanar collinear $\rightarrow$ Forces acting in a plane and having same lines of action.



Coplanar concurrent $\rightarrow$ forces are acting in a plane and these forces intersect at a common point O.



Coplanar parallel $\rightarrow$ All the forces are in the same plane and they are parallel.



Coplanar like parallel $\rightarrow$ All the forces are in the same plane and they are acting in the same direction.

Coplanar Non-concurrent Non parallel $\rightarrow$ All the forces are acting in the same plane, neither they are parallel or intersect each other.

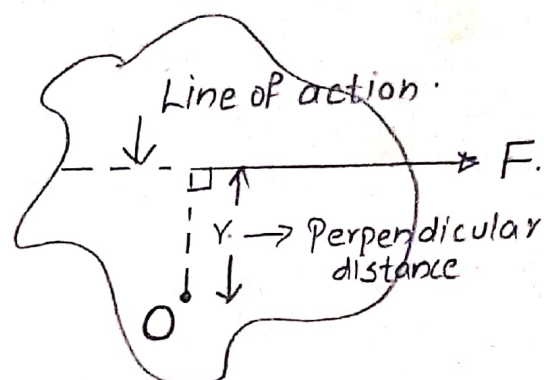
2. MOMENT OF A FORCE.

(12)

The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Let F = A force acting on a body.

r = $\perp$ distance from the point O on the line of action of force F .



Moment of the force F about O .

(CM)

$$M = F \times r$$

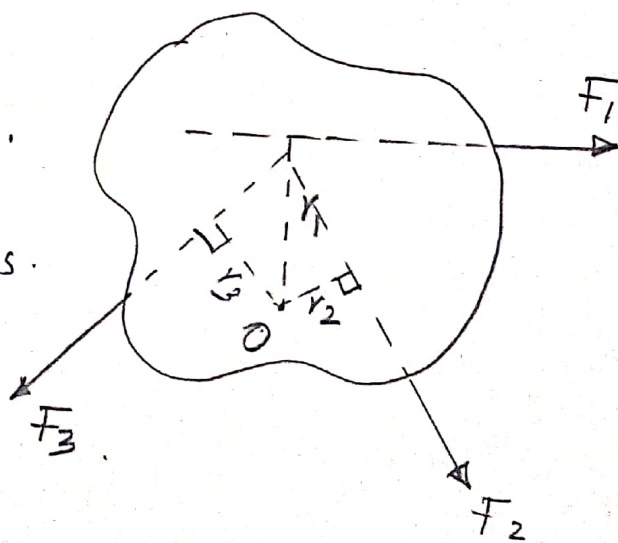
The tendency of this moment is to rotate the body in the clockwise direction about O . Hence this moment is called clockwise moment.

The fig shows a body on which three forces F_1 , F_2 & F_3 are acting. Suppose it is required to find the resultant moments if these forces about point O .

Let r_1 = $\perp$ distance from O on the line of action of force F_1 .

r_2 = $\perp$ distance from O on the lines of action of force F_2 .

r_3 = $\perp$ distance from O on the line of action of force F_3 .



Moment of F_1 about $O = F_1 \times r_1$ (↺)

Moment of F_2 about $O = F_2 \times r_2$ (↺)

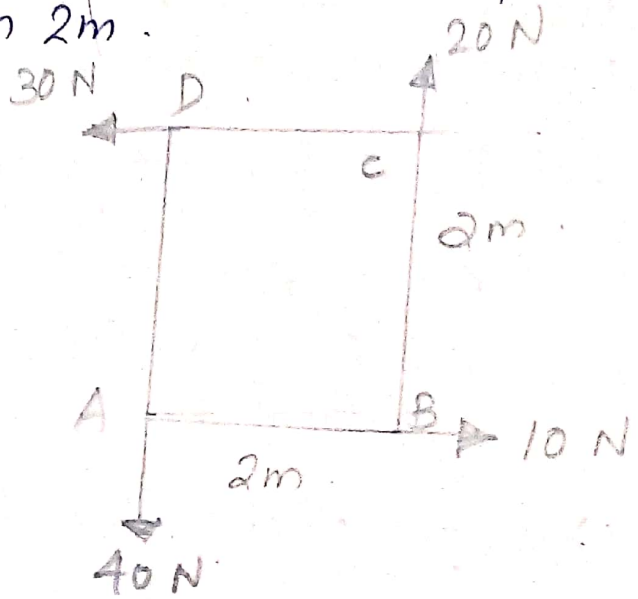
Moment of F_3 about $O = F_3 \times r_3$ (↻)

1. Four forces of magnitude 10 N , 20 N , 30 N & 40 N are acting resp along the four sides of a square $ABCD$ as shown in fig. Determine the resultant moment about the point A . Each side of the square is given 2 m .

The forces at A and B passes through point A .

Hence $\perp$ distance from A on the lines of action of these forces will be zero.

$\Rightarrow$ Moments about A will be zero.



Moment of the force at C about point $A = 20 \times 2 = 40\text{ Nm}$ ($\uparrow$).

Moment of the force at D about $A = 30 \times 2$.

$\therefore$ Resultant moment of all forces about $A = 40 + 60 = 100\text{ Nm}$ ($\uparrow$)

PRINCIPLE OF MOMENTS (OR VARIGNON'S PRINCIPLE) (13)

It states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

Alg. to Varignon's principle; the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

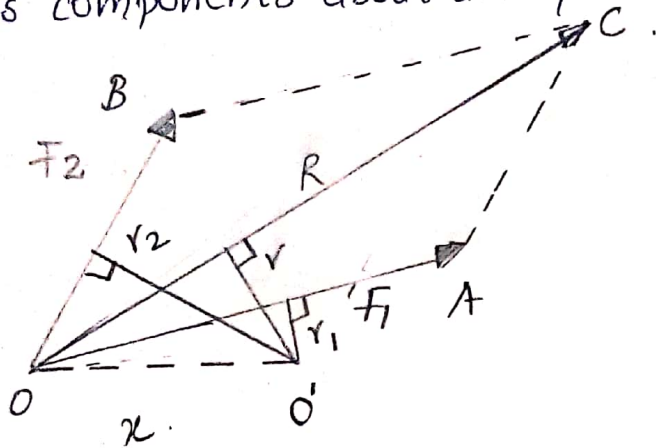


Fig shows 2 forces F_1 and F_2 acting at point O. These forces are represented in magnitude and direction by OA & OB. Their resultant is represented in mag. and direction by OC, which is the diagonal of the parallelogram OACB. Let O' is the point in the plane about which moments of F_1 , F_2 & R are to be determined. From O' , draw $\perp$ on OA, OC & OB.

Let $r_1 = \perp$ distance bet F_1 & O'

$r = \perp$ distance bet R & O'

$r_2 = \perp$ distance bet F_2 & O'

Alg. to varignon's theorem,

Moment of R about O' must be equal to the algebraic sum of moments of F_1 & F_2 about O'

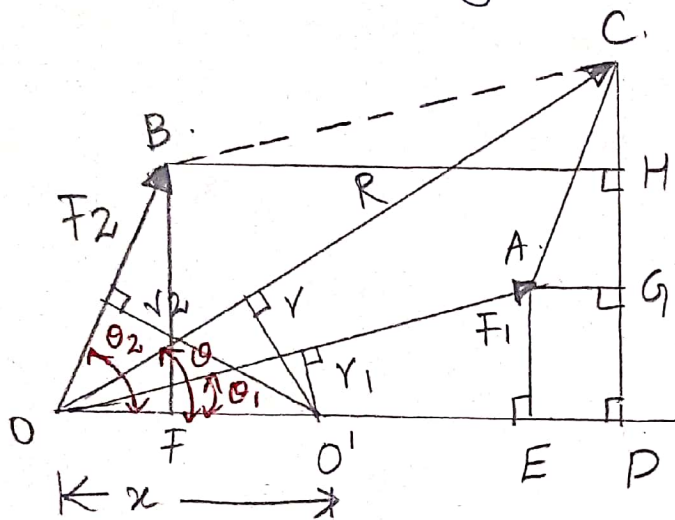
$$R \times r = F_1 \times r_1 + F_2 \times r_2.$$

Join OO' and produce it to D . From points C, A & B draw $\perp$ on OD meeting at D, E and F resp. From A & B also draw $\perp$ on CD meeting the line CD at G & H resp.

Let $\theta_1 =$ Angle made by F_1 with OD .

θ = Angle made by R with OD .

θ_2 = Angle made by F_2 with OD.



$$CH = GD = AE.$$

$$BF = HD$$

$$\sin \theta_1 = \frac{AE}{F_1}$$

$$AE = GD = CH = F_1 \sin \theta_1$$

$$\cos \theta_1 = \frac{OE}{F_1}$$

$$OE = F_1 \cos \theta_1$$

$$\sin \theta_2 = \frac{BF}{F_2}$$

$$BF = HD = F_2 \sin \theta_2$$

$$\cos \theta_2 = \frac{OF}{F_2}$$

$$OF = F_2 \cos \theta_2$$

$$\sin \theta = \frac{CD}{R}$$

$$CD = R \sin \theta$$

$$\cos \theta = \frac{OD}{R}$$

$$OD = R \cos \theta$$

Let OO' be x .

(11)

$$\sin \theta_1 = \frac{r_1}{x}$$

$$\sin \theta_2 = \frac{r_2}{x}$$

$$\sin \theta = \frac{r}{x}$$

$$r_1 = x \sin \theta_1$$

$$r_2 = x \sin \theta_2$$

$$r = x \sin \theta$$

Moment of R about O'

$$= R \times r$$

$$= (R \times \sin \theta) x$$

$$= CD x$$

$$= (CH + HD) x$$

$$= (F_1 \sin \theta_1 + F_2 \sin \theta_2) x$$

$$= F_1 x \sin \theta_1 + F_2 x \sin \theta_2$$

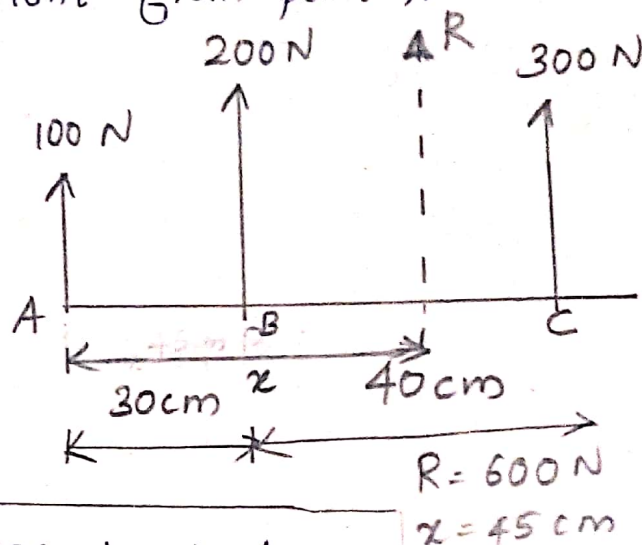
$$\boxed{R \times r = F_1 r_1 + F_2 r_2}$$

1. Three force (like parallel) forces 100N, 200N & 300N are acting at points A, B & C resp on a straight line ABC. The distances are $AB = 30\text{ cm}$ & $BC = 40\text{ cm}$. Find the resultant & also the distance of the resultant from point A on line ABC.

$$\text{Resultant, } R = 100 + 200 + 300$$

$$R = 600\text{ N}$$

Let the resultant be located at a distance of $x\text{ cm}$ from A.



Algebraic sum of Moment about A of all forces about A.

$$= 200 \times 30 + R \times x + 300 \times 70$$

$$= 27000 + R \times x$$

$$600 \times x = 27,000$$

$$x = 45\text{ cm}$$

Algebraic sum of moments of all forces about A

$$= 200 \times 30 + 300 \times 70$$

$$= 27,000\text{ Nm}$$

Moment of resultant R about A = $R \times x$

But algebraic sum of moments of all forces about A = Moment of resultant R about A

$$600 \times x = 27,000$$

$$x = \frac{27,000}{600} = 45\text{ cm}$$

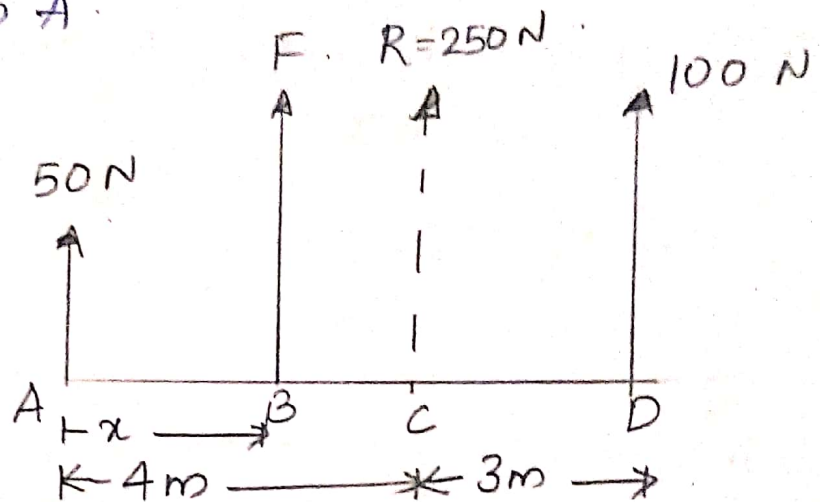
2. The 3 like parallel forces of magnitude 50 N , F and 100 N are shown in fig. If the resultant $R = 250\text{ N}$ and is acting at a distance of 4 m from A, then find.

- (i) Mag of force F .
 (ii) Distance of F from A.

(i) $R = 50 + F + 100$

$$250 = 50 + F + 100$$

$$F = 100\text{ N}$$



- (ii) Let F be located at a distance x from A.

Algebraic sum of
 Moment of all forces about A = Moment of the resultant R about A.

$$F \times x + 100 \times 7 = 250 \times 4$$

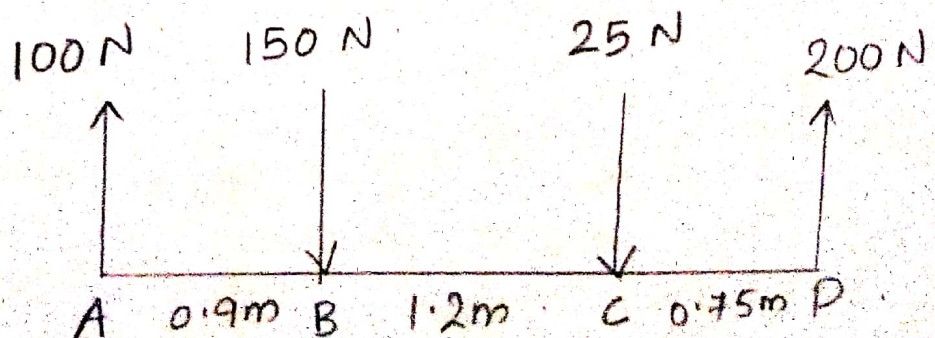
$$100 \times x + 700 = 1000$$

$$100x = 300$$

$$x = \underline{\underline{3\text{ m}}}$$

3. Four parallel forces of magnitudes 100 N , 150 N , 25 N & 200 N are shown in fig. Determine the magnitude of the resultant and also the distance of the resultant from point A.

$$\begin{aligned} R &= 100 + 200 \\ &\quad - 150 - 25 \\ &= \underline{\underline{125\text{ N}}} \end{aligned}$$



Algebraic sum of all forces about A

$$\begin{aligned} &= -150 \times 0.9 - 25 \times 2.1 + 200 \times 2.85 \\ &= \underline{\underline{382.5 \text{ Nm}}} \end{aligned}$$

Moment of the resultant R about A

$$= 125 \times x$$

$$125x = 382.5$$

$$x = \underline{\underline{3.06 \text{ m}}}$$

4. Four forces of magnitude 10 N, 20 N, 30 N and 40 N are acting resp along the four sides of a square ABCD as shown in the fig. Determine the mag., direction & position of the resultant force.

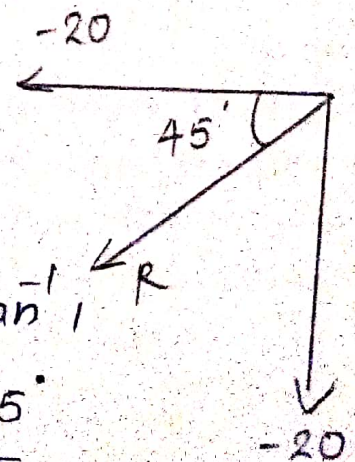
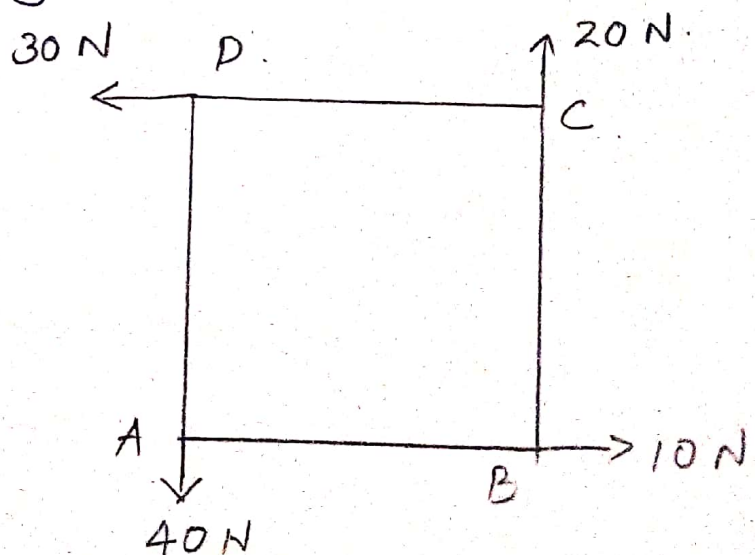
$$\begin{aligned} \Sigma H &= 10 - 30 \\ &= -20 \text{ N} \end{aligned}$$

$$\begin{aligned} \Sigma V &= -40 + 20 \\ &= -20 \text{ N} \end{aligned}$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$\begin{aligned} &= \sqrt{(-20)^2 + (-20)^2} \\ &= \underline{\underline{20\sqrt{2} \text{ N}}} \end{aligned}$$

$$\tan \theta = \frac{\Sigma V}{\Sigma H} = \theta = \tan^{-1} \frac{-20}{-20} = \tan^{-1} 1 = \underline{\underline{45^\circ}}$$



(16)

Position of the resultant force.

Let L be the $\perp$ distance bet A & line of action of the resultant force.

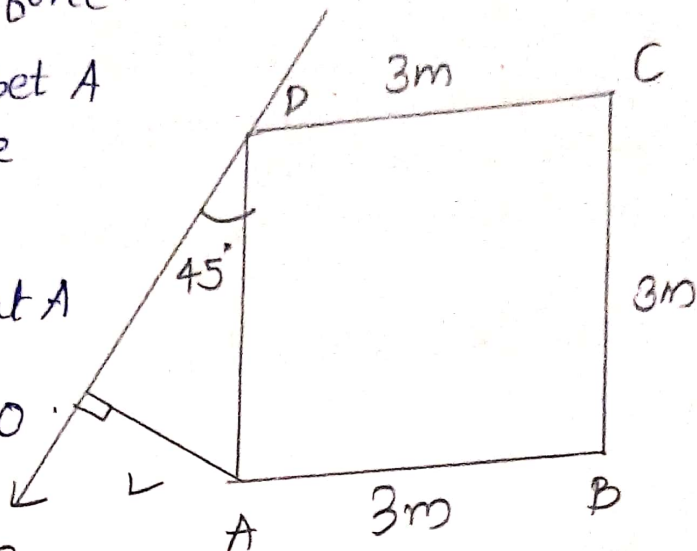
Moment of all forces about A

$$20 \times 3 + 30 \times 3 \neq R \times L = 0$$

$$60 + 90 + 20\sqrt{2} \times L = 0$$

$$100 + 20\sqrt{2} \times L = 0$$

$$20\sqrt{2} \times L = 100$$



$$20 \times 3 + 30 \times 3 = 60 + 90 = 150 \text{ Nm}$$

Moment of R about A.

$$R \times L$$

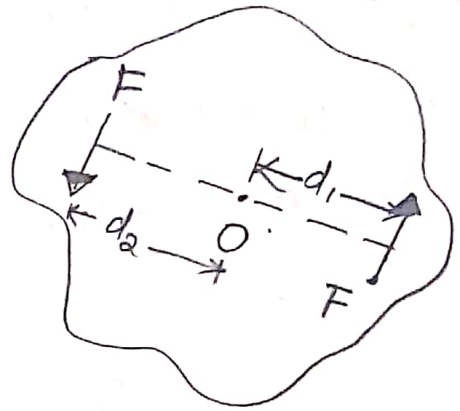
$$150 = 20\sqrt{2} \times L$$

$$L = \frac{150}{20\sqrt{2}} = \underline{\underline{5.3 \text{ m}}}$$

MOMENT OF A COUPLE

Two forces F and $-F$ of the same magnitude, parallel lines of action and opposite direction, are said to form a couple. Hence the sum of components of two forces in any direction is zero. The forces tend to rotate the body.

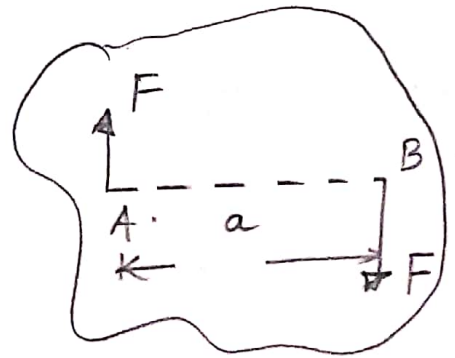
$$M_O = Fd_1 + Fd_2.$$



When 2 equal and opposite parallel forces act on a body, at some distance apart, the two forces form a couple which has a tendency to rotate the body.

The perpendicular distance between the parallel forces is known as the Arm of the couple.

F - equal & opposite force acting in the opposite direction.



2 forces will form a couple which has a tendency to rotate the body in clockwise direction.

Moment of the couple = Product of either one of the forces and $\perp$ dist between the forces.

$$M = F \times a \cdot \text{Nm}$$

1. A system of parallel forces are acting on a rigid bar as shown in fig. Reduce this system to:

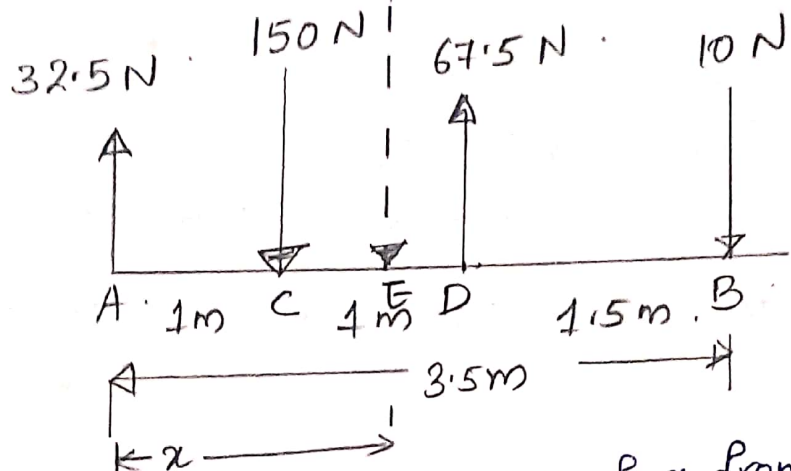
(17)

(i) a single force.

(ii) a single force and a couple at A.

(iii) a single force and a couple at B.

$$\begin{aligned} (i) \quad R &= 32.5 + 67.5 \\ &\quad - 150 - 10 \\ &= -60 \text{ N} \\ &= \underline{\underline{-60 \text{ N}}} \end{aligned}$$



Let the resultant be located at a distance of x from A.

Moment of the resultant about the point A = $-60x$.

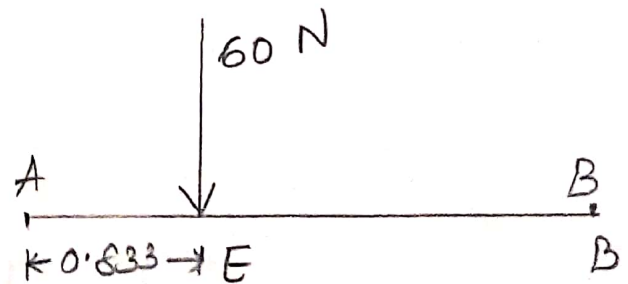
Algebraic sum of moments of all forces abt A.

$$= -150 \times 1 + 67.5 \times 2 - 10 \times 3.5$$

$$= -50 \text{ Nm}$$

$$-60x = -50$$

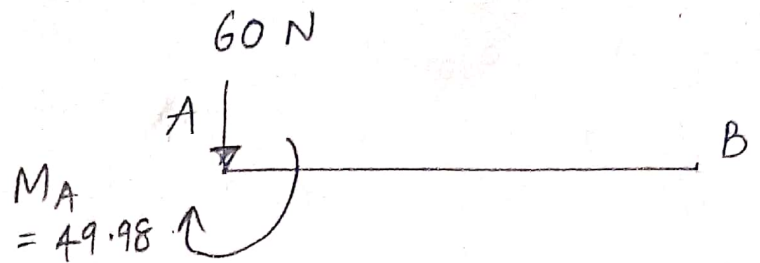
$$x = \frac{50}{60} = \underline{\underline{0.833 \text{ m}}}$$



(ii) The resultant force R acting at point E can be replaced by an equal force applied at point A in the same direction together with a couple.

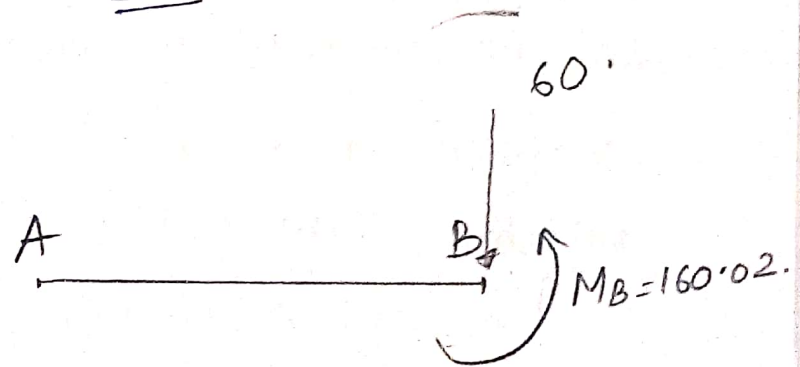
$$\text{Moment of the couple} = -60 \times 0.833$$

$$= -49.98 \text{ Nm} \quad (\curvearrowright)$$

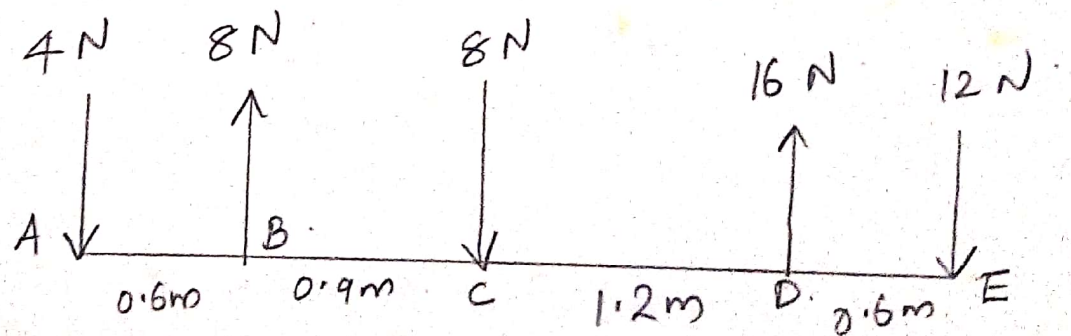


(iii) $BE = 3.5 - AE$
 $= 3.5 - 0.833 = 2.667 \text{ m}$

Moment of the couple $= 60 \times 2.667$
 $= 160.02 \text{ Nm}$



2. Determine the resultant of the parallel force system.



Resultant $= 8 + 16 - 4 - 8 - 12$
 $= 0$

As the resultant force on the system is zero, there will be 2 possibilities.

The system has a resultant couple
 or
 system is in equilibrium.

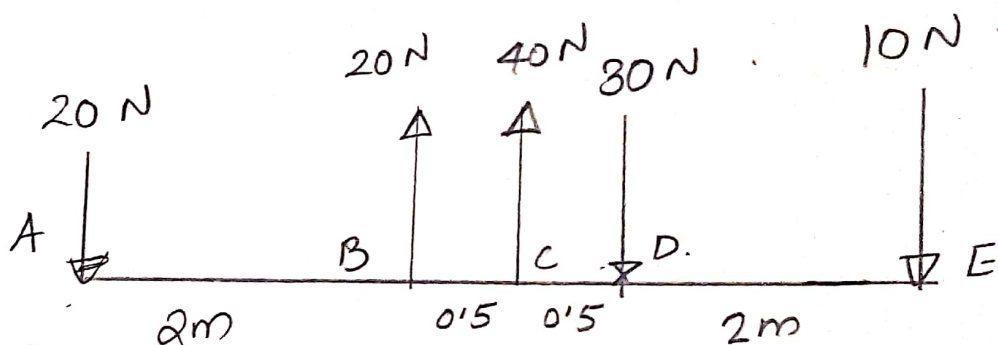
Algebraic sum of moments of all forces about A. (18)

$$\begin{aligned}
 &= 4 \times 0 + 8 \times 0.6 - 8 \times (0.6 + 0.9) + 16 \times (0.6 + 0.9 + 1.2) \\
 &\quad - 12 \times (0.6 + 0.9 + 1.2 + 0.6) \\
 &= -3.6 \text{ Nm. (2)}
 \end{aligned}$$

As the algebraic sum of moments of all forces is not zero, the system will have a resultant couple.

3. Determine the resultant of the parallel forces acting on a body as shown in fig.

$$\begin{aligned}
 R &= 20 + 40 \\
 &\quad - 20 - 30 - 10 \\
 &= 0
 \end{aligned}$$

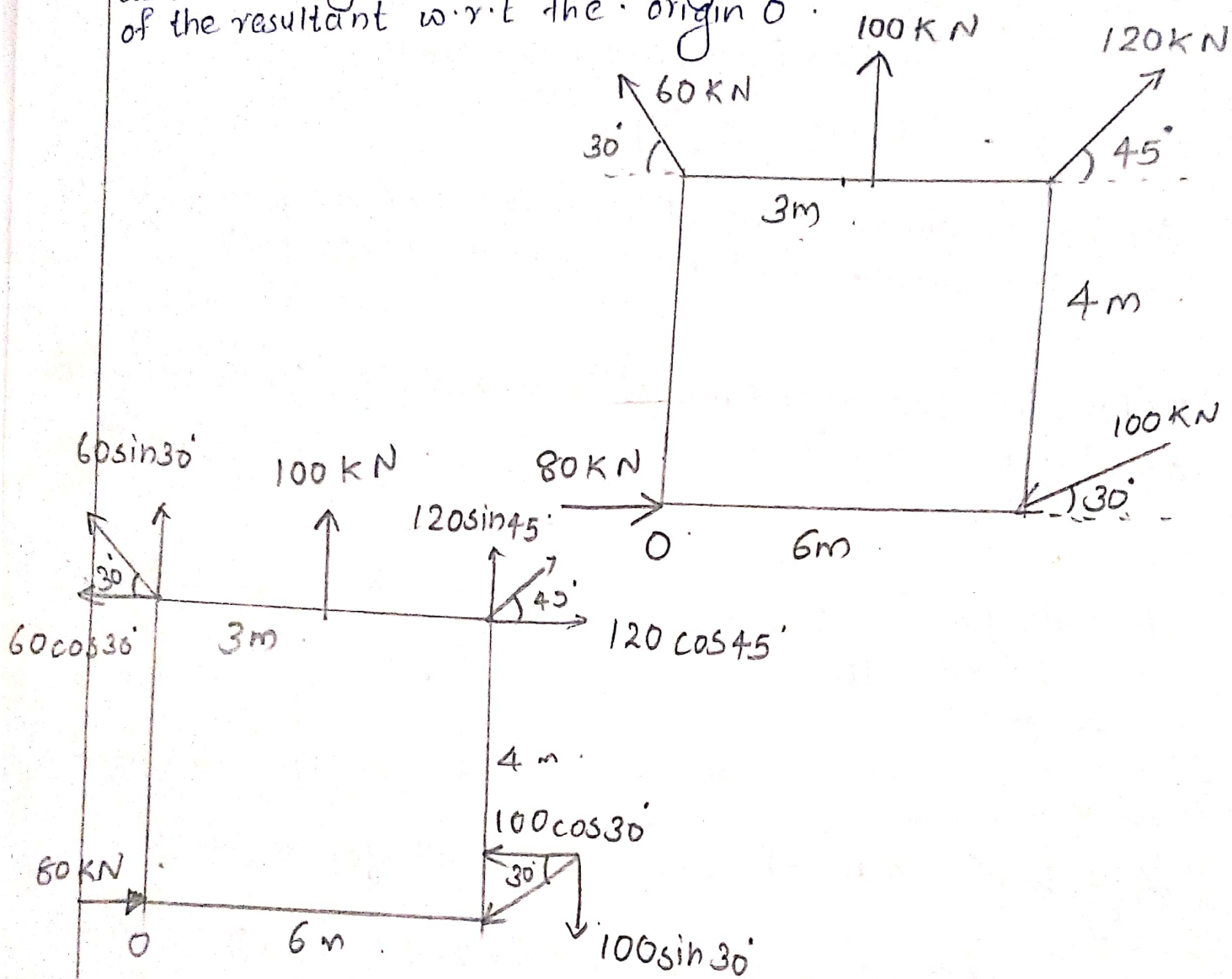


Algebraic sum of moments of all forces at A.

$$\begin{aligned}
 &= 20 \times 0 + 20 \times 2 + 40 \times 2.5 - 30 \times 3 - 10 \times 5 \\
 &= 0
 \end{aligned}$$

As the resultant force on a body is zero, & resultant moment is zero, the body is in equilibrium.

A plate measuring $6\text{m} \times 4\text{m}$ is acted upon by a set of forces as shown in fig. Find out the magnitude, direction & position of the resultant w.r.t the origin O.

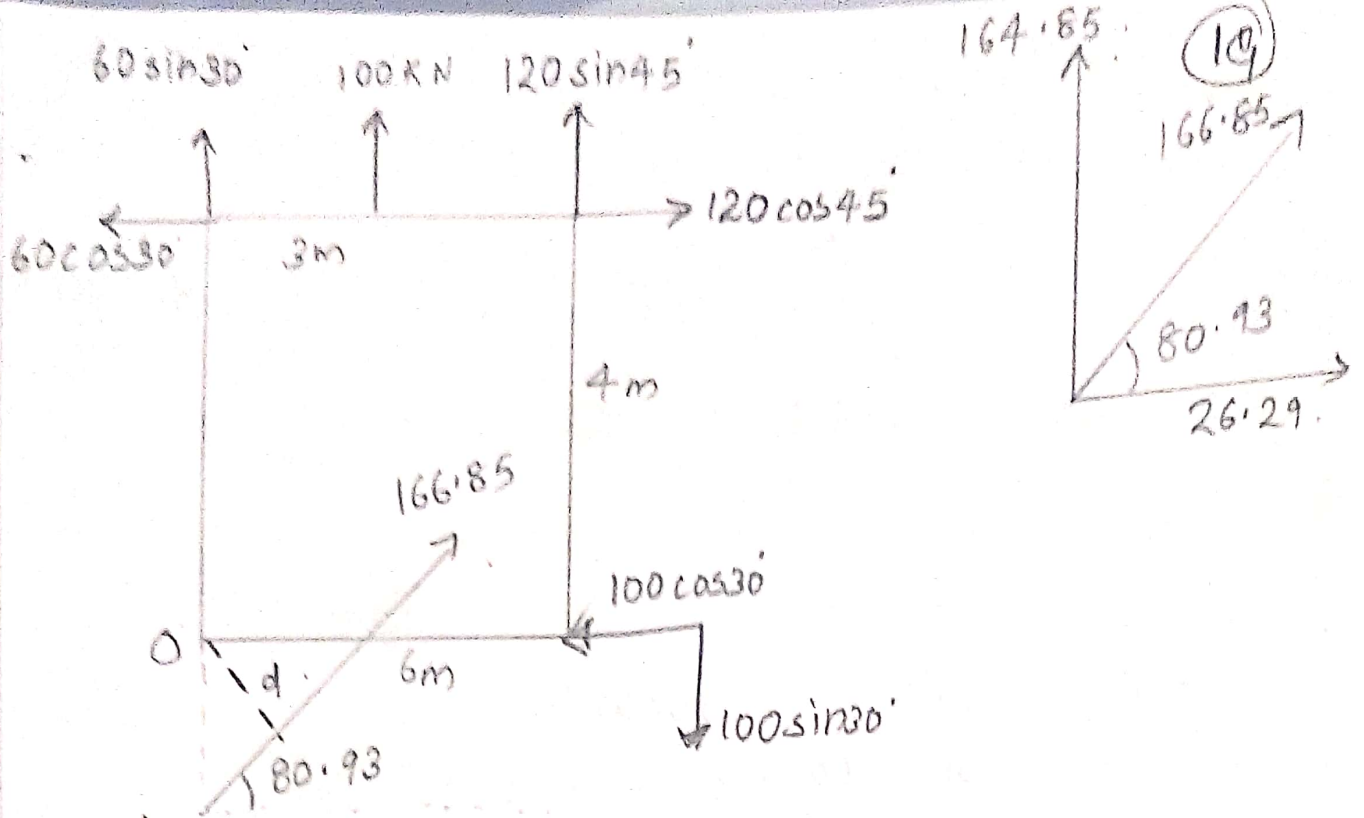


$$\begin{aligned}\Sigma H &= 80 + 120\cos 45^\circ - 100\cos 30^\circ - 60\cos 30^\circ \\ &= 26.29 \text{ kN } (\rightarrow)\end{aligned}$$

$$\begin{aligned}\Sigma V &= 120\sin 45^\circ + 100 + 60\sin 30^\circ - 100\sin 30^\circ \\ &= 164.85 \text{ kN } (\uparrow)\end{aligned}$$

$$\begin{aligned}R &= \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{26.29^2 + 164.85^2} = \sqrt{27,866.68} \\ &= 166.93 \text{ kN}\end{aligned}$$

$$\theta = \tan^{-1} \frac{\Sigma V}{\Sigma H} = 80.93^\circ$$



Moment about the point O .

$$166.85 \times d = -100 \sin 30^\circ \times 6 + 120 \sin 45^\circ \times 6 - 120 \cos 45^\circ \times 4 + 100 \times 3 + 60 \cos 30^\circ \times 4$$

$$166.85 \times d = 377.55 \quad (\uparrow)$$

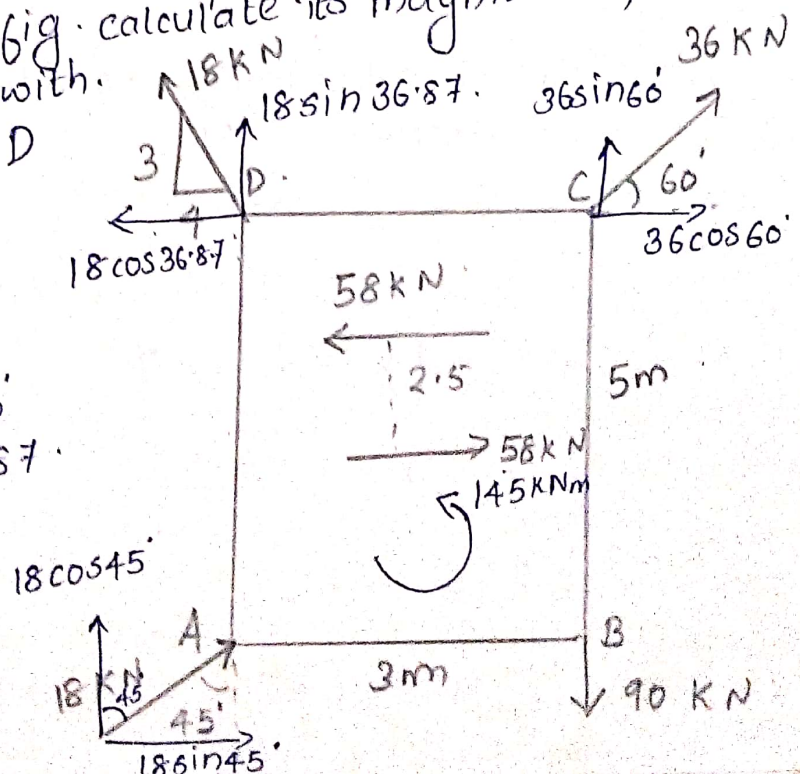
$$d = 2.26 \text{ m}, \quad d = \underline{\underline{2.26 \text{ m}}}$$

Find out the resultant of the coplanar non-concurrent force system shown in fig. calculate its magnitude & direction and locate its position with respect to sides AB & AD .

$$\theta = \tan^{-1} \frac{3}{4} = 36.87^\circ$$

$$\Sigma H = 18 \sin 45^\circ + 36 \cos 60^\circ - 18 \cos 36.87^\circ$$

$$= \underline{\underline{16.33 \text{ kN}}}$$

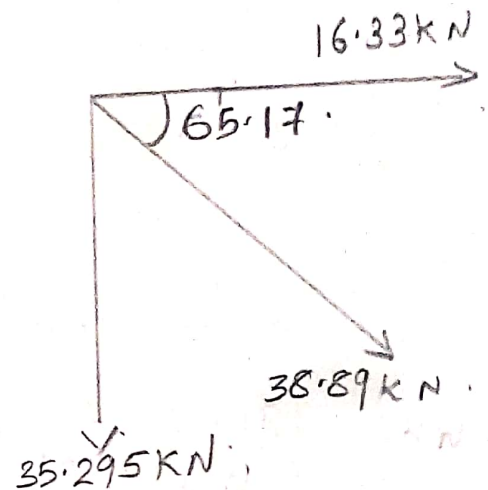


$$\begin{aligned} \Sigma V &= 18 \cos 45^\circ - 90 + 36 \sin 60^\circ + 18 \sin 36.87^\circ \\ &= ~~18.47 \text{ kN}~~ - 35.295 \text{ kN} \end{aligned}$$

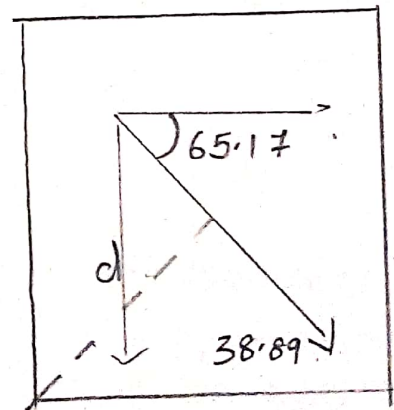
$$R = \sqrt{\Sigma V^2 + \Sigma H^2} = \sqrt{(-35.295)^2 + (16.33)^2} = \underline{\underline{38.89 \text{ kN}}}$$

$$\theta = \tan^{-1} \frac{\Sigma V}{\Sigma H} = \tan^{-1} \frac{-35.295}{16.33} = ~~11.38^\circ~~ \underline{\underline{65.17^\circ}}$$

$$\begin{aligned} \text{Distance along AB, } \bar{x} &= \frac{\Sigma M_A}{\Sigma V} \\ &= \frac{-90 \times 3 + 145 - 5 \times 36 \cos 60^\circ + 3 \times 36 \sin 60^\circ + 5 \times 18 \cos 36.87^\circ}{-35.295} \\ &= \frac{-49.469}{-35.295} = \underline{\underline{1.402 \text{ m}}} \end{aligned}$$



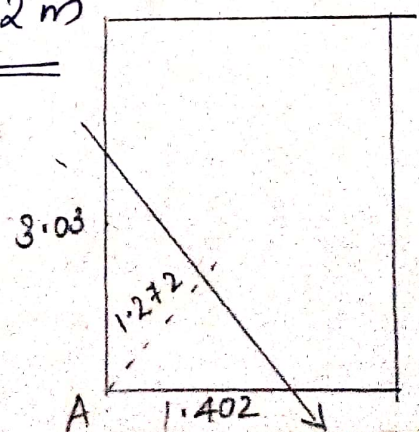
$$\begin{aligned} \text{Distance along AD, } \bar{y} &= \frac{\Sigma M_A}{\Sigma H} \\ &= \frac{-49.469}{16.33} = \underline{\underline{-3.03 \text{ m}}} \end{aligned}$$



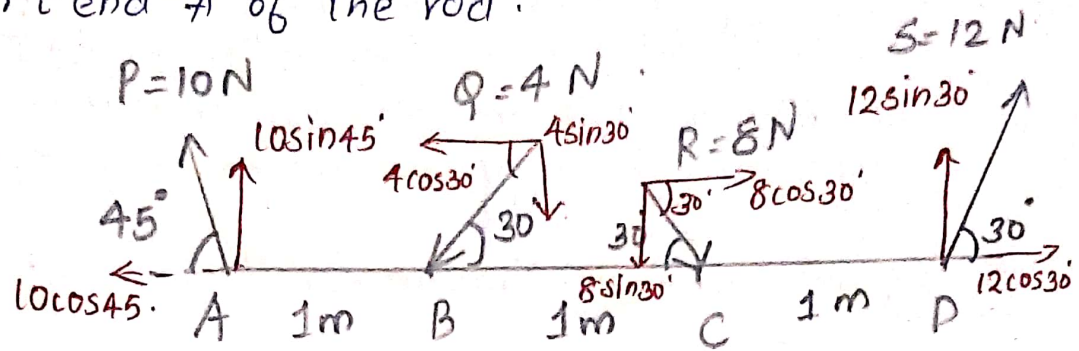
d - dist from the origin to the resultant.

$$-38.89 \times d = -90 \times 3 - 36 \cos 60^\circ \times 5 + 36 \sin 60^\circ \times 3 + 18 \cos 36.87^\circ \times 5 + 145$$

$$\frac{\Sigma M_A}{R} \quad d = \frac{49.469}{38.89} = \underline{\underline{1.272 \text{ m}}}$$



ABCD is a weightless rod under the action of 4 forces P, Q, R, S. Calculate the resultant, magnitude and direction if $P = 10\text{ N}$, $Q = 4\text{ N}$, $R = 8\text{ N}$ and $S = 12\text{ N}$. Also locate the point of intersection w.r.t end A of the rod.



$$\Sigma H = 8\cos 30^\circ + 12\cos 30^\circ - 4\sin 30^\circ - 10\cos 45^\circ$$

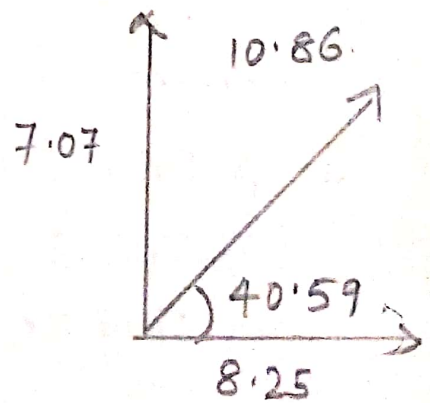
$$= \underline{\underline{8.25\text{ N}}}$$

$$\Sigma V = 10\sin 45^\circ + 12\sin 30^\circ - 4\sin 30^\circ - 8\sin 30^\circ$$

$$= \underline{\underline{7.07\text{ N}}}$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{8.25^2 + 7.07^2} = 10.86\text{ N}$$

$$\theta = \tan^{-1} \frac{\Sigma V}{\Sigma H} = 40.59^\circ$$



Moment of Resultant abt A

$$= 7.07 \times x \quad \text{--- (1)}$$

Moment of all forces abt A.

$$-4\sin 30^\circ \times 1 - 8\sin 30^\circ \times 2$$

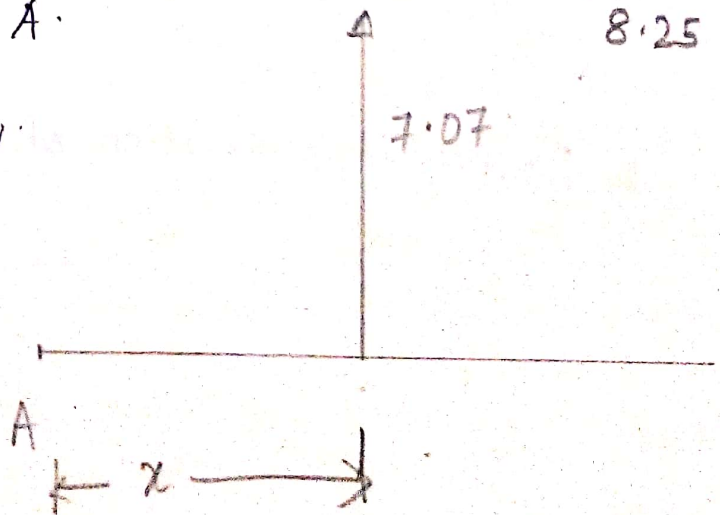
$$+ 12\sin 30^\circ \times 3$$

$$= 8 \quad \text{--- (2)}$$

Equating (1) & (2)

$$7.07x = 8$$

$$x = \underline{\underline{1.13\text{ m}}}$$



EQUILIBRIUM.

When some external forces (which may be concurrent or parallel) are acting on a stationary body, the body may start moving or may start rotating about any point. But if the body does not start moving and also does not start rotating about any point, then the body is said to be in equilibrium.

Equations of Equilibrium for Concurrent Force System:-

$$\sum H = 0.$$

$$\sum V = 0.$$

Equations of Equilibrium for Non-Concurrent Force system.

$$\sum H = 0, \sum V = 0, \sum M = 0.$$

EQUILIBRANT.

The equilibrant of a system of forces is a single force which act along with the other forces to keep the body in equilibrium.

Method of finding out equilibrant.

- 1) Find $\sum H$.
- 2) Find $\sum V$.
- 3) Find $R = \sqrt{\sum H^2 + \sum V^2}$, $\theta = \tan^{-1} \frac{\sum V}{\sum H}$.
- 4) Reverse the direction of the resultant.

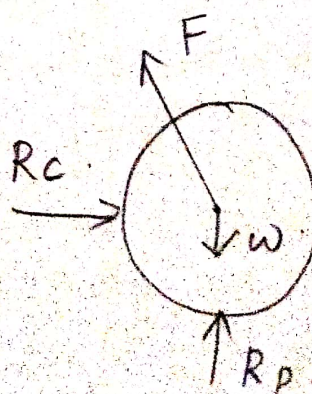
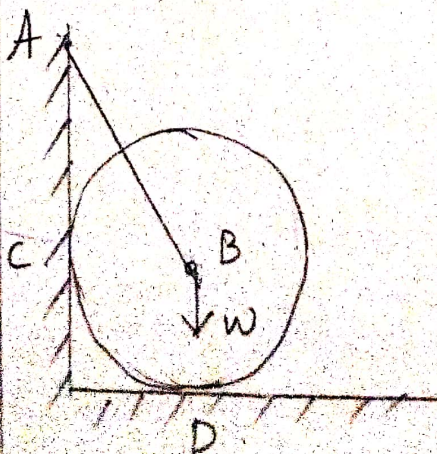
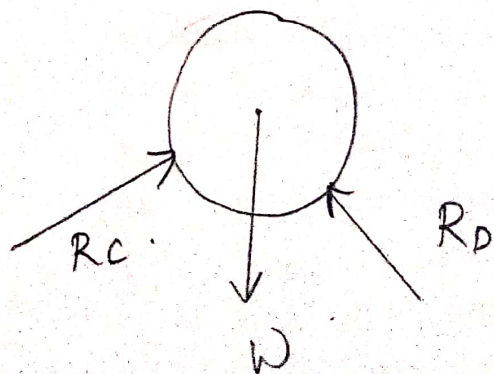
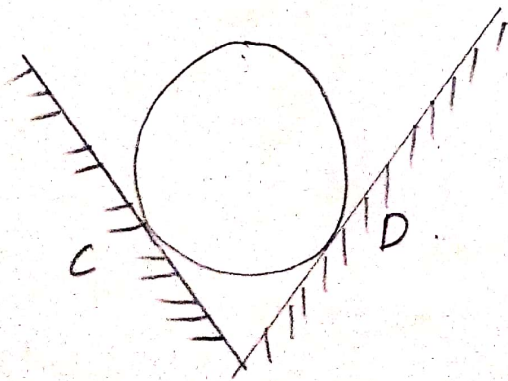
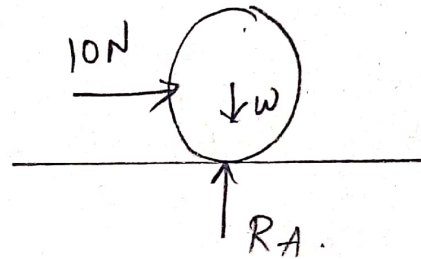
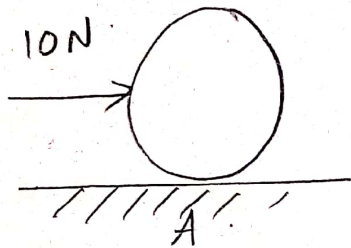
FREE BODY DIAGRAM.

(21)

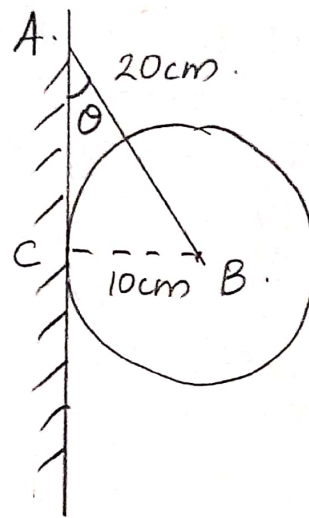
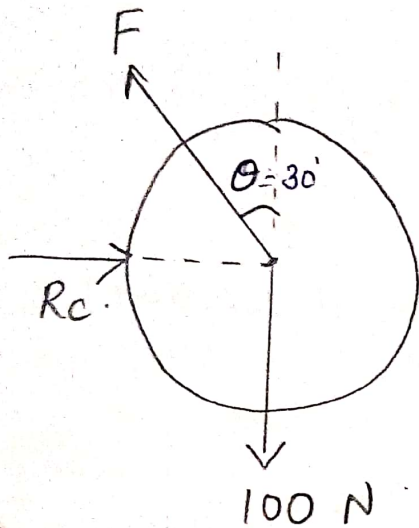
Representation of a body that has been released from all the constraints.

Steps for drawing free body diagram

1. Represent the body to some scale.
2. Mark the external forces.
3. Mark the weight of the body acting through the centre of gravity.
4. Release the constraints.
5. Mark reactions at each of the released constraint.



1. A circular roller of weight 100 N and radius 10 cm hangs by a tie rod AB = 20 cm and rests against a smooth vertical wall at C as shown in fig. Determine.
- the force F in the tie rod
 - the reaction R_c at point C.



In ΔABC , $\sin \theta = \frac{10}{20}$, $\theta = 30^\circ$.

$$\sum H = R_c - F \sin 30^\circ = 0$$

$$R_c = F \sin 30^\circ$$

$$\sum V = F \cos 30^\circ - 100 = 0$$

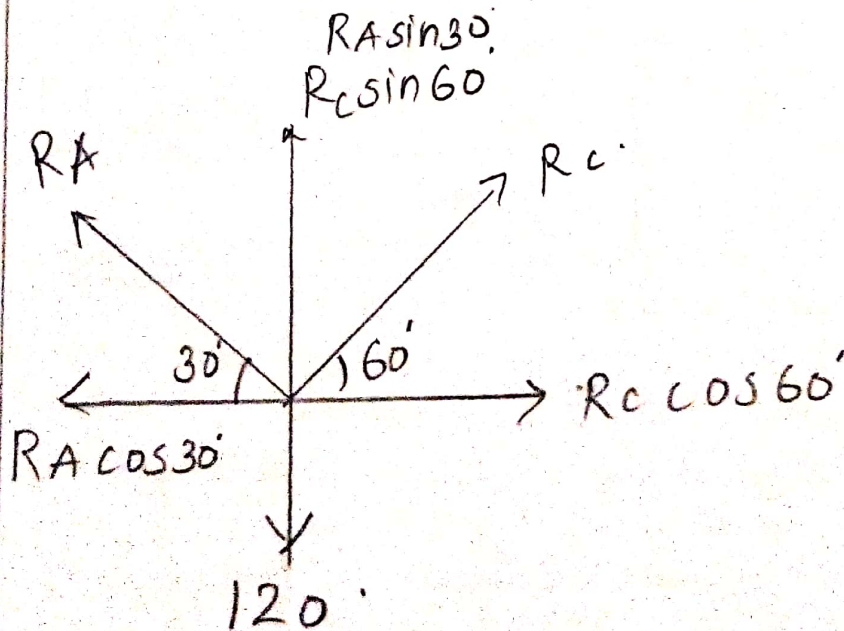
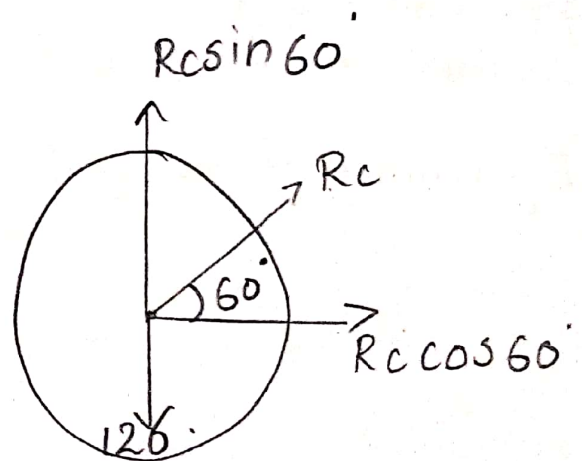
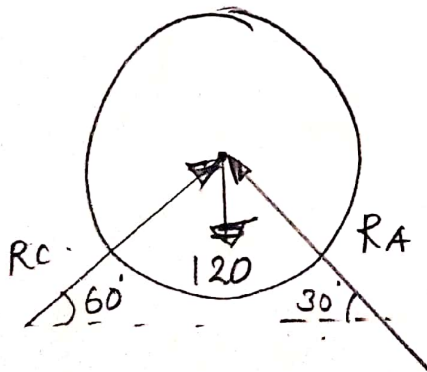
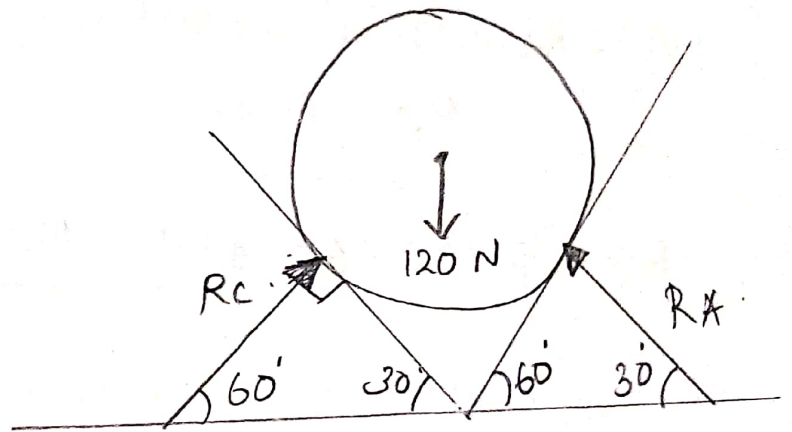
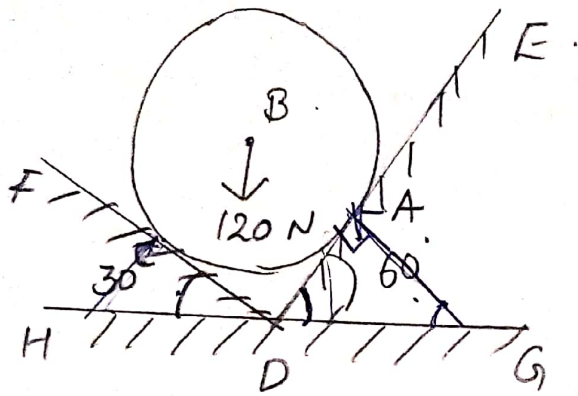
$$F \cos 30^\circ = 100$$

$$F = \frac{100}{\cos 30^\circ} = \underline{\underline{115.47 \text{ N}}}$$

$$R_c = 115.47 \sin 30^\circ$$

$$= \underline{\underline{57.73 \text{ N}}}$$

2. A ball of weight 120 N rests in a right angled groove as shown in Fig. The sides of the groove are inclined to an angle of 30° & 60° to the horizontal. If all the surfaces are smooth, then determine the reactions R_A & R_C @ the points of contact.



$$\sum H = 0$$

$$R_C \cos 60^\circ - R_A \cos 30^\circ = 0$$

$$R_C \cos 60^\circ = R_A \cos 30^\circ$$

$$R_C = 1.732 R_A$$

$$\sum V = 0$$

$$R_A \sin 30^\circ + R_C \sin 60^\circ = 120$$

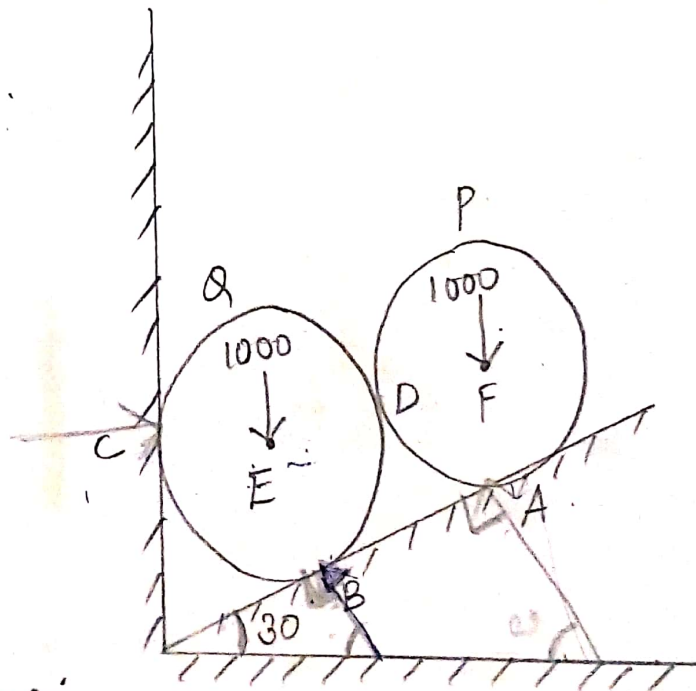
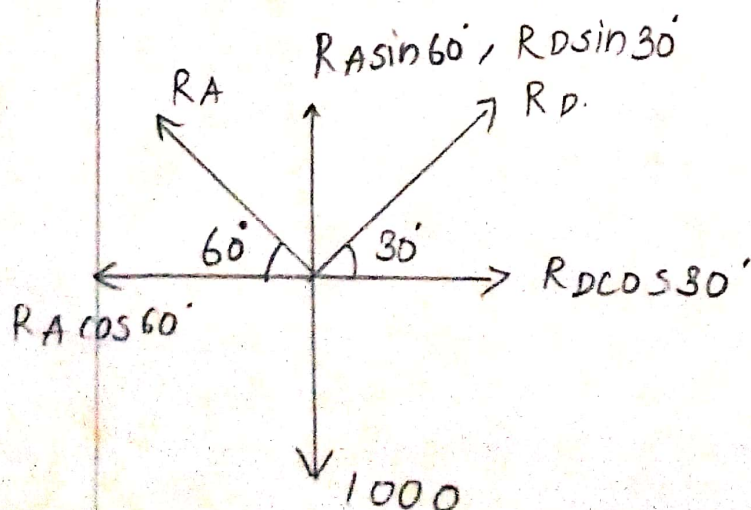
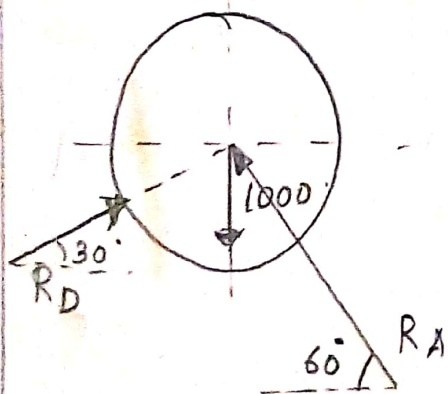
$$1.732 R_A \sin 60^\circ + \cancel{1.732} R_A \sin 30^\circ = 120$$

$$R_A = 60 \text{ N}$$

$$R_C = 1.732 \times 60 = \underline{\underline{103.92 \text{ N}}}$$

3. Two identical rollers, each of weight $w = 1000 \text{ N}$, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B & C. Assume all the surfaces to be smooth.

Equilibrium of roller P.



$$\Sigma H = 0$$

$$R_D \cos 30^\circ - R_A \cos 60^\circ = 0 \quad R_D = 0.577 R_A$$

(23)

$$\Sigma V = 0$$

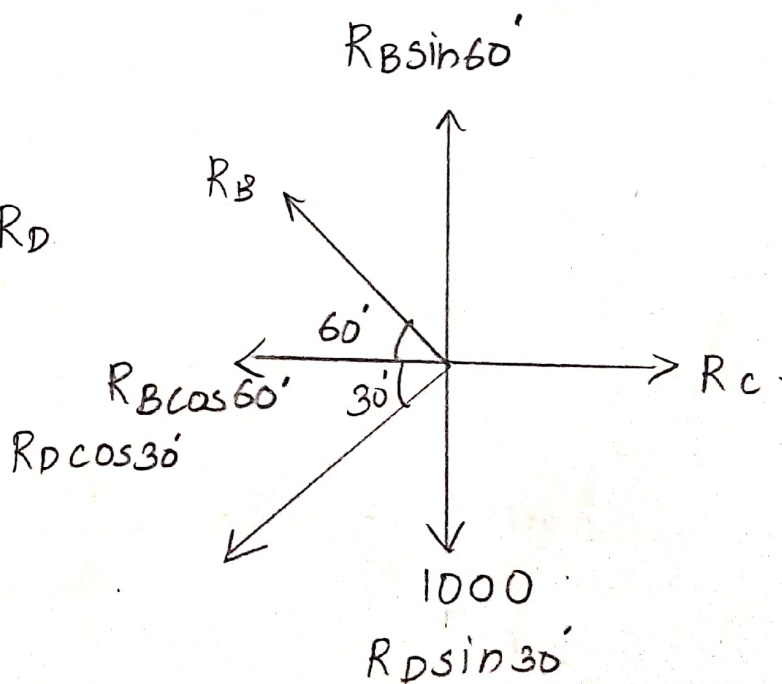
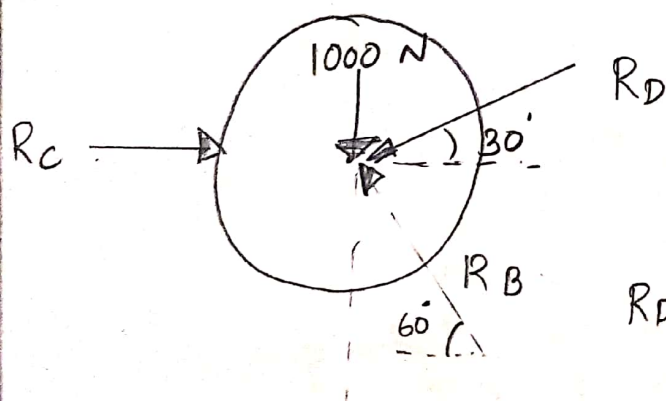
$$R_A \sin 60^\circ + R_D \sin 30^\circ = 1000$$

$$R_A \sin 60^\circ + 0.577 R_A \sin 30^\circ = 1000$$

$$R_A = 866.17 \text{ N}$$

$$R_D = 0.577 \times 866.17 = 499.78 \text{ N}$$

Equilibrium of roller C.



$$\Sigma H = 0$$

$$R_C - R_B \cos 60^\circ - R_D \cos 30^\circ = 0 \quad \text{--- (1)}$$

$$\Sigma V = 0$$

$$R_B \sin 60^\circ - 1000 - R_D \sin 30^\circ = 0$$

$$R_B \sin 60^\circ - 1000 - 499.78 \sin 30^\circ = 0$$

$$R_B = 1443.24 \text{ N}$$

$$R_C = 1154.44 \text{ N}$$

4. Two spheres, each of weight 1000 N and of radius 25 cm rest in a horizontal channel of width 90 cm as in fig. Find the reactions on the points of contact A, B & C.

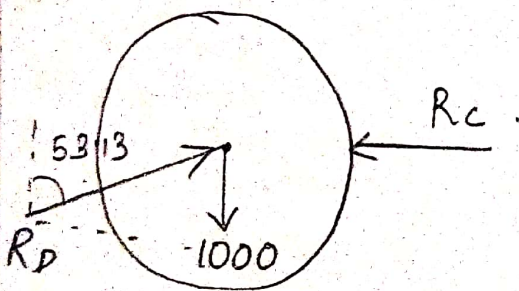
Consider ΔEFG .

$$\sin \theta = \frac{40}{50}, \theta = 53.13$$

~~$$\cos \theta = \frac{40}{50}$$~~

$$\cos \theta = \frac{30}{50}$$

FBD of ball 2.



$$\sum H = 0$$

$$R_D \sin 53.13 = R_C$$

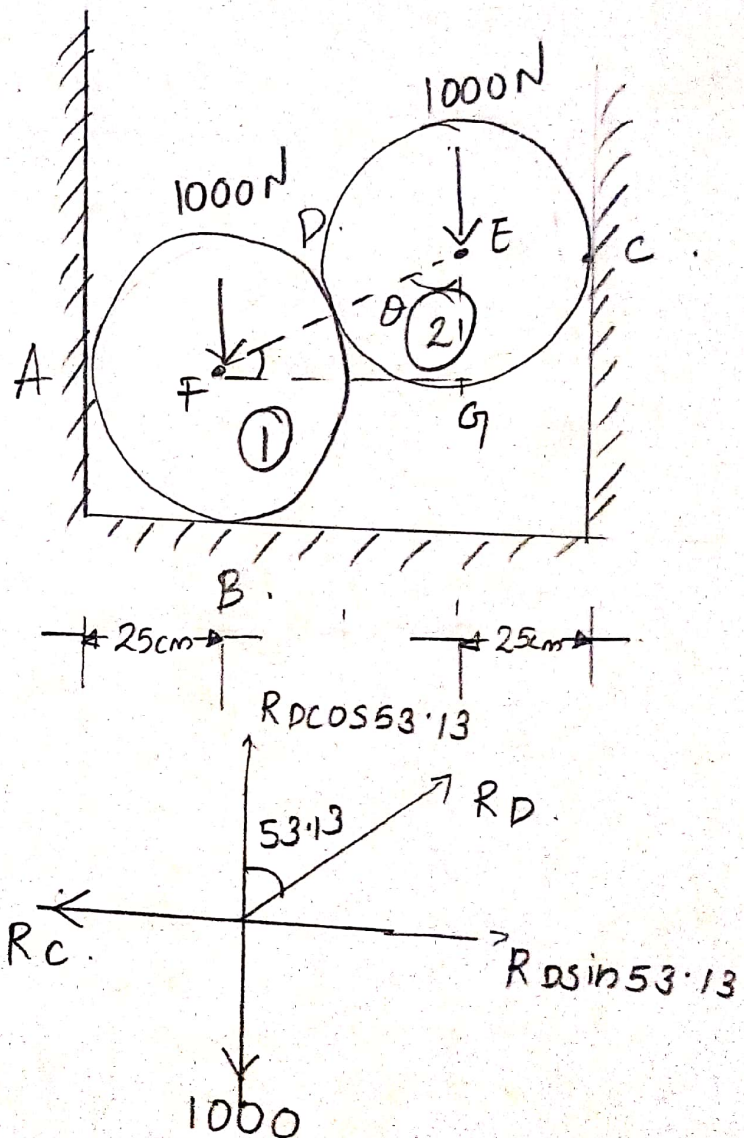
$$\sum V = 0$$

$$R_D \cos 53.13 = 1000$$

$$R_D = 1666.66 \text{ N}$$

$$R_C = 1666.66 \sin 53.13$$

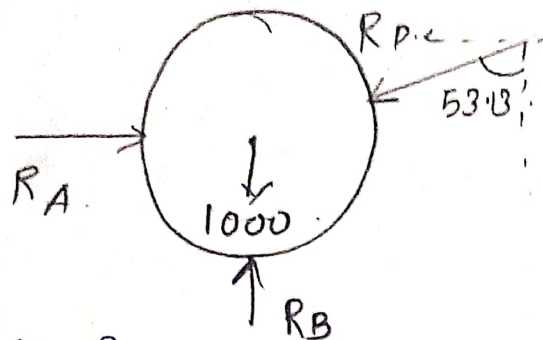
$$= 1333.33 \text{ N}$$



$$R_D = 1666.66 \text{ N}$$

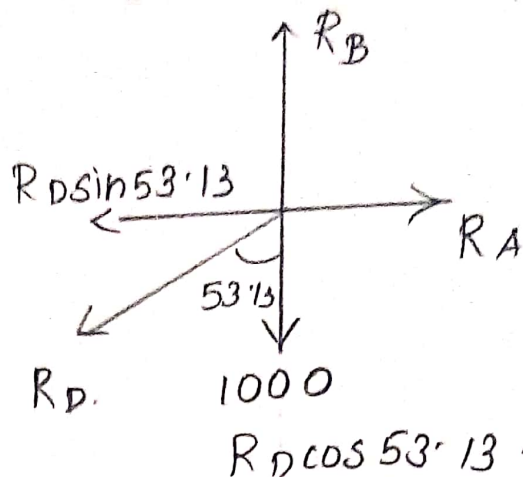
$$R_C = 1333.33 \text{ N}$$

FBD of ball 1



$$\sum H = 0$$

$$\begin{aligned} R_A &= R_D \sin 53.13 \\ &= 1666.66 \sin 53.13 \\ &= 1333.33 \text{ N} \end{aligned}$$



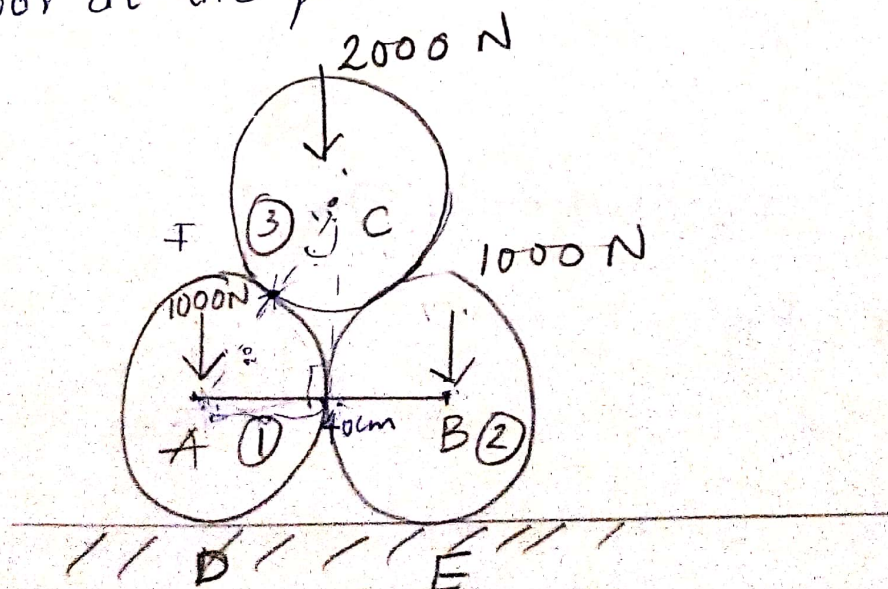
$$\begin{aligned} R_B &= 1999.06 \text{ N} \\ R_A &= 1333.33 \text{ N} \end{aligned}$$

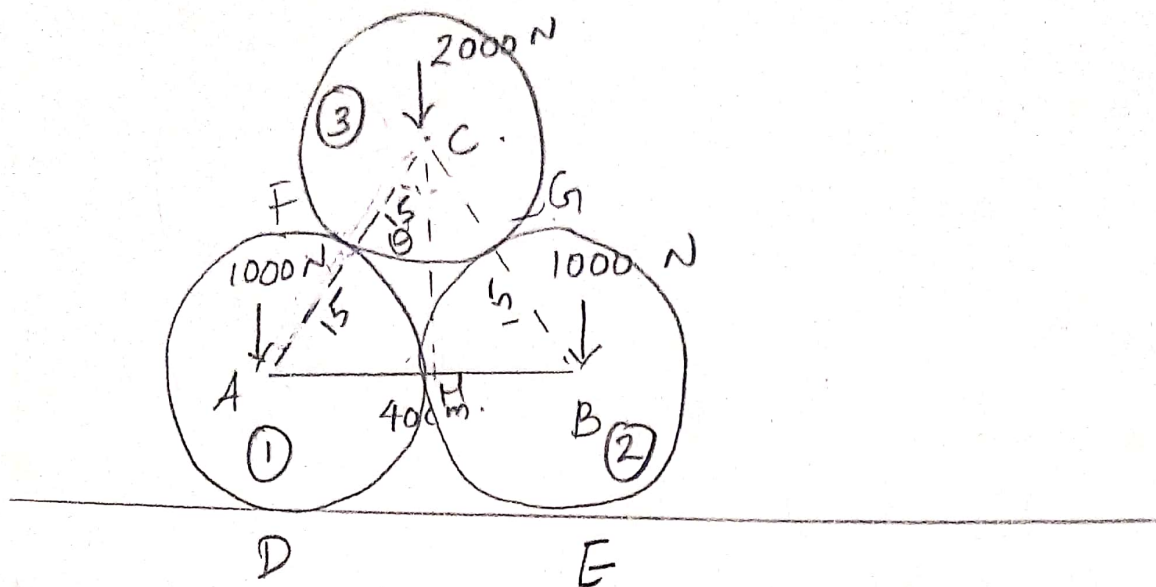
$$\sum V = 0$$

$$1000 + R_D \cos 53.13 = R_B$$

$$\begin{aligned} R_B &= 1000 + 1666.66 \cos 53.17 \\ &= 1999.067 \text{ N} \end{aligned}$$

5. Two smooth circular cylinders, each of weight $w = 1000 \text{ N}$ and radius 15 cm , are connected at their centres by a string AB of length $= 40 \text{ cm}$ and rest upon a horizontal plane, supporting above them a third cylinder of weight $= 2000 \text{ N}$ and radius 15 cm as shown in fig. Find the force T in the string AB and the pressure produced on the floor at the points of contact D & E .



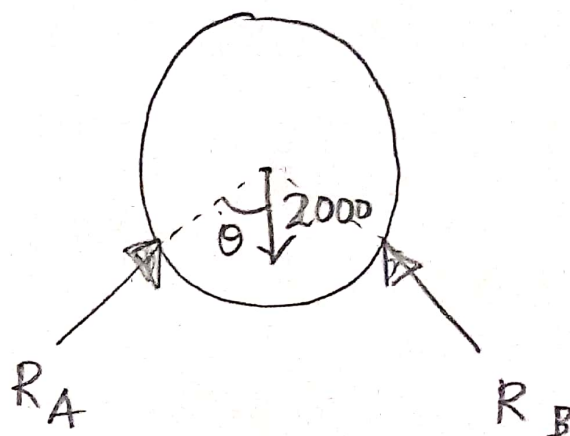
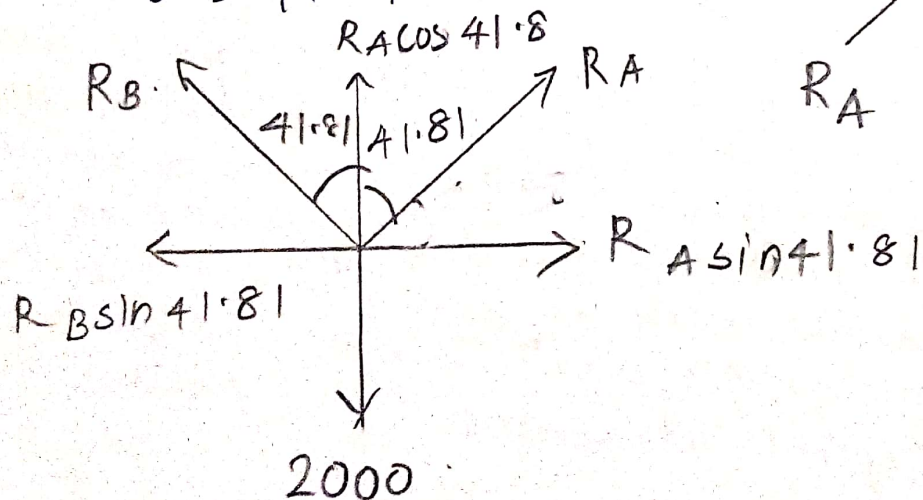


Equilibrium of point 3.

Consider ACH.

$$\sin \theta = \frac{20}{30}$$

$$\theta = 41.81^\circ$$



$$\sum H = 0$$

$$R_A \sin 41.81^\circ = R_B \sin 41.81^\circ \quad \text{--- (1)}$$

$$R_A = R_B$$

$$\sum V = 0$$

$$R_A \cos 41.81^\circ + R_B \cos 41.81^\circ = 2000 \quad \text{--- (2)}$$

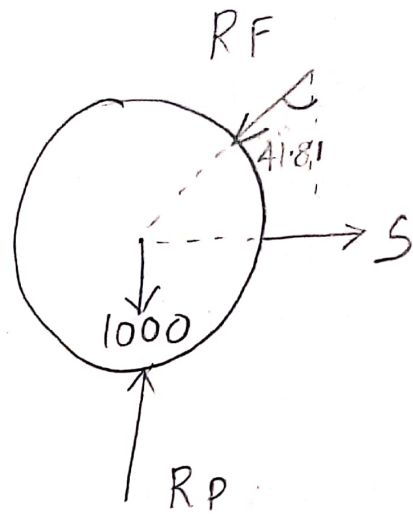
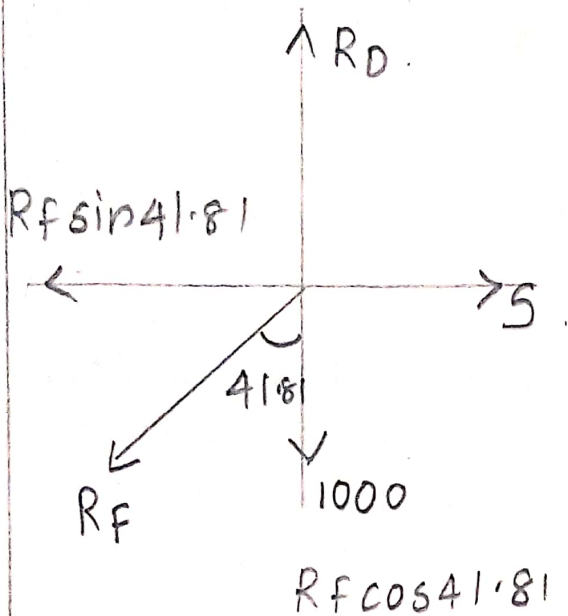
$$2 R_A \cos 41.81^\circ = 2000$$

$$R_F = \underline{\underline{1341.63 \text{ N}}}$$

$$R_G = \underline{\underline{1341.63 \text{ N}}}$$

(25)

Equilibrium of ball (1)



$$\Sigma H = 0$$

$$S - R_F \sin 41.81 = 0$$

$$S = 1341.63 \sin 41.81$$

$$= \underline{\underline{894.41 \text{ N}}}$$

$$\Sigma V = 0$$

$$R_D = 1000 + 1341.63 \cos 41.81$$

$$= \underline{\underline{1999.99 \text{ N}}}$$

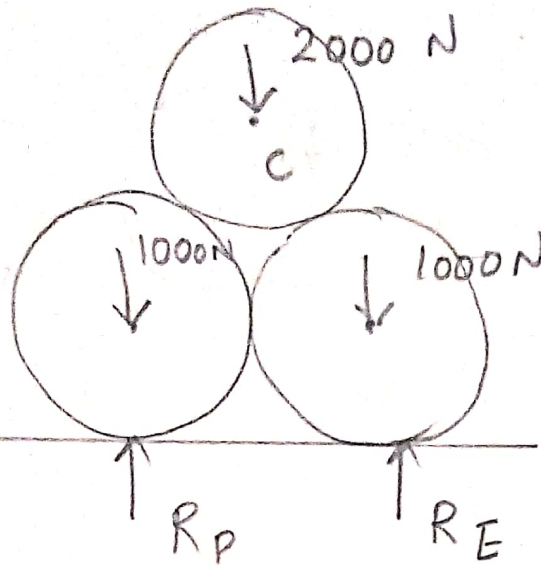
Equilibrium of cylinders 1, 2 & 3.

In the case only vertical forces exist.

$$R_D + R_E = 4000$$

$$R_E = 4000 - 1999.99$$

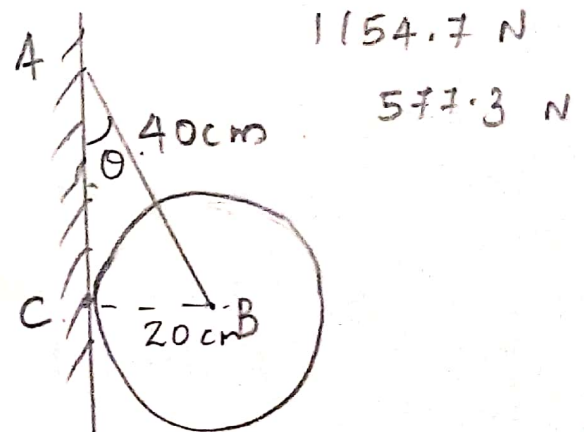
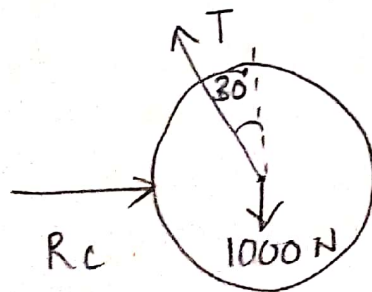
$$= \underline{\underline{2000 \text{ N}}}$$



6. A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB = 40 cm and rests against a smooth vertical wall @ C as shown in fig. Determine the tension in the tie rod and reaction R_C at point C.

$$\sin \theta = \frac{20}{40}$$

$$\theta = 30^\circ$$



$$\Sigma H = 0$$

$$R_C = T \sin 30^\circ$$

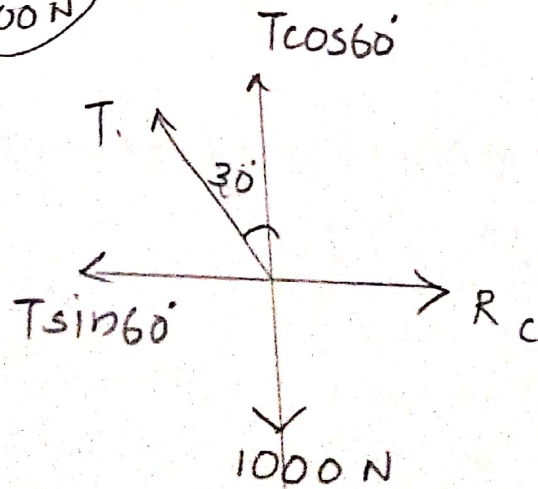
$$\Sigma V = 0$$

$$T \cos 30^\circ = 1000$$

$$T = \underline{\underline{1154.7 \text{ N}}}$$

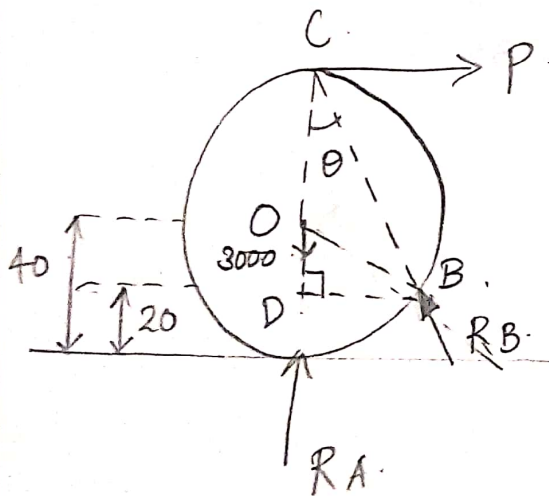
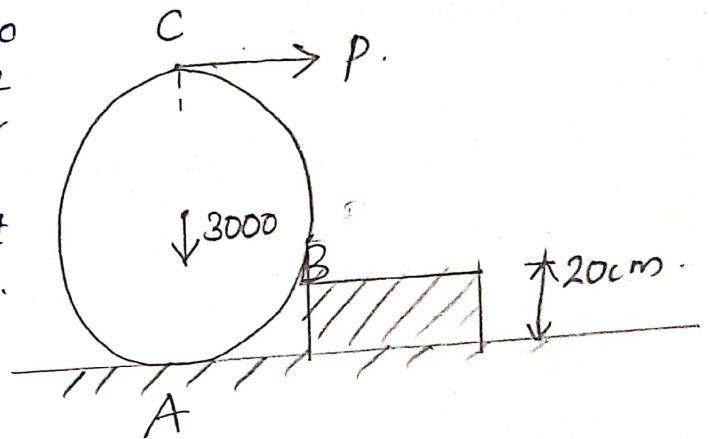
$$R_C = 1154.7 \sin 30^\circ$$

$$= \underline{\underline{577.35 \text{ N}}}$$



7. A roller of radius 40cm weighing 3000N is to be pulled over a rectangular block of height 20cm as shown in the fig. by a horizontal force P applied at the end of the string moved round the circumference of the roller. Find the magnitude of the horizontal force which will just turn the roller over the corner of the rectangular block. Also find out the mag & direction of reactions at A & B. Assume all the surfaces as smooth.

When the roller is about to turn over the corner of the rectangular block, the roller lifts at the point A and then there will be no contact bet the roller & the point A. Hence R_A at point A will become zero.



$$OB = 40\text{cm}$$

$$OD = 20\text{cm}$$

$$BD = \sqrt{40^2 - 20^2} = 34.64$$

For the equilibrium, these 3 forces should pass through a common point. As the force P & weight W is passing through C , hence R_B must also pass through C .

$$\tan \theta = \frac{BD}{CD}, \quad \theta = \tan^{-1} \left(\frac{34.64}{60} \right) = 30^\circ$$

$$\Sigma H = 0$$

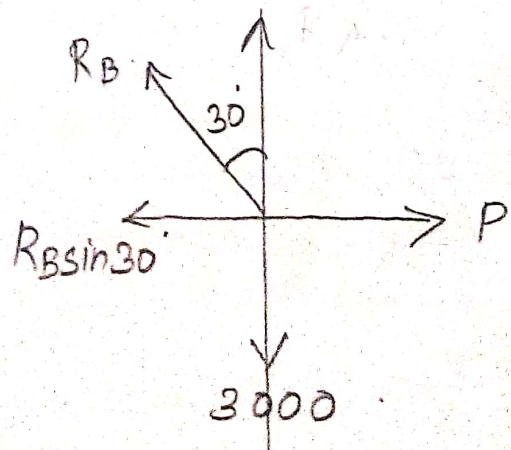
$$P - R_B \sin 30^\circ = 0$$

$$\Sigma V = 0$$

$$R_B \cos 30^\circ = 3000$$

$$R_B = 3464.10\text{ N}$$

$$P = 1732.05\text{ N}$$



- 8 If the force P is applied horizontally at the centre of the roller, what would be the magnitude of this force? Also determine the least force and its line of action at the roller centre, for turning the roller over the rectangular block.

Consider $\triangle ODB$.

$$\sin \theta = \frac{BD}{OB}$$

$$\theta = \sin^{-1} \left(\frac{34.64}{40} \right) = 60^\circ$$

$$\Sigma H = 0$$

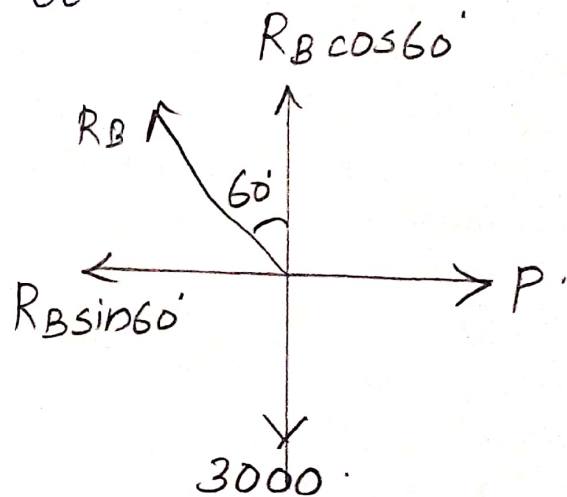
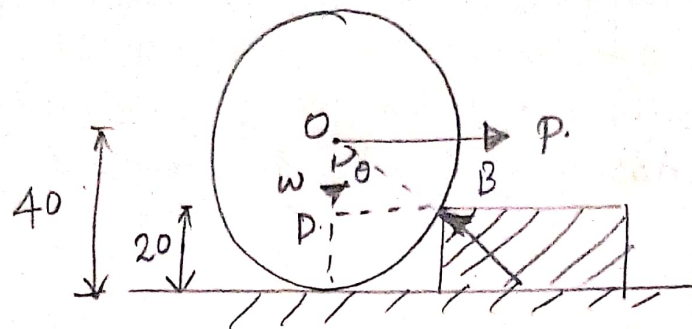
$$P = R_B \sin 60^\circ$$

$$\Sigma V = 0$$

$$R_B \cos 60^\circ = 3000$$

$$R_B = 6000 \text{ N}$$

$$P = 5196.15 \text{ N}$$



3 Consider $\triangle OCB$.

$$\sin \alpha = \frac{BC}{OB}$$

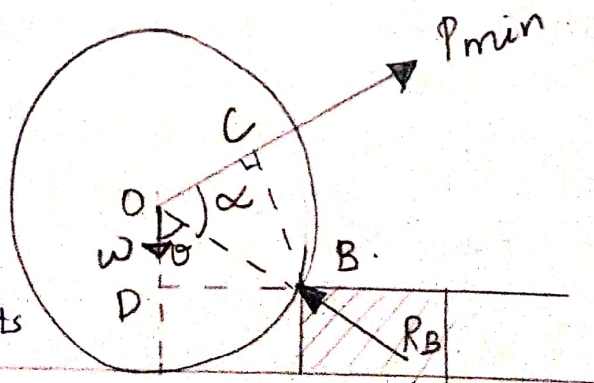
$$CB = OB \sin \alpha$$

~~$\Sigma M_B = 0$~~ Taking moments of all forces about B.

$$P_{\min} \times OB \sin \alpha - 3000 \times 34.64 = 0$$

$$P_{\min} = \frac{3000 \times 34.64}{40 \sin \alpha}$$

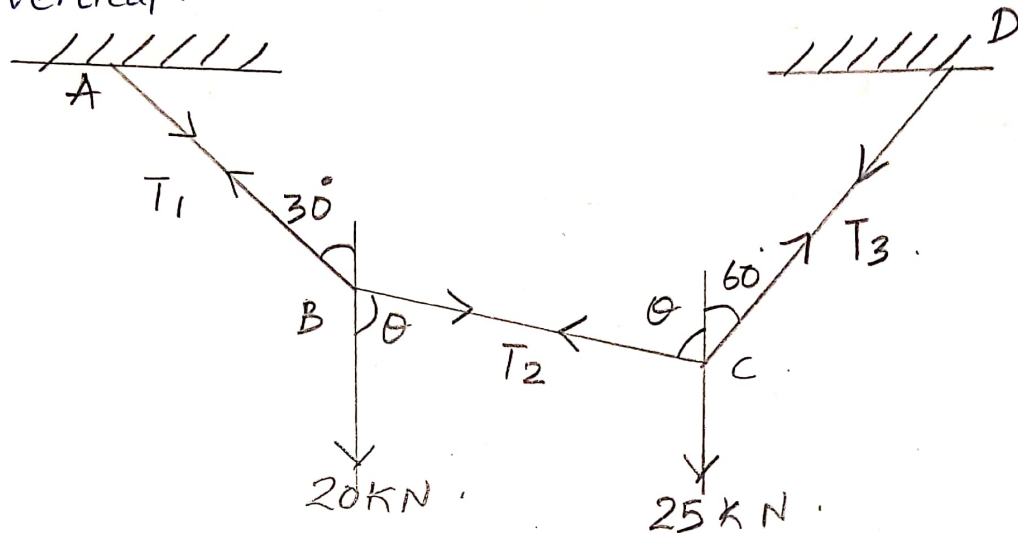
when $\alpha = \frac{\pi}{2}$
 $\sin \alpha$ will be max



$$P_{\min} = \underline{2598 \text{ N}}$$

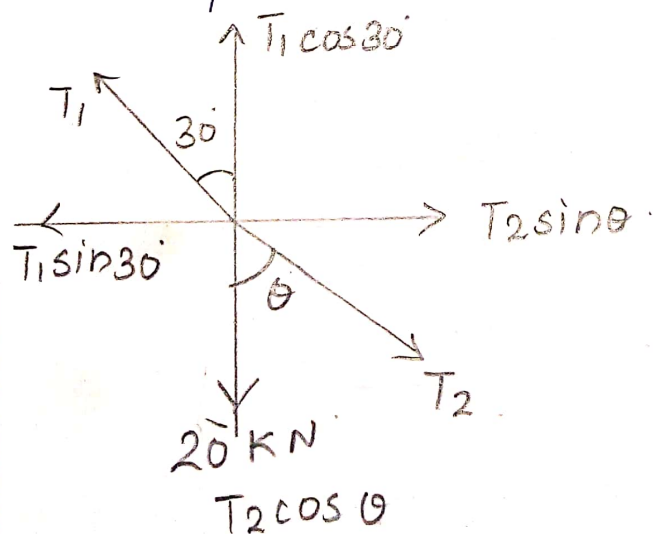
(27)

9. A wire is fixed at 2 points A & B as shown in fig. 2 weights 20 kN & 25 kN are supported at B & C resp. when equilibrium is reached, it is found that the inclination of AB is 30° and that of CD is 60° to the vertical. Find out the tension in the segments AB, BC & CD of the rope & also the inclination of BC to the vertical.



Let T_1 , T_2 & T_3 be the tensions in the parts of the strings AB, BC & CD resp. Let the inclination of BC with the vertical be θ .

FBD of point B.



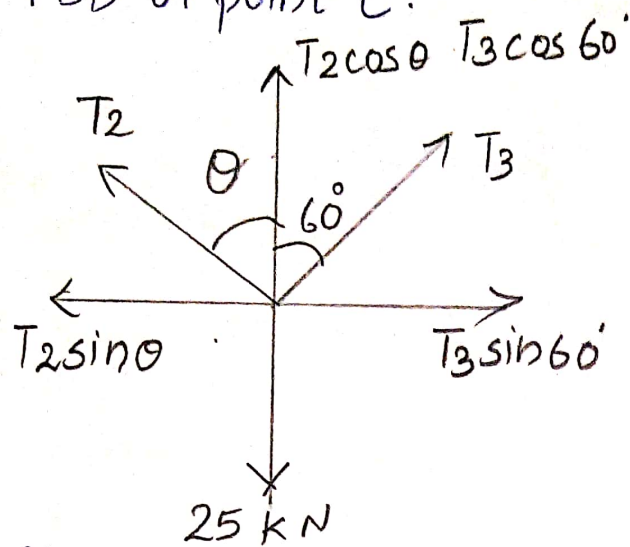
$$\Sigma H = 0$$

$$T_2 \sin \theta = T_1 \sin 30^\circ \quad \text{--- (1)}$$

$$\Sigma V = 0$$

$$T_1 \cos 30^\circ = 20 + T_2 \cos \theta \quad \text{--- (2)}$$

FBD of point C.



$$\sum H = 0$$

$$T_3 \sin 60^\circ = T_2 \sin \theta \quad \text{--- (3)}$$

$$\sum V = 0$$

$$T_2 \cos \theta + T_3 \cos 60^\circ = 25 \quad \text{--- (4)}$$

Equating (1) & (3)

$$T_3 \sin 60^\circ = T_1 \sin 30^\circ$$

$$T_1 = T_3 1.732 \quad \text{--- (5)}$$

Sub (5) in (2); Equating (2) & (4)

$$1.732 T_1 \cos 30^\circ - 20 = 25 - T_3 \cos 60^\circ$$

$$1.732 T_3 \cos 30^\circ + T_3 \cos 60^\circ = 45$$

$$T_3 (1.999) = 45$$

$$\boxed{T_3 = 22.5 \text{ kN}}$$

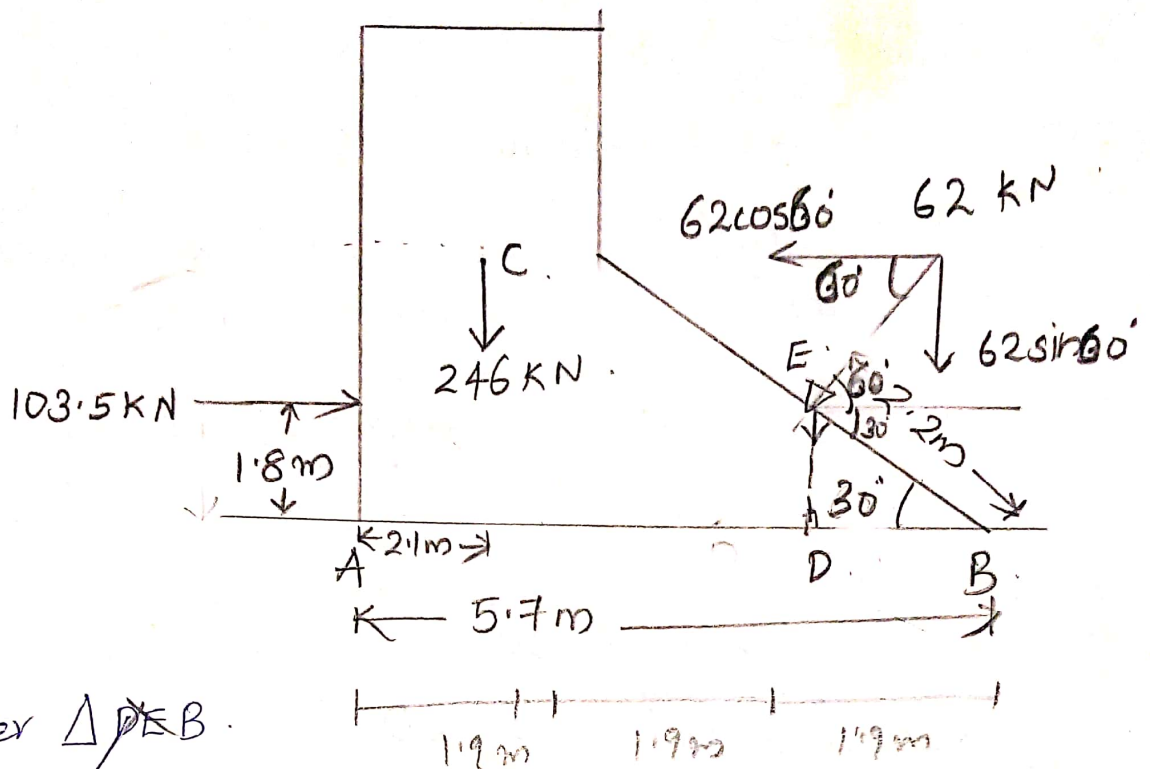
$$T_1 = 22.5 \times 1.732$$

$$\boxed{T_1 = 38.97 \text{ kN}}$$

Sub T_3 in (3)

$$T_2 = \frac{22.5 \sin 60^\circ}{\sin \theta}$$

10. Determine the resultant of the three forces acting on the dam shown in fig and locate its intersection with the base AB. For a good design, this intersection should occur within the middle third of the base. Comment whether it is a good design or not.



Consider $\triangle BDE$.

$$\cos 30^\circ = \frac{BE}{BD}$$

$$BD = \frac{2.2}{\cos 30^\circ}$$

Consider $\triangle BDE$

$$\cos 30^\circ = \frac{BD}{2.2}$$

$$BD = 1.905 \text{ m}$$

$$\sin 30^\circ = \frac{DE}{2.2}$$

$$DE = 1.1 \text{ m}$$

$\Sigma H = 0$

$$103.5 - 62 \cos 60^\circ = 72.5 \text{ kN}$$

$\Sigma V = 0$

$$-246 - 62 \sin 60^\circ = -299.693 \text{ kN}$$

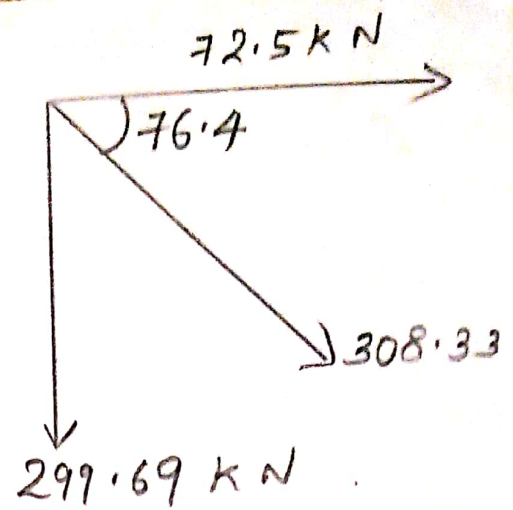
$$R = \sqrt{EH^2 + EV^2}$$

$$= \sqrt{72.5^2 + (-299.693)^2}$$

$$= \underline{\underline{308.33 \text{ kN}}}$$

$$\theta = \tan^{-1} \frac{EV}{EH} = \tan^{-1} \frac{299.693}{72.5}$$

$$= \underline{\underline{76.4}}$$



$$\sum M_A = -308.33 \times d$$

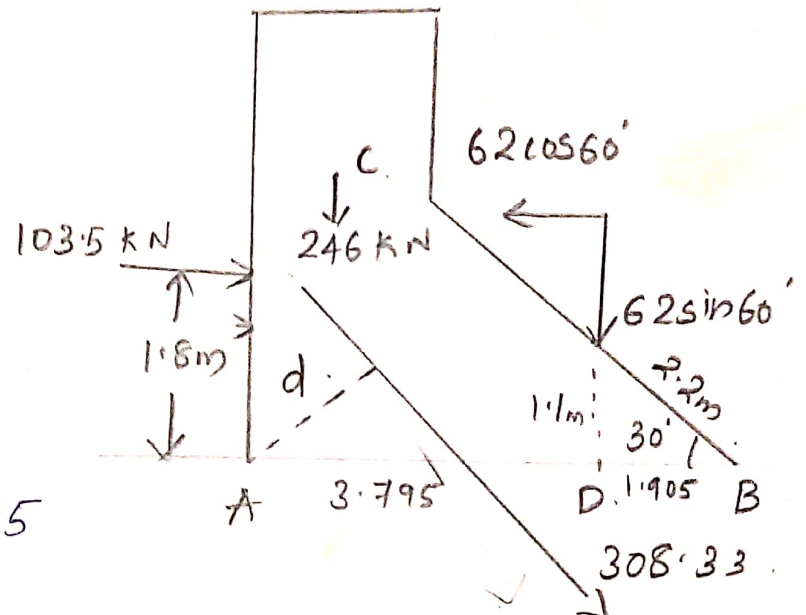
$$= -308.33 \times 1.8$$

$$= -554.994$$

$$+ 62 \cos 60^\circ \times 1.1$$

$$- 246 \times 2.1$$

$$- 62 \sin 60^\circ \times 3.795$$



$$d = \frac{-499.967}{-308.33} = 1.621$$

$$d = \frac{-872.567}{-308.33}$$

$$= \underline{\underline{2.83 \text{ m}}}$$

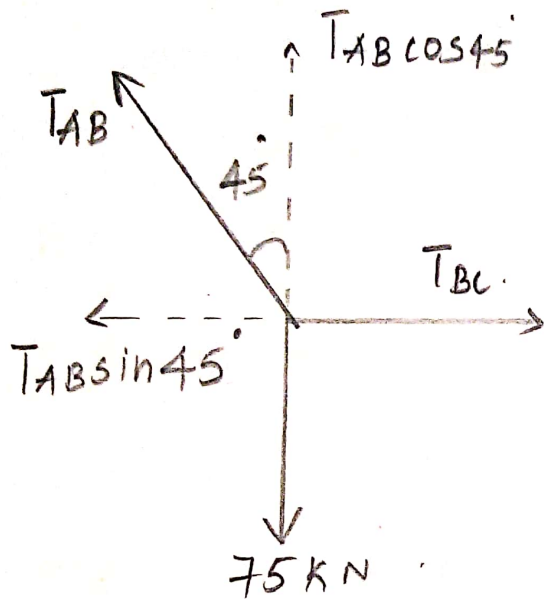
$$\bar{x} = \frac{\sum M_A}{\sum V} = \frac{-872.567}{-299.693}$$

$$= \underline{\underline{2.91 \text{ m}}}$$

The middle third region is between 1.9 to 3.8m from A.
 Since the point of intersection of the resultant is 2.91m from A,
 Hence the design is safe.

11. AB & CD are 2 strings hung on a horizontal ceiling at A & B. A weight of 75 N is hung from point B. Calculate the mag of the weight that should be hung from point C, so that the portion BC of the string is horizontal. The point B is 0.6 m from point A & point C is 0.35 m from point D. Also calculate the tension in the portions AB, BC & CD of the string.

FBD at point B.



$$\Sigma H = 0$$

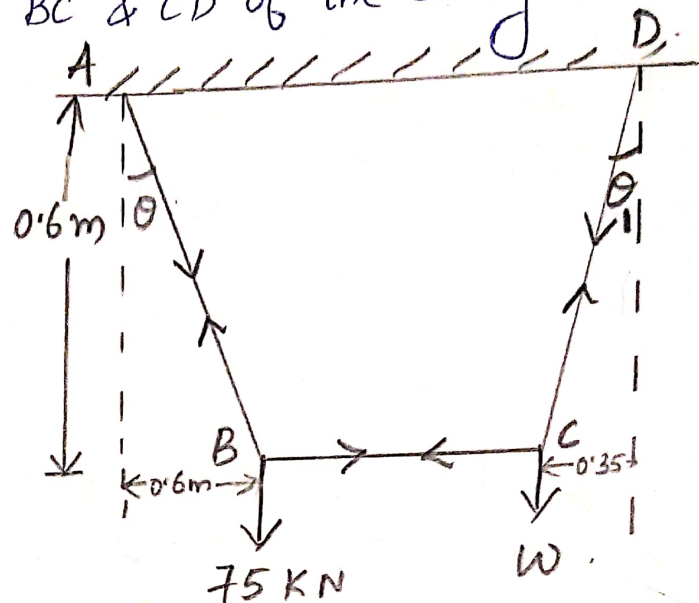
$$T_{BC} - T_{AB} \sin 45^\circ = 0$$

$$\Sigma V = 0$$

$$T_{AB} \cos 45^\circ - 75 = 0$$

$$T_{AB} = \underline{\underline{106.066 \text{ kN}}}$$

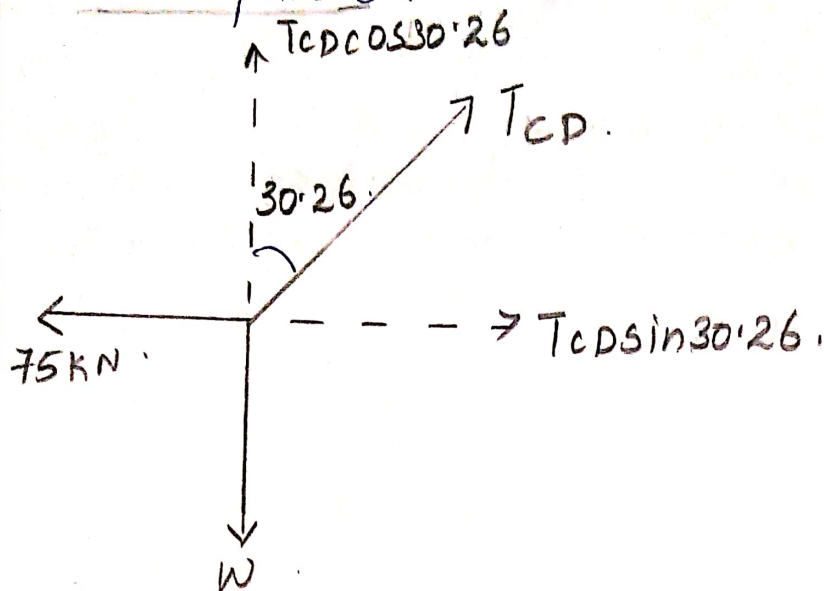
$$T_{BC} = \underline{\underline{75 \text{ kN}}}$$



$$\tan \theta = \frac{0.6}{0.6}$$

$$\theta = 45^\circ$$

FBD at point C.



$$\tan \theta_1 = \frac{0.35}{0.6}$$

$$\theta_1 = \cancel{39.74}^\circ$$

$$30.26$$

$$\sum H = 0$$

$$T_{CD} \sin 30.26 - 75 = 0$$

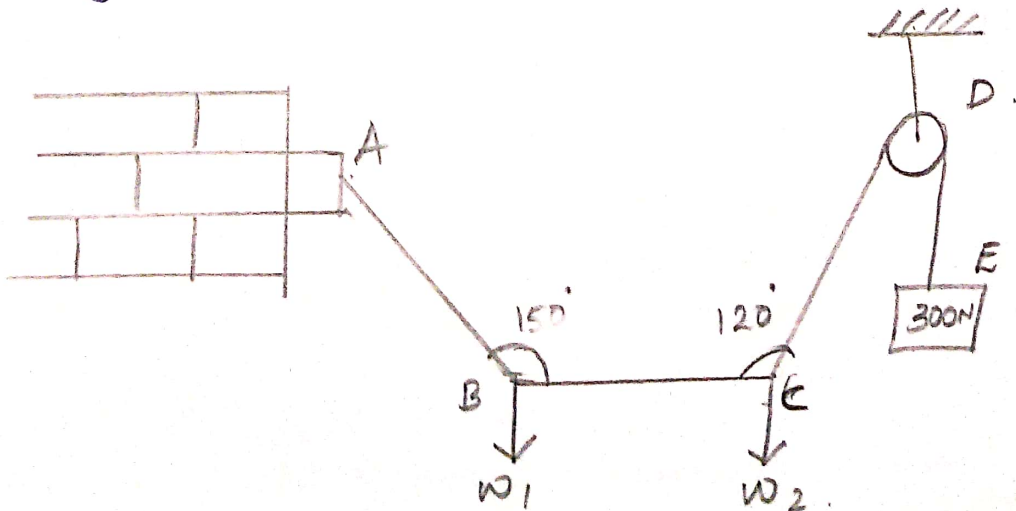
$$T_{CD} = 148.83 \text{ kN}$$

$$\sum V = 0$$

$$T_{CD} \cos 30.26 - W = 0$$

$$W = 128.55 \text{ N}$$

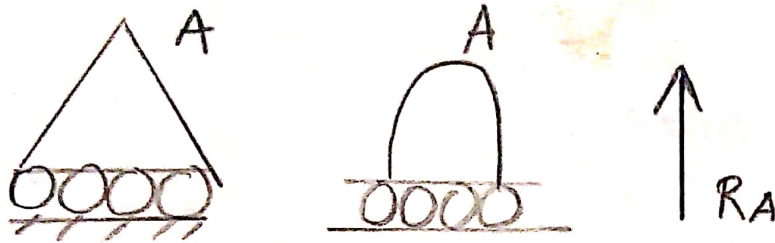
12. A light string ABCDE whose extreme point A is fixed; has weights w_1 & w_2 attached to it at points B & C. It passes around a smooth small pulley carrying a weight of 300N & at the end E. calculate the weights w_1 & w_2 & tension in the string.



MODULE II

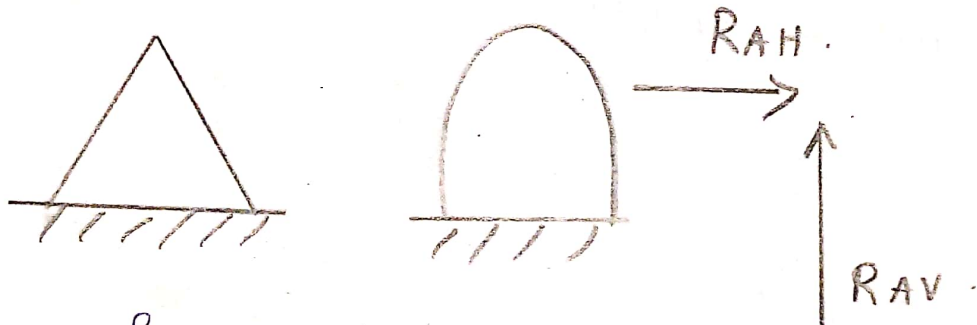
TYPES OF SUPPORT.

1) ROLLER SUPPORT.



1 plane of constraint.
eg:- Skating shoe.

2) HINGED SUPPORT.



2 planes of constraints.
eg:- windows.

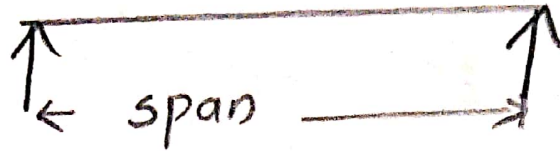
3) FIXED SUPPORT



No movement in any direction.
3 planes of constraints.
eg:- wall.

TYPES OF BEAMS.

1) SIMPLY SUPPORTED BEAM.



2) CANTILEVER BEAM.



3) OVERHANGING BEAM.



4) CONTINUOUS BEAM.



TYPES OF LOADING.

1) CONCENTRATED LOADING.

Load is acting at a single point.

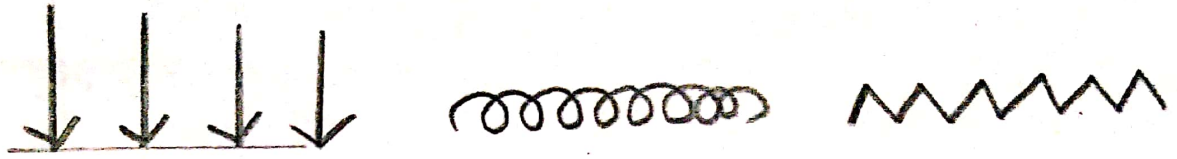
Unit — N & KN.

Point of application is the point itself.

eg:- A man standing on an inclined ladder.



4) UNIFORMLY DISTRIBUTED LOADING.



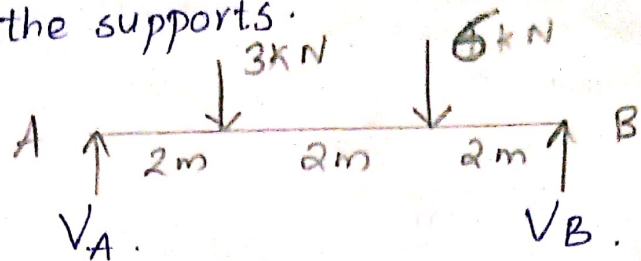
Unit $\rightarrow$ N/m , KN/m .

Point of application = $\frac{\text{Span}}{2}$ from both sides .

Net loading = Span $\times$ load .

1. Determine the reaction at the supports.

Assume the system to be in equilibrium.



$$\sum H = 0$$

$$\sum V = 0$$

$$V_A + V_B - 3 - 6 = 0$$

$$V_A + V_B = 9$$

$$\sum M_A = -3 \times 2 - 6 \times 4 + V_B \times 6 = 0$$

$$-6 - 24 + V_B \times 6 = 0$$

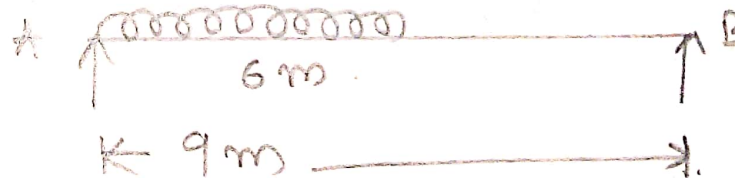
$$6V_B = 30$$

$$V_B = 5 \text{ kN}$$

$$V_A = 9 - 5$$

$$V_A = 4 \text{ kN}$$

2. A simply supported beam AB of length 9m carries a UDL of 10 kN/m for a distance of 6m from the free end. Calculate the reactions at A & B. 10 kN/m



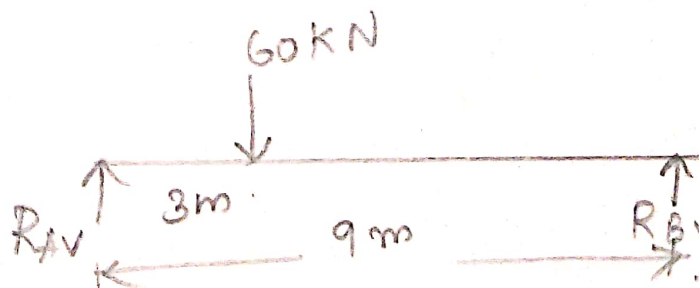
$$R_{AV} + R_{BV} = 60 \text{ kN}$$

$$\sum M_A = 0$$

$$-60 \times 3 + R_{BV} \times 9 = 0$$

$$R_{BV} = \frac{180}{9}$$

$$= 20 \text{ kN}$$



$$R_{AV} = 40 \text{ kN}$$

$$R_{BV} = 20 \text{ kN}$$

3. Find the reactions at the supports.

$$\sum V = 0$$

$$V_A + V_B = 50 + 10 \times 4 + 40$$

$$= 130 \text{ kN}$$

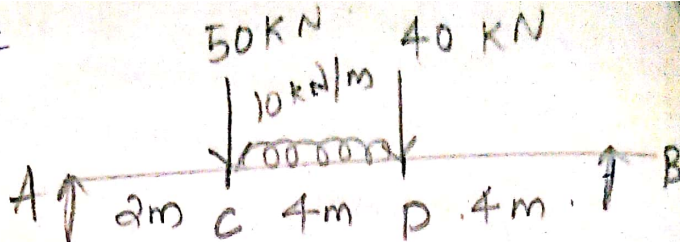
$$\sum M_A = -50 \times 2 - 40 \times 6 - 10 \times 4 \times (2+2) + V_B \times 10$$

$$-100 - 240 - 160 + 10V_B = 0$$

$$10V_B = 500$$

$$V_B = 50 \text{ kN}$$

$$V_A = 80 \text{ kN}$$

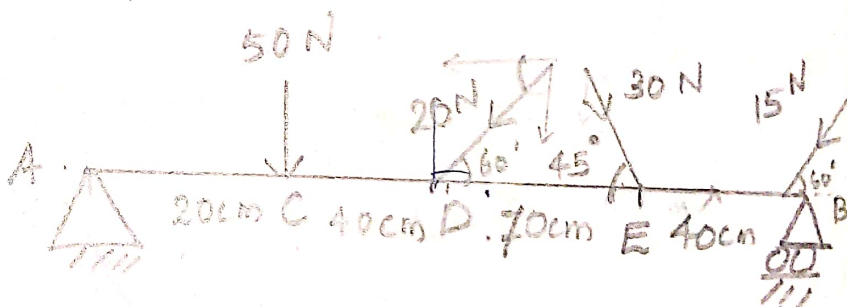


4. Determine the reactions at the supports.

$$\sum H = 0$$

$$H_A - 20 \cos 60^\circ + 30 \cos 45^\circ - 15 \cos 60^\circ = 0$$

$$H_A = -3.71 \text{ kN}$$



$$\sum V = 0$$

$$V_A + V_B - 20 \sin 60^\circ - 30 \sin 45^\circ - 15 \sin 60^\circ = 0$$

$$-50$$

$$V_A + V_B = 101.52 \text{ kN}$$

$$\sum M_A = 0$$

$$-20 \sin 60^\circ \times 60 - 30 \sin 45^\circ \times 100 - 15 \sin 60^\circ \times 170 + V_B \times 170 = 0$$

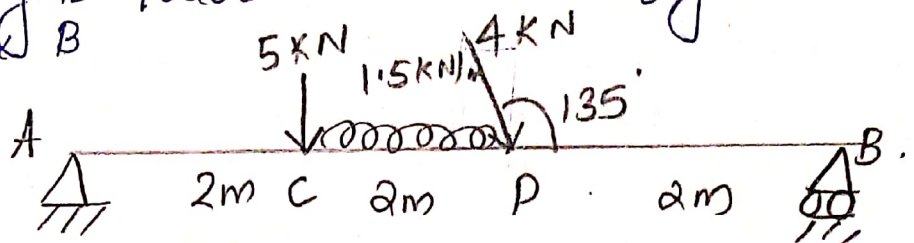
$$-50 \times 20$$

$$V_B = 41.2 \text{ N}$$

$$V_A = 101.52 - 41.2 = 60.32 \text{ N}$$

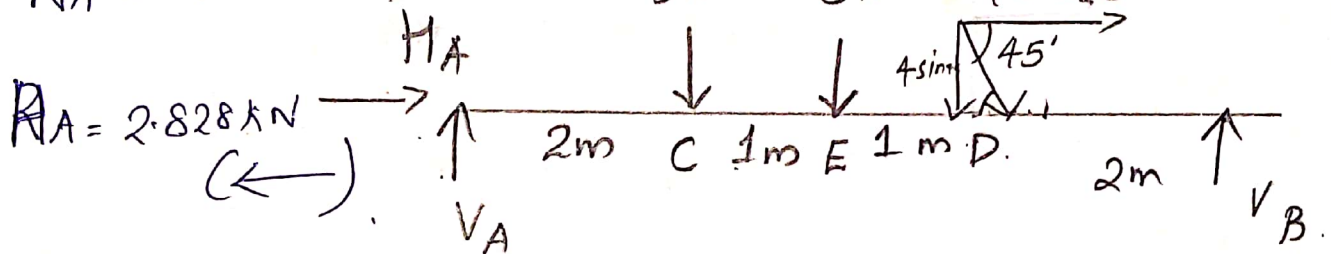
A beam AB 6m long is loaded as shown in fig. Determine the reactions at A & B

$$\Sigma H = 0$$



$$R_A + 4 \cos 45^\circ = 0$$

$$R_A = -2.828 \text{ kN}$$



$$\Sigma V = 0$$

$$-5 - 3 - 4 \sin 45^\circ + V_B + V_A = 0$$

$$V_A + V_B = 10.828 \text{ kN}$$

$$\Sigma M_A = 0$$

$$-5 \times 2 - 3 \times 3 - 4 \sin 45^\circ \times 4 + V_B \times 6 = 0$$

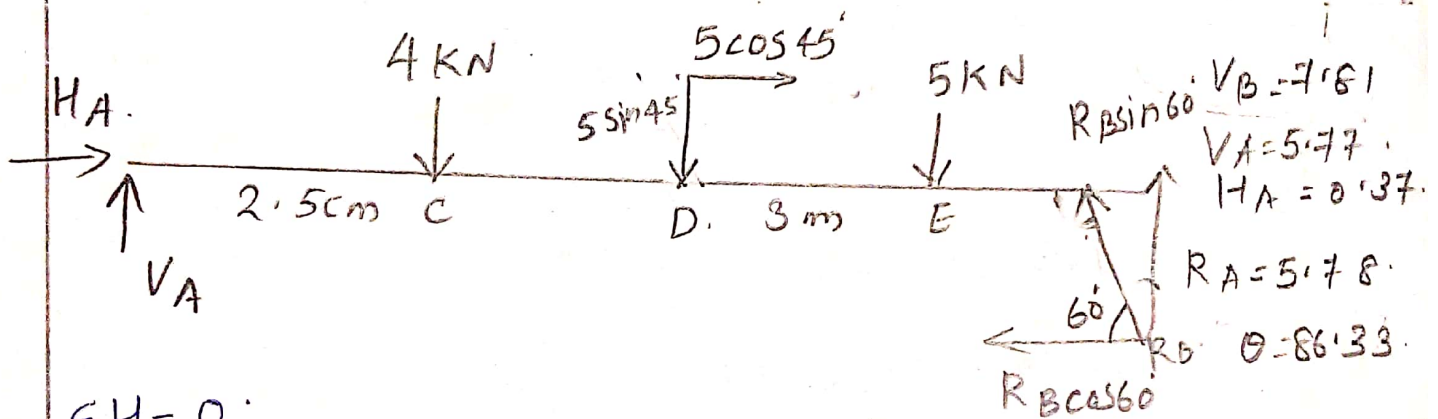
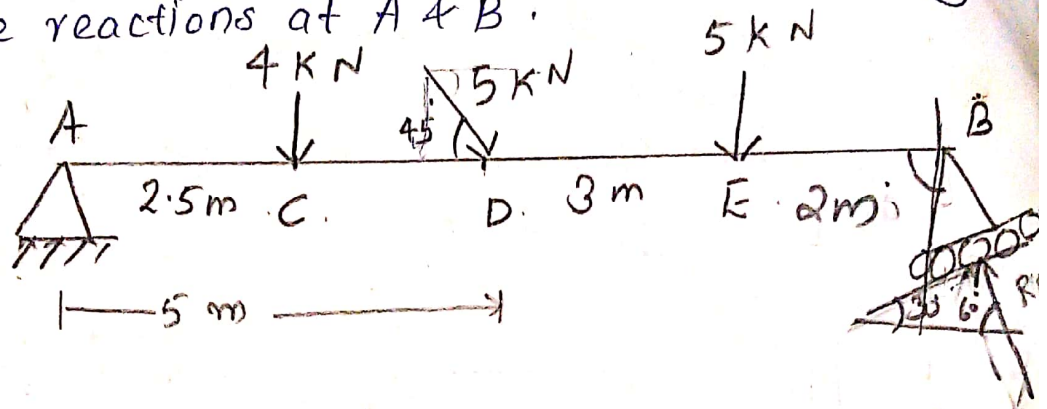
$$6 V_B = 30.313$$

$$V_B = 5.052 \text{ kN}$$

$$V_A = 10.828 \text{ kN} - 5.052 \text{ kN}$$

$$V_A = 5.775 \text{ kN}$$

6. A beam AB 10m long is hinged at A and supported on rollers over a smooth surface inclined at 30° to the horizontal at B. The beam is loaded as shown in Fig. Determine the reactions at A & B.



$$\Sigma H = 0$$

$$H_A + 5 \cos 45^\circ - R_B \cos 60^\circ = 0 \quad \text{--- (1)}$$

$$\Sigma V = 0$$

$$V_A - 4 - 5 \sin 45^\circ + 5 + R_B \sin 60^\circ = 0 \quad \text{--- (2)}$$

$$\Sigma M_A = 0$$

$$4 \times 2.5 + 5 \sin 45^\circ \times 5 - 5 \sin 45^\circ \times 8 + R_B \sin 60^\circ \times 10 = 0$$

$$\boxed{R_B = 7.81 \text{ kN}}$$

Sub R_B in (2)

$$V_A = 4 + 5 \sin 45^\circ + 5 - 7.81 \sin 60^\circ$$

$$\boxed{V_A = 5.77 \text{ kN}}$$

$$H_A = R_B \cos 60^\circ - 5 \cos 45^\circ$$

$$\boxed{H_A = 0.37 \text{ kN}}$$

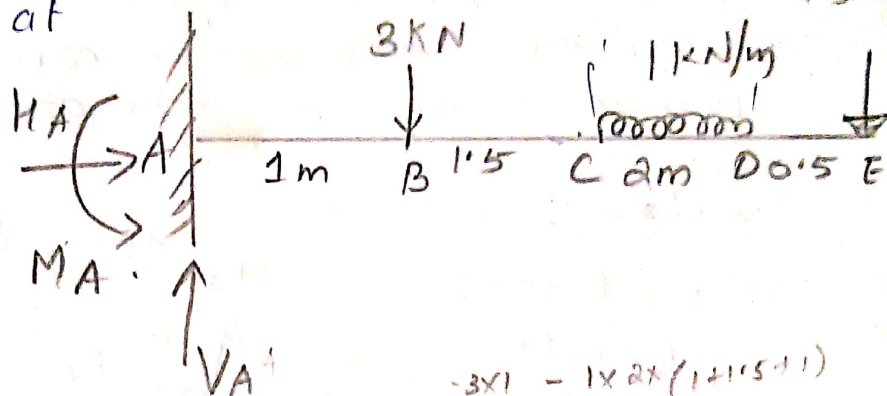
7.

Find the reactions at the support.

$$\sum V = 0$$

$$V_A - 3 - 1 \times 2 - 2.5 = 0$$

$$V_A = 7.5 \text{ kN}$$



$$-3 \times 1 - 1 \times 2 \times (1 + 1.5 + 1) - 2.5 \times$$

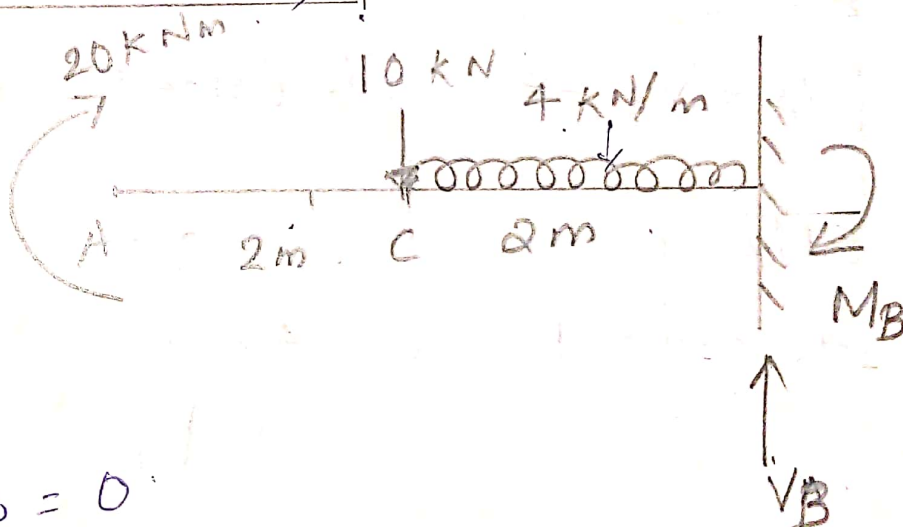
$$\sum M_A = 0$$

$$-3 \times 1 + 1 \times 2 \times (1 + 1.5 + 1) - 2.5 \times 5 + M_A = 0$$

$$M_A = 22.5 \text{ kNm}$$

8.

Find the reactions at the support.



$$\sum V = 0$$

$$-10 - 4 \times 2 + V_B = 0$$

$$V_B = 18 \text{ kN}$$

$$\sum M_B = 0$$

$$-20 + 10 \times 2 + 4 \times 2 \times 1 - M_B = 0$$

$$M_B = 8 \text{ kNm}$$

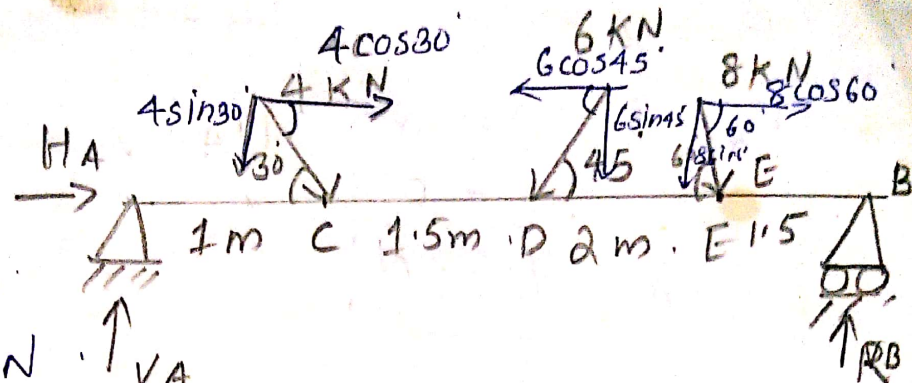
9. A beam AB of span 6m is hinged at A and supported on rollers at end B and carries load as shown in Fig. Determine the reactions at A & B.

$$\sum H = 0$$

$$H_A + 4\cos 30^\circ - 6\cos 45^\circ + 8\cos 60^\circ = 0$$

$$H_A = -3.22 \text{ kN}$$

$$H_A = 3.22 \text{ kN} (\leftarrow)$$



$$V_A = 5.87 \text{ kN}$$

$$V_B = 3.22 \text{ kN}$$

$$R_B = 7.3 \text{ kN}$$

$$\sum V = 0$$

$$V_A + V_B - 4\sin 30^\circ - 6\sin 45^\circ - 8\sin 60^\circ = 0$$

$$\sum M_A = 0$$

$$-4\sin 30^\circ \times 1 - 6\sin 45^\circ \times 2.5 - 8\sin 60^\circ \times 4.5 + R_B \times 6 = 0$$

$$6V_B = 43.783$$

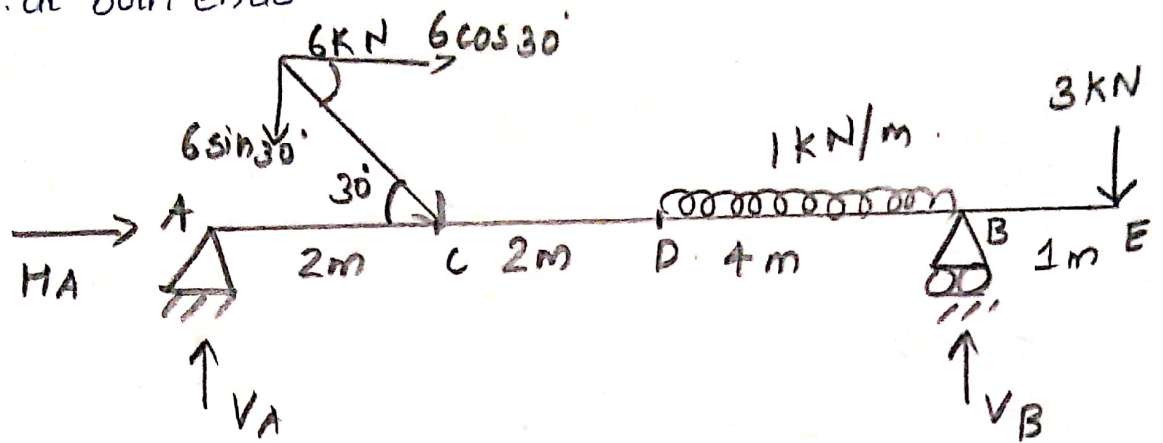
$$V_B = 7.3 \text{ kN}$$

$$V_A = 13.17 - 7.3$$

$$V_A = 5.87 \text{ kN}$$

10.

A beam is loaded as shown in fig. Determine the reactions at both ends.



$$\sum V = 0$$

$$V_A + V_B - 6 \sin 30^\circ - 1 \times 4 - 3 = 0$$

$$V_A + V_B = 10 \text{ kN}$$

$$\sum H = 0$$

$$H_A + 6 \cos 30^\circ = 0$$

$$H_A = -5.196 \text{ kN}$$

$$H_A = 5.196 \text{ kN} (\leftarrow \rightarrow)$$

$$\sum M_A = 0$$

$$-6 \sin 30^\circ \times 2 - 1 \times 4 (4 + 2) + V_B \times 8 - 3 \times 9 = 0$$

$$8V_B - 57 = 0$$

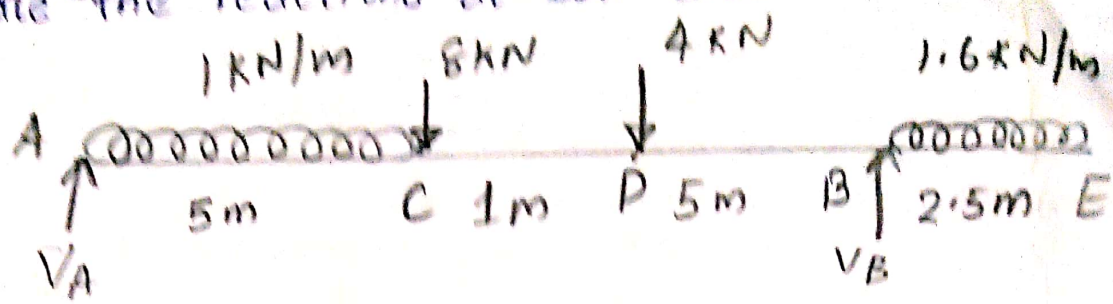
$$V_B = \frac{57}{8} = 7.125 \text{ kN}$$

$$V_B = 7.125 \text{ kN}$$

$$V_A = 10 - 7.125$$

$$V_A = 2.875 \text{ kN}$$

11. An overhanging beam carries the loads as shown in fig. Calculate the reactions at both ends.



$$R_A = 10 \text{ kN}$$

$$R_B = 11 \text{ kN}$$

$$\Sigma V = 0$$

$$V_A + V_B - 1 \times 5 - 8 - 4 - 1.6 \times 2.5 = 0$$

$$V_A + V_B = 21$$

$$\Sigma M_A = 0$$

$$-1 \times 5 \times 2.5 - 8 \times 5 - 4 \times 6 + V_B \times 11 - 1.6 \times 2.5 (11 + 1.25) = 0$$

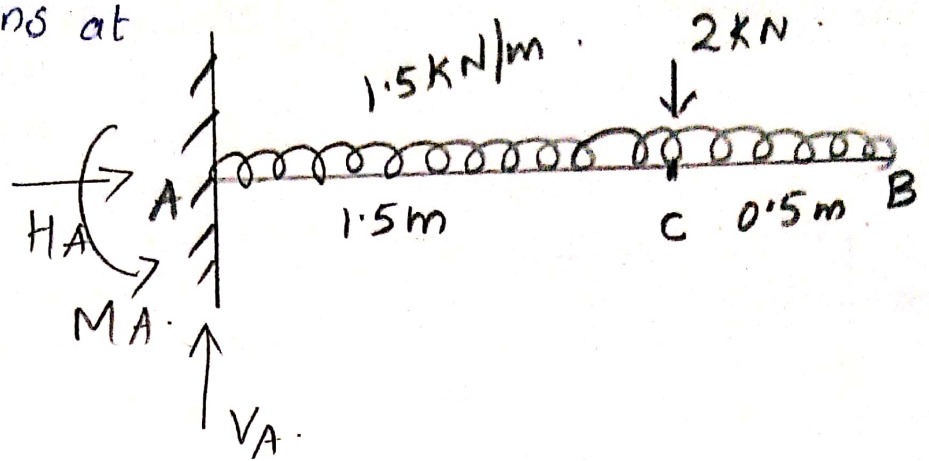
$$11V_B = 125.5$$

$$V_B = 11.41 \text{ kN}$$

$$V_A = 21 - 11.41$$

$$V_A = 9.59 \text{ kN}$$

12. find the reactions at the support.



$$\sum H = 0$$

$$H_A = 0$$

$$\sum V = 0$$

$$V_A - (1.5 \times 2) - 2 = 0$$

$$\boxed{V_A = 5 \text{ kN}}$$

$$\sum M_A = 0$$

$$-(1.5 \times 2 \times 1) - 2 \times 1.5 + M_A = 0$$

$$\boxed{M_A = 6 \text{ kNm}}$$

VECTOR SPACE

TYPES OF VECTORS .

1. Free vector.
A vector that can be placed anywhere in the space & can be moved parallel to itself is known as a free vector.
2. Sliding vector
The vector that can be moved along its line of action, so that its magnitude and direction remains the same. eg:- A force acting on a rigid body can be named as a sliding vector.
3. Bound vector.
A bound vector is a vector with a well defined point of application. It is therefore specified by magnitude, direction & point of application.
4. Equal vector
2 vectors are said to be equal, if they have same magnitude & direction.
5. Equivalent vectors.
2 vectors are said to be equivalent, if they produce the same effect on a rigid body.

COMPONENTS OF A FORCE.

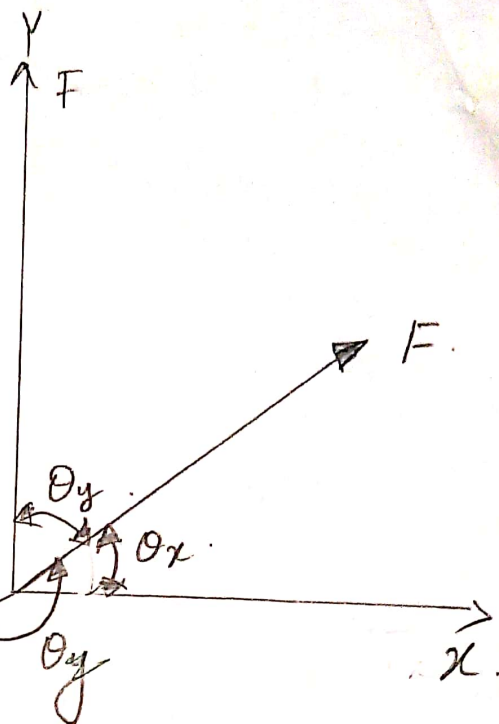
Consider a force F acting at the origin O of the system of rectangular co-ordinates x, y & z . The force F has 3 components.

F_x, F_y , & F_z .

Let θ_x = angle made by the force F with x -axis.

θ_y = angle made by the force F with y -axis.

θ_z = angle made by the force F with z -axis.



$$\cancel{F_x} = \cancel{F \cos \theta_x}, \quad \cos \theta_x = \frac{F_x}{F}$$

$$\cos \theta_y = \frac{F_y}{F}$$

$$\cos \theta_z = \frac{F_z}{F}$$

$$\Rightarrow F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

The magnitude of the resultant force is given by.

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

The cosines of θ_x , θ_y & θ_z are known as the direction cosines of the force F . If l , m , n represents the direction cosines, we have.

$$l = \cos \theta_x.$$

$$m = \cos \theta_y.$$

$$n = \cos \theta_z.$$

$$l^2 + m^2 + n^2 = 1.$$

$$\boxed{\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1}$$

Let the vectors i , j & k represents vectors of unit length in the positive x , y & z directions. Then the force vector.

$$\vec{F} = Fx\hat{i} + Fy\hat{j} + Fz\hat{k}$$

$$\boxed{\vec{F} = F\cos\theta_x\hat{i} + F\cos\theta_y\hat{j} + F\cos\theta_z\hat{k}}$$

1. A force F is acting at the origin in a direction defined by angles $\theta_y = 65^\circ$ and $\theta_z = 40^\circ$. Given that, the x component of the force is -90 kN . Determine the other component & the value of θ_x .

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\cos^2 \theta_x + \cos^2 65 + \cos^2 40 = 1$$

$$\cos^2 \theta_x = \cancel{0.19277} \cdot 0.2346$$

$$\cos \theta_x = \pm 0.484 \quad \cos \theta_x = -0.484$$

$$\theta_x = 61.05, \quad 118.94$$

Given $F_x = -90$, $F \cos \theta_x = -90$.
 But F cannot be negative, $\cos \theta_x$ should be negative
 $\theta_x = 118.94^\circ$, $\Rightarrow F = \frac{-90}{\cos 119} = \underline{185.63}$

$$F_y = F \cos \theta_y \\ = 185.63 \cos 65 = \underline{78.45 \text{ N}}$$

$$F_z = F \cos \theta_z \\ = 185.63 \cos 40 = \underline{142.20 \text{ N}}$$

2. A magnitude of the component of a force along x, y & z axis are -4.5 kN , 5 kN , & -3 kN resp. find the magnitude of the force and its inclination with x, y and z axis.

$$F_x = F \cos \theta_x = -4.5$$

$$F_y = F \cos \theta_y = 5$$

$$F_z = F \cos \theta_z = -3$$

$$\begin{aligned} \text{Magnitude of } F &= \sqrt{F_x^2 + F_y^2 + F_z^2} \\ &= \sqrt{(-4.5)^2 + 5^2 + (-3)^2} \\ &= \sqrt{54.25} = \underline{7.365} \end{aligned}$$

$$F_x = F \cos \theta_x$$

$$\cos \theta_x = \frac{-4.5}{7.36}, \quad \theta_x = \underline{127.69^\circ}$$

$$\cos \theta_y = \frac{5}{7.36}, \quad \theta_y = \underline{47.2^\circ}$$

$$\cos \theta_z = \frac{-3}{7.36}, \quad \theta_z = \underline{114.05^\circ}$$

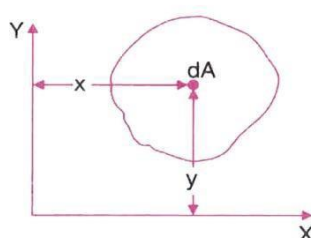
Difference between CENTRE OF GRAVITY and CENTROID :

- The term centre of gravity applies to bodies with mass and weight, and centroid applies to plane areas.
- Centre of gravity of a body is the point through which the resultant gravitational force (weight) acts for any orientation of body whereas centroid is the point in the plane area such that the moment of that area about any axis through that point is zero.

MOMENT OF INERTIA

Moment of inertia is a purely mathematical term which gives a quantitative estimate of the relative distribution of the area with respect to some reference axis.

Consider a thin lamina of area A as shown in figure. Let dA be an elemental area in the plane figure.



Let 'x' be the distance of the elemental area dA from Yaxis and 'y' be the distance of elemental area dA from X axis.

The **moment of inertia** of the area about X-axis , $I_{XX} = \sum \square \square \cdot \square^2$

$dA \cdot y$ is known as the **first moment of area** about Y axis. The first moment of area is used to determine the centroid of the area.

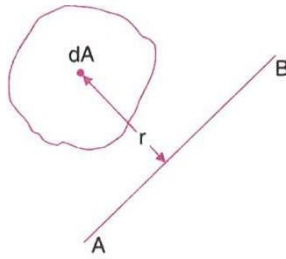
If the moment of area is again multiplied by the perpendicular distance between dA and Yaxis, then the quantity

$dA y^2$ is known as **moment of moment of area** or **second moment of area** or **area moment of inertia** about Yaxis.

Similarly,

$\therefore$ **second moment of area** or **area moment of inertia** of area A about Y axis , $I_{YY} = \sum \square \square \cdot \square^2$

In general, If an elemental area 'dA' is considered and 'r' is the distance of the elemental area from a reference axis AB,



Then the moment of inertia of the entire area about reference axis is given by

$$I_{AB} = \sum \square \square \cdot \square^2 = \int \square \square \square^2$$

If instead of area, the mass (m) of the body is taken into account, then the second moment is known as **second moment of mass** or **mass moment of inertia**.

∴ if m is the mass of the body and x and y are the perpendicular distances of its centre of gravity from Y and X axes,

Mass moment of inertia about Y axis = mx^2

Mass moment of inertia about X axis = my^2

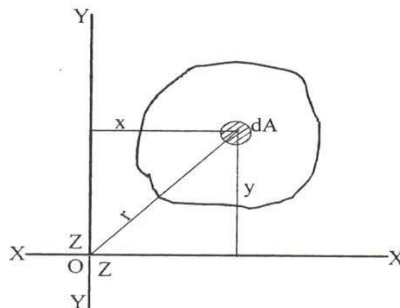
“Hence the product of area(or mass) and the square of the distance of the centroid (or centre of gravity) of the area(or mass) from an axis is known as Moment of Inertia of the area(or mass) about that axis.”

Moment of Inertia is represented by **I**. Moment of inertia about X-axis is represented by **I_{xx}** and about Y-axis is represented as **I_{yy}** .

If **G** is the centroid of the body, the axis passing through the centroid of the body is known as centroidal axis and the moment of inertia about the centroidal axis is given by **I_G** .

Since it is a term obtained by multiplying area by the square of the distance, its unit in SI is given as m^4 .

POLAR MOMENT OF INERTIA

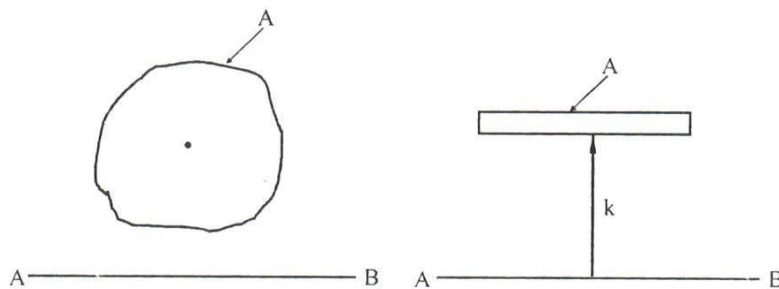


Moment of inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia. It is denoted by J or I_{ZZ} . Thus the moment of inertia about an axis perpendicular to the plane of the area at O in figure is called polar moment of inertia at O and is given by,

$$I_{ZZ} = \sum \square \square . \square^2$$

RADIUS OF GYRATION

Consider an area which has a moment of inertia I with respect to reference axis AB.



Let us assume that this area is compressed to a thin strip of negligible width parallel to axis AB. For this strip to have the same moment of inertia I with respect to same reference axis AB, the strip should be placed at a distance “ k ” from the axis AB such that $I = Ak^2$, where $k = \sqrt{I/A}$ is called the radius of gyration.

Radius of gyration of a body or a given area about an axis is a distance such that its square multiplied by the area gives moment of inertia of the area about the given axis. It is a mathematical term given by the relation

$$k = \sqrt{I/A}, \quad \text{where } k = \text{radius of gyration}$$

I = Moment of Inertia

A = Cross sectional area.

Suffixes with the moment of inertia also accompany the term radius of gyration k .

ie $k_{XX} = \sqrt{I_{XX}/A}$

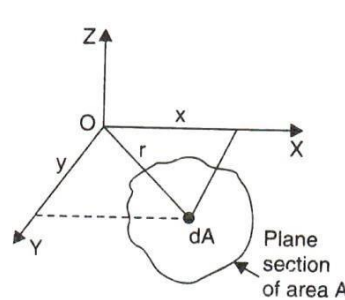
$$k_{YY} = \sqrt{I_{YY}/A}$$

PERPENDICULAR AXIS THEOREM

“The moment of inertia of an area about an axis perpendicular to its plane at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point lying in the plane of the area”

If I_{XX} and I_{YY} be the moment of inertia of a plane area about two mutually perpendicular axis X-X and Y-Y in the plane of the area, then the moment of inertia of the area I_{ZZ} about the axis Z-Z is given by

$I_{ZZ} = I_{XX} + I_{YY}$ (I_{ZZ} is also known as polar moment of inertia).



Proof:

Let A plane area A is lying in plane X-Y is shown in figure.

Let x = distance of dA from Y axis

y = distance of dA from X axis

R = distance of dA from Z axis.

Then $r^2 = x^2 + y^2$

Now moment of inertia of dA about X-axis = $dA \times y^2$

moment of inertia of total area A about X-axis = $\sum dA \times y^2$

moment of inertia of entire area A about Y-axis = $\sum dA \times x^2$

moment of inertia of total area A about Z-axis = $\sum dA \times r^2$

$$\sum dA \times r^2 = \sum dA (x^2 + y^2)$$

$$= \sum dA \times x^2 + \sum dA \times y^2$$

$$= I_{XX} + I_{YY}$$

$$I_{ZZ} = I_{XX} + I_{YY}$$

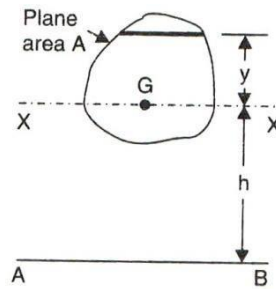
PARALLEL AXIS THEOREM

“if the moment of inertia of a plane area about an axis in the plane of an area through the C.G of the plane area be represented by I_G , then the moment of inertia of the given plane area about a parallel axis AB in the plane of area at a distance ‘ h ’ from the C.G is given by

$$I_{AB} = I_G + Ah^2$$

Proof:

Consider an elemental parallel strip dA at a distance y from centroidal axis,



$$\text{Then } I_{AB} = \sum (y+h)^2 \cdot dA$$

$$= \sum (y^2 + 2yh + h^2) \cdot dA$$

$$= \sum (y^2) dA + \sum (2yh) dA + \sum (h^2) dA$$

$$\text{Here, 1. } \sum (y^2) dA = I_G,$$

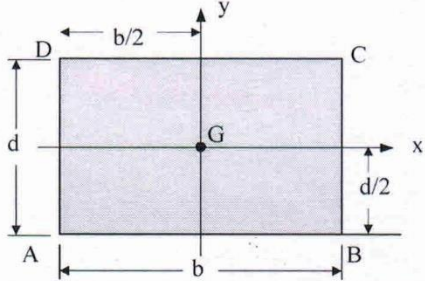
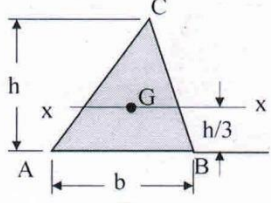
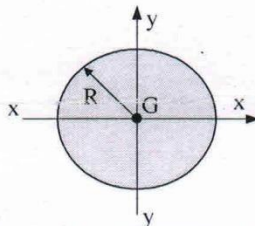
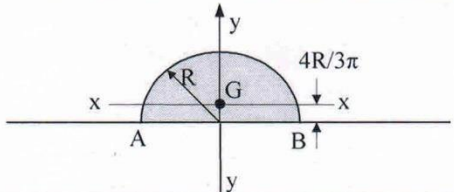
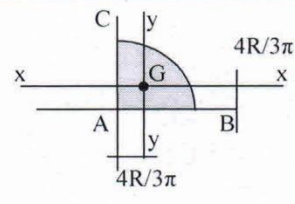
2. $\sum (2yh) dA = 2h \cdot \sum y \cdot dA$, where $\sum y \cdot dA$ is the moment of the total area about X-X axis, which is equal to zero because X-X is the centroidal axis.

$$3. \sum (h^2) dA = h^2 \cdot \sum dA = h^2 \cdot A$$

$$\text{Therefore } I_{AB} = I_G + A h^2$$

Moment of Inertia Formulas

Note: In the table below, I_{Gxx} and I_{Gyy} indicates the moment of inertia is taken about axes that passes through the centroid, denoted as 'G'. I_{AB} denotes the moment of inertia about base AB.

Rectangle: $I_{Gxx} = \frac{1}{12}bd^3 \quad I_{AB} = \frac{1}{3}bd^3$ $I_{Gyy} = \frac{1}{12}db^3 \quad I_{BC} = \frac{1}{3}db^3$ $Area = bd$	
Triangle: $I_{Gxx} = \frac{1}{36}bh^3 \quad I_{AB} = \frac{1}{12}bh^3$ $Area = \frac{1}{2}bh$	
Circle: $I_{Gxx} = I_{Gyy} = \frac{1}{4}\pi R^4 = \frac{\pi D^4}{64}$ $Area = \pi R^2$	
Semi-circle: $I_{Gxx} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4 = 0.11R^4 \quad I_{Gyy} = \frac{1}{8}\pi R^4$ $I_{AB} = \frac{1}{8}\pi R^4 \quad Area = \frac{\pi R^2}{2}$	
Quarter Circle: $I_{Gxx} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)R^4 = 0.055R^4$ $I_{AB} = \frac{1}{16}\pi R^4 \quad Area = \frac{\pi R^2}{4}$	

FRICTION

- ★ When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called the force of friction and is always acts in the direction opposite to the direction of motion.
- ★ Friction acts parallel to the surface of contact and depends upon the nature of surface of contact.
- ▲ The property of the bodies by virtue of which a force is exerted by a stationary body on the moving body to resist the motion of the moving body is called FRICTION.

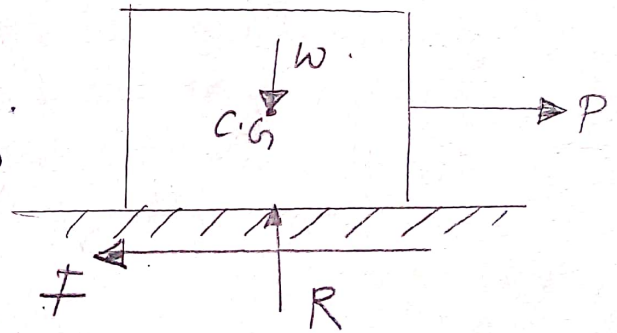
Consider a solid body placed on a horizontal plane surface as shown in the fig.

Let W = weight of body acting through C.G. downward.

R = Normal reaction of body acting through C.G. upward

P = Force acting on the body through C.G. and parallel to the horizontal surface.

If P is small, the body will not move as the force of friction acting on the body in the direction opposite to P will be more than P . But if the magnitude of P goes on increasing, a stage comes, when the solid body is on the point of motion. At this stage, the force of friction acting on the body is called the limiting force of friction and is denoted by F .



Resolving the forces on the body vertically and horizontally.

$$R = W$$

$$P = F$$

If the mag of P is further increased, the body will start moving. The force of friction acting on the body when the

1. Co-efficient of Friction (μ).

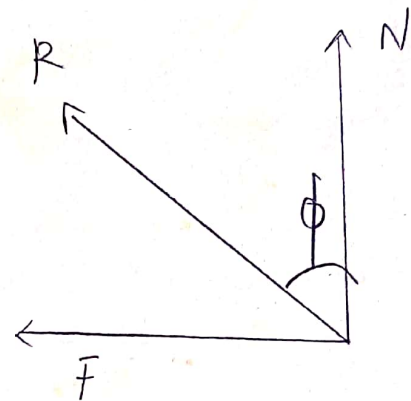
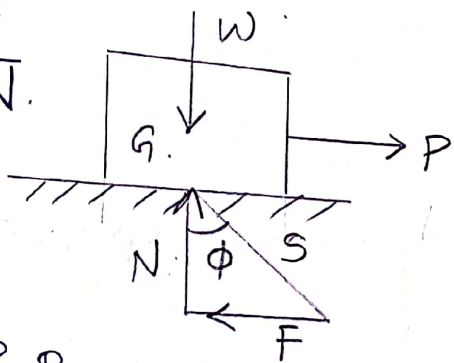
It is defined as the ratio of limiting force of friction (F_{lim}) to the normal reaction (N) between 2 bodies

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F_{lim}}{N}$$

2. Angle of Friction (ϕ).

It is defined as the angle made by the resultant of the normal reaction (N) and limiting force of friction (F_{lim}) with the normal reaction (N)

$$\tan \phi = \frac{F}{N}$$



3. Angle of Repose.

Consider the block of weight w resting on an inclined plane which makes an angle θ with the horizontal. When θ is small, the block will rest on the plane. If θ is increased gradually, a stage is reached at which the body starts sliding down the plane. The angle θ for which the motion is impending is called the Angle of Repose.

Thus, the max inclination of the plane on which a body free from external forces can repose is called Angle of Repose.

4. Cone of Friction.

It is defined as the right circular cone with vertex at the point of contact of the 2 bodies, axis in the direction of normal reaction (N) and semi-vertical angle equal to angle of friction ϕ .

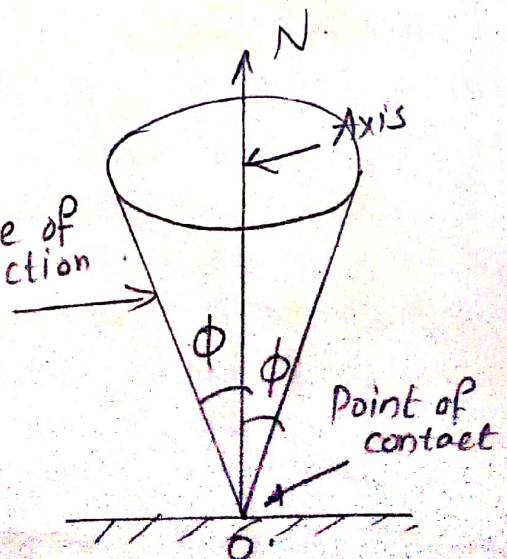


Fig shows the cone of friction in which,

O = Point of contact between 2 bodies.

N = Normal reaction and also axis of the cone of friction.

ϕ = Angle of friction.

LAWS OF SOLID OR COULOMB FRICTION

1. The force of friction acts in the opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. When the surface is on the point of motion, the force of friction is max and this max frictional force is called limiting frictional force.
4. The limiting frictional force bears a constant ratio to the normal reaction between 2 surfaces.
5. The limiting frictional force does not depend upon the shape and areas of surfaces in contact.

1. A body of weight 70 N is placed on a rough horizontal plane. To just move the body on the horizontal plane, a push of 20 N inclined at 20° to the horizontal plane is required. Find the co-efficient of friction.

$\Sigma H = 0$; Resolving the forces along the plane.

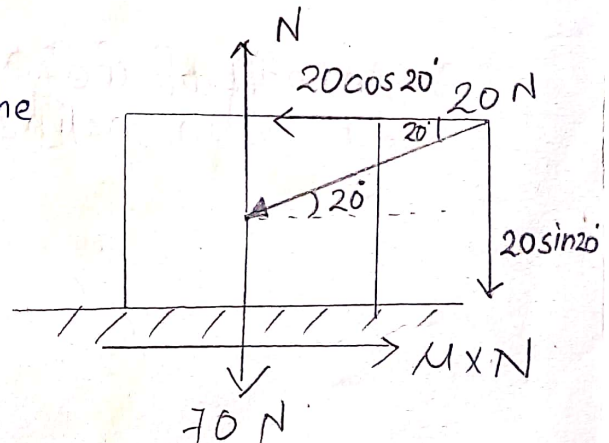
$$\mu N = 20 \cos 20^\circ = 18.79 \text{ N} \quad \text{--- (1)}$$

Resolving the forces ~~para~~ normal to the plane.

$$\begin{aligned} N &= 70 + 20 \sin 20^\circ \\ &= 76.84 \text{ N} \quad \text{--- (2)} \end{aligned}$$

Sub (2) in (1).

$$\mu = 18.79 / 76.84 = \boxed{0.244}$$

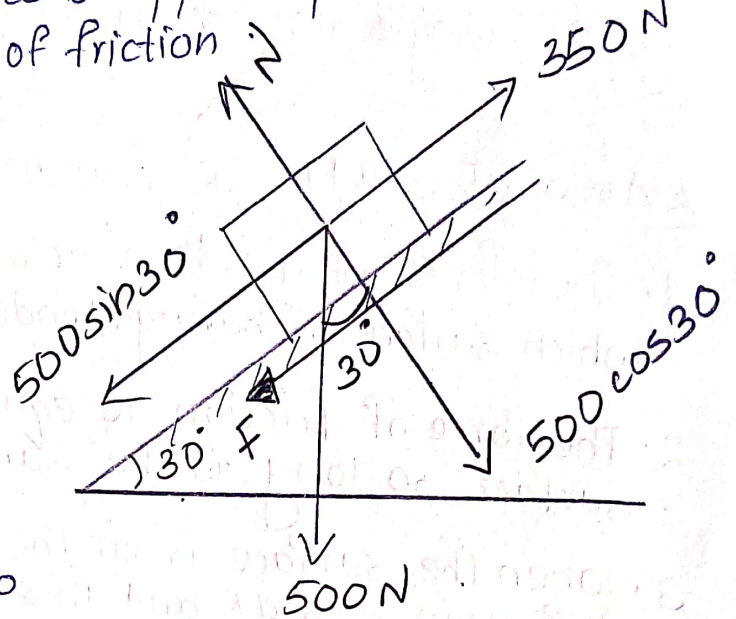


PROBLEMS ON INCLINED PLANE

1. A body of weight 500 N is pulled up an inclined plane by a force of 350 N . The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the co-efficient of friction.

Resolving the forces along the plane.

$$350 = 500 \sin 30^\circ + F \quad \text{--- (1)}$$



Resolving the forces $\perp$ to the plane.

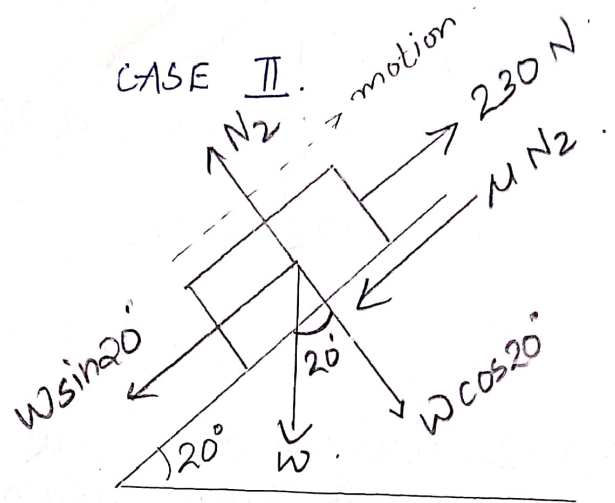
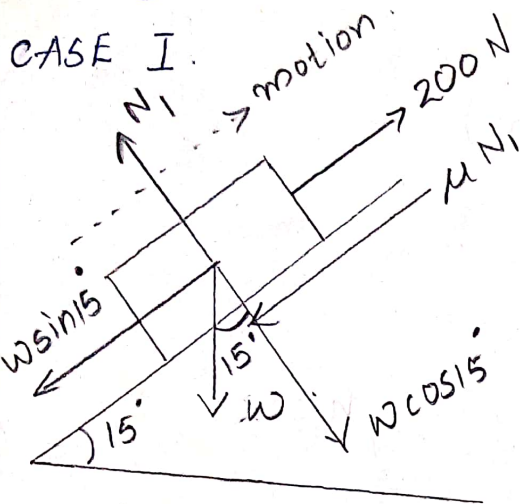
$$N = 500 \cos 30^\circ \quad \text{--- (2)}$$
$$= \underline{\underline{433\text{ N}}}$$

sub (2) in (1)

$$\mu \times 433 = 350 - 500 \sin 30^\circ$$
$$= \underline{\underline{0.23}}$$

2. An effort of 200 N is required just to move a certain body up an inclined

An effort of 200N is required just to move a certain body up an inclined plane of angle 15° . The force is acting parallel to the plane. If the angle of inclination is made 20° , the effort required again parallel to the plane, is found to be 230N. Find the weight of the body and co-efficient of friction.



Resolving the forces parallel to the plane.

$$200 = W \sin 15^\circ + \mu N_1 \quad \text{--- (1)}$$

$$230 = \mu N_2 + W \sin 20^\circ \quad \text{--- (3)}$$

Resolving the forces perpendicular to the plane.

$$N_1 = W \cos 15^\circ \quad \text{--- (2)}$$

$$N_2 = W \cos 20^\circ \quad \text{--- (4)}$$

sub (4) in (3).

Sub (2) in (1).

$$230 = \mu (W \cos 20^\circ) + W \sin 20^\circ$$

$$200 = W \sin 15^\circ + \mu (W \cos 15^\circ)$$

$$200 = W (\sin 15^\circ + \mu \cos 15^\circ) \quad \text{--- (A)}$$

$$230 = W (\sin 20^\circ + \mu \cos 20^\circ) \quad \text{--- (B)}$$

$A \div B$.

$$\frac{200}{230} = \frac{W (\sin 15^\circ + \mu \cos 15^\circ)}{W (\sin 20^\circ + \mu \cos 20^\circ)}$$

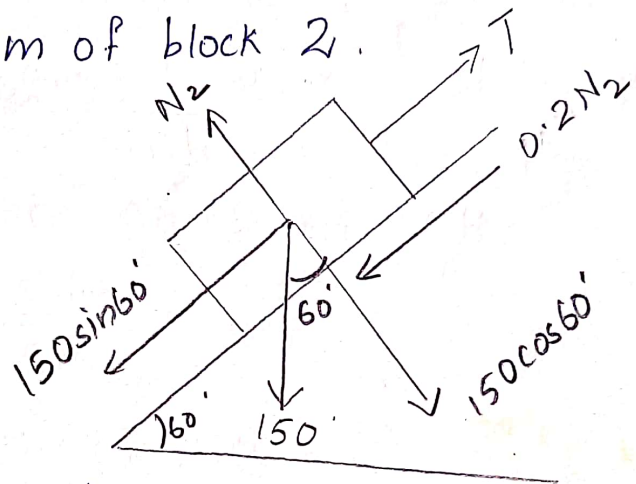
Resolving the forces parallel to the plane.

$$P \cos 60^\circ = 0.2 N_1 + T \quad \text{--- (1)}$$

Resolving the forces perpendicular to the plane.

$$N_1 + P \sin 60^\circ = 100 \quad \text{--- (2)}$$

Consider the equilibrium of block 2.



Resolving the forces along the plane.

$$T = 150 \sin 60^\circ + 0.2 N_2 \quad \text{--- (3)}$$

Resolving the forces perpendicular to the plane.

$$N_2 = 150 \cos 60^\circ \quad \text{--- (4)}$$

$$\boxed{N_2 = 75}$$

Sub (4) in (3).

$$T = 150 \sin 60^\circ + 0.2 \times 75$$

$$\boxed{T = 144.9}$$

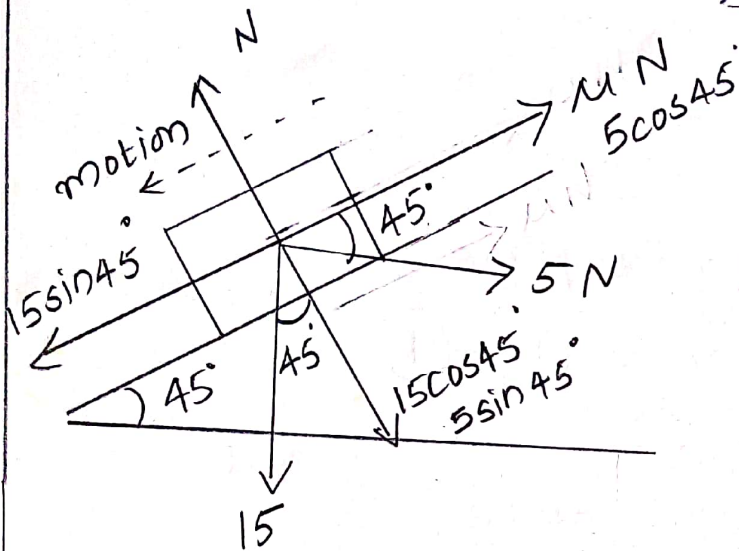
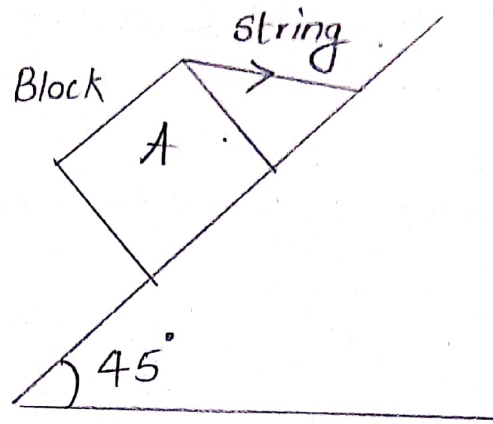
Sub T in (1).

$$144.9 = P \cos 60^\circ - 0.2 N_1$$

$$100 = P \sin 60^\circ + N_1$$

Sub N_1 in (1).

If the string is removed, the block will slide down the plane.



Resolving the forces along the plane.

$$15 \sin 45^\circ = \mu N + 5 \cos 45^\circ \quad \text{--- (1)}$$

Resolving the forces perpendicular to the plane.

$$N = 15 \cos 45^\circ + 5 \sin 45^\circ$$

$$N = 14.14 \text{ N} \quad \text{--- (2)}$$

Sub (2) in (1)

$$\mu = \frac{15 \sin 45^\circ - 5 \cos 45^\circ}{14.14}$$

$$\boxed{\mu = 0.5}$$

$$\boxed{N = 14.14 \text{ N}}$$

$$F = \mu N = 0.5 \times 14.14, F = \boxed{7.07 \text{ N}}$$

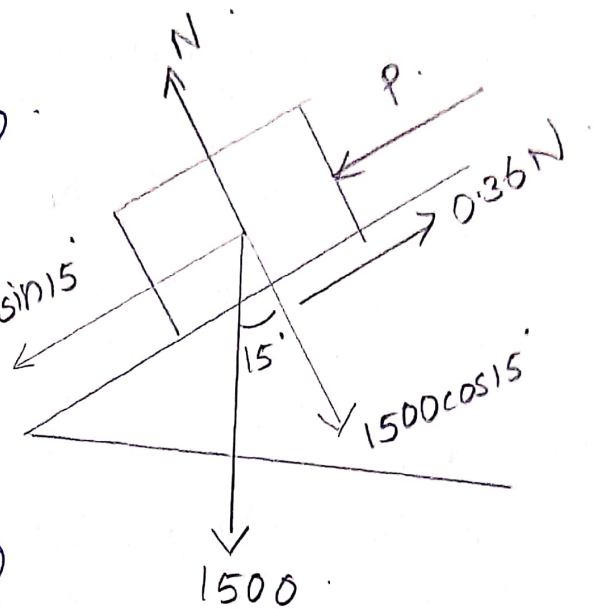
Resolving the forces along the plane.

$$0.36 N - P - 1500 \sin 15^\circ = 0 \quad \text{--- (3)}$$

Resolving the forces perpendicular along the plane.

$$N - 1500 \cos 15^\circ = 0$$

$$N = 1448.88 \text{ N} \quad \text{--- (4)}$$

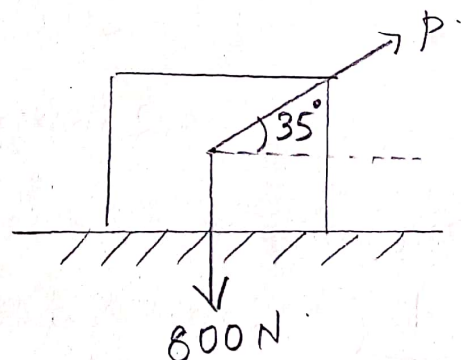
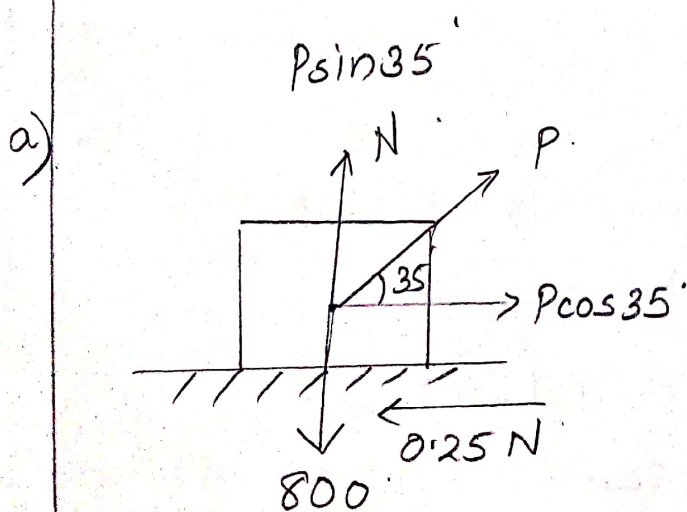


sub (4) in (3)

$$0.36 \times 1448.88 - 1500 \sin 15^\circ = P$$

$$P = \underline{\underline{133.36 \text{ N}}}$$

6. A block having a weight of 800 N lying on a horizontal floor is just dragged by a force inclined at 35° to the floor. Find
- value of P .
 - inclination with horizontal line so that P is min.
- Find the value of $P_{\min}$. $\mu = 0.25$.



Resolving the forces along the plane.

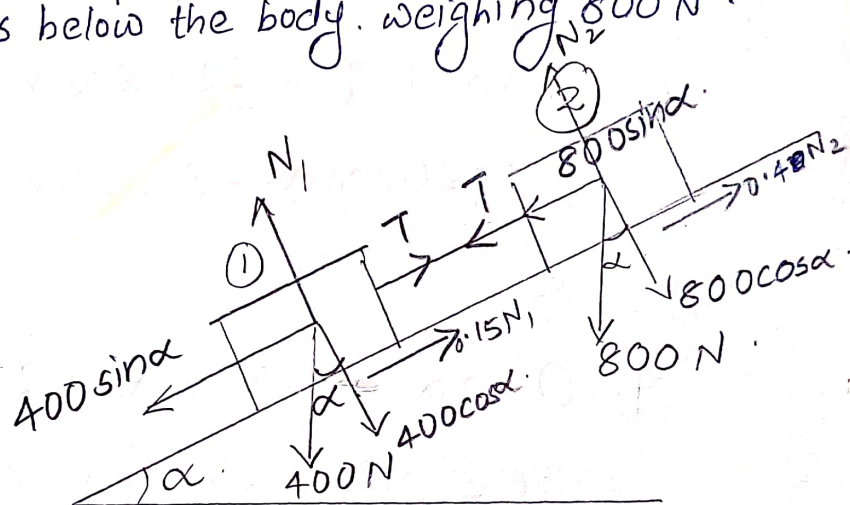
$$P \cos 35^\circ - 0.25 N = 0$$

$$P = 200$$

$$0.25 \sin 14.03 + \cos 14.03.$$

$$= \underline{\underline{194.03 \text{ N}}}$$

7. A cord connects 2 bodies of weights 300 N and 800 N. The 2 bodies are placed on an inclined plane and cord is parallel to inclined plane. The co-efficients of friction for the weight of 400 N is 0.15 and that for 800 N is 0.4. Determine the inclination of the plane to the horizontal and the tension in the cord when the motion is about to take place, down the inclined plane. The body weighing 400 N is below the body weighing 800 N.



FBD of block 1.

Resolving the forces along the plane.

$$T + 0.15 N_1 - 400 \sin \alpha = 0 \quad \text{--- (1)}$$

Resolving the forces $\perp$ plane.

$$N_1 = 400 \cos \alpha \quad \text{--- (2)}$$

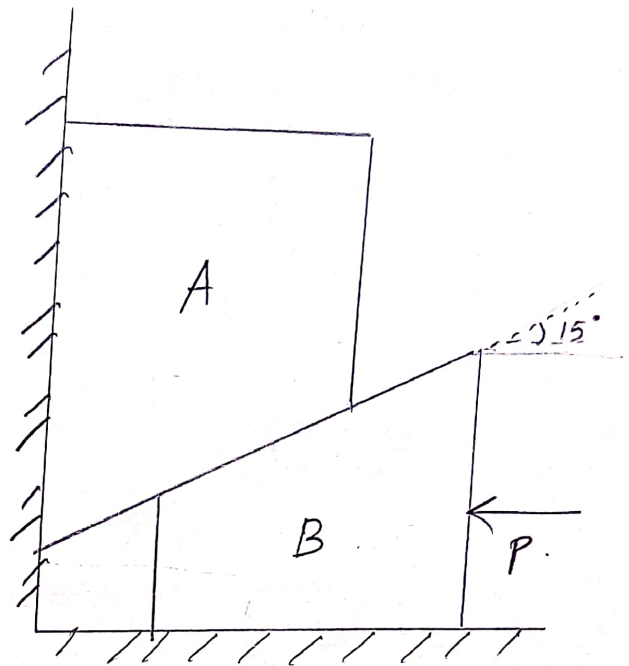
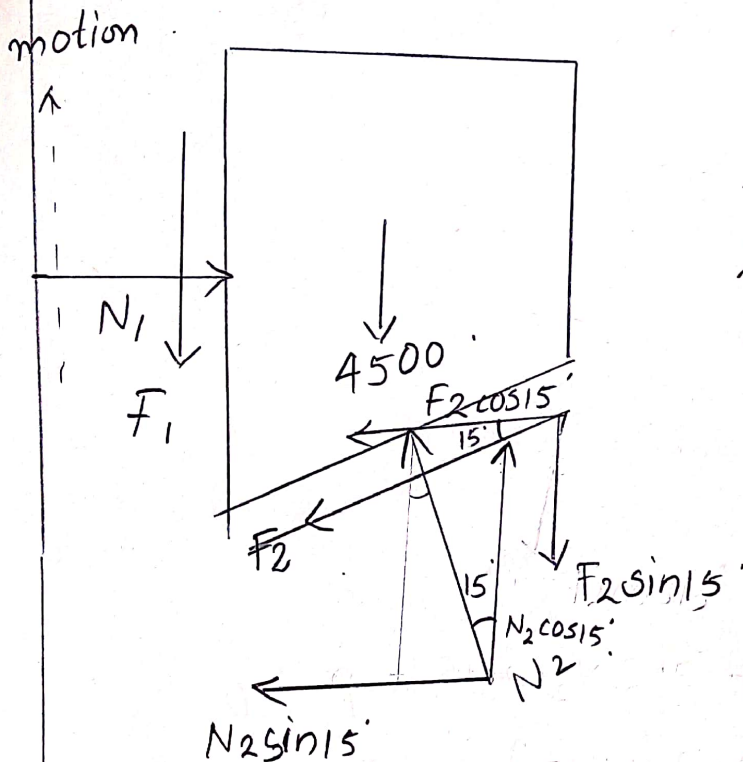
$$T + 0.15 \times 400 \cos \alpha - 400 \sin \alpha = 0.$$

$$T = 400 \sin \alpha - 60 \cos \alpha \quad \text{--- A.}$$

WEDGE FRICTION

Determine the horizontal force 'P' required for wedge B to raise the block 'A' of weight 4500 N, if the coefficient of friction on all surfaces is 0.2.

FBD of block A.



$$\Sigma H = 0$$

$$N_1 - N_2 \sin 15^\circ - F_2 \cos 15^\circ = 0$$

$$N_1 - N_2 \sin 15^\circ - 0.2 N_2 \cos 15^\circ = 0$$

$$N_1 - 0.452 N_2 = 0 \quad \text{--- (1)}$$

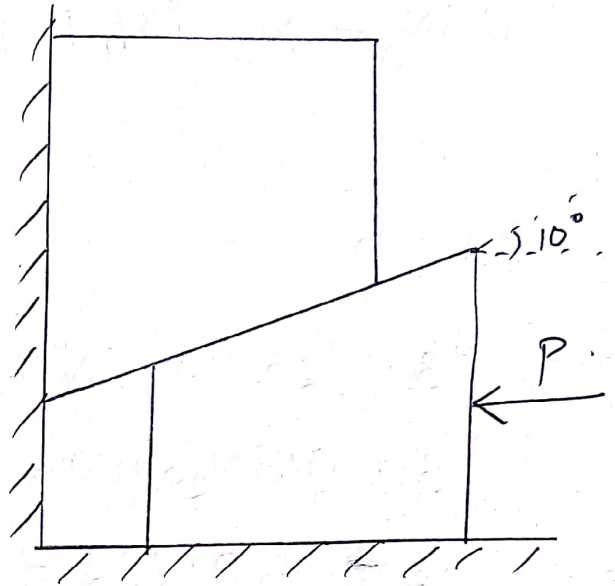
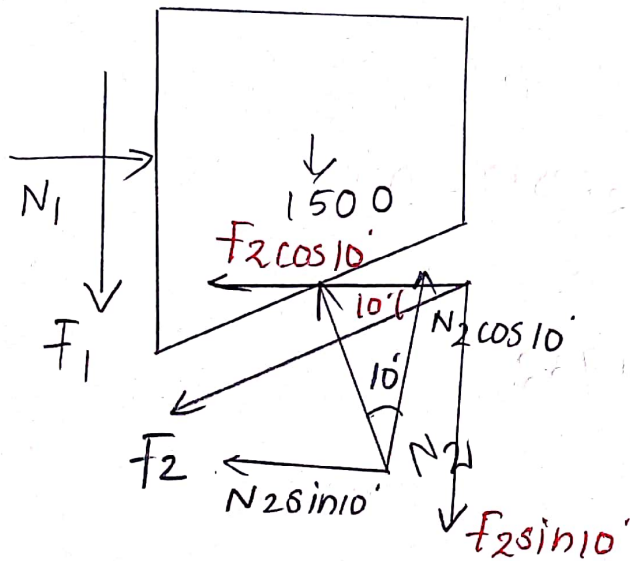
$$\Sigma V = 0$$

$$N_2 \cos 15^\circ - 0.2 N_2 \sin 15^\circ - 4500 - 0.2 N_1 = 0$$

$$-0.2 N_1 + 0.914 N_2 = 4500 \quad \text{--- (2)}$$

A block is lying on a 10° wedge on a horizontal floor & leaning against a vertical wall & weighing 1500 N is to be raised by applying a horizontal force P to the wedge. Assuming coefficient of friction between all the surfaces in contact to be 0.3 , determine the minimum horizontal force to be applied to raise the block.

FBD of block.



$$\sum H = 0$$

$$N_1 - N_2 \sin 10^\circ - F_2 \cos 10^\circ = 0$$

$$N_1 - N_2 \sin 10^\circ - 0.3 N_2 \cos 10^\circ = 0$$

$$N_1 - N_2 (0.469) = 0 \quad \text{--- (1)}$$

$$\sum V = 0$$

$$-F_1 + F_2 \sin 10^\circ + N_2 \cos 10^\circ - 1500 = 0$$

$$-0.3 N_1 - 0.3 N_2 \sin 10^\circ + N_2 \cos 10^\circ - 1500 = 0$$

$$-0.3 N_1 + 0.93 N_2 = 1500 \quad \text{--- (2)}$$

$$N_1 = 889.043\text{ N}$$

$$N_2 = 1895.6\text{ N}$$

MODULE V

NEWTON'S 2ND LAW OF MOTION

Let a body of mass 'm' is moving with a velocity 'u' along a straight line. It is acted upon a force F and the velocity of the body becomes 'v' in time t .

u - initial velocity of the body.

v - final velocity of the body.

m - mass of the body.

a - uniform linear acceleration.

F - Force acting on the body, which changes the velocity u to v in time t .

t - time in sec to change the velocity from u to v

Initial momentum of the body = mu .

Final momentum of the body = mv

$$\begin{aligned}\therefore \text{Change of momentum} &= \text{Final momentum} - \text{Initial momentum} \\ &= mv - mu \\ &= m(v - u).\end{aligned}$$

$$\begin{aligned}\text{Rate of change of momentum} &= \frac{\text{Change of momentum}}{\text{time}} \\ &= \frac{m(v - u)}{t}\end{aligned}$$

$$\frac{v - u}{t} = a$$

$$= ma.$$

But according to Newton's 2nd Law of motion, the rate of change of momentum is directly proportional to the external force acting on the body.

$$F \propto ma.$$

$$F = kma.$$

K - constant of proportionality.

1. A force of 100N acts on a body having a mass of 4 kg. If the initial velocity of the body is 5m/s, determine:
 (i) acceleration produced in the direction of force.
 (ii) distance moved by the body in 10s.

$$F = 100 \text{ N}$$

$$m = 4 \text{ kg}$$

$$t = 10 \text{ s}$$

$$u = 5 \text{ m/s}$$

$$1) a = \frac{F}{m} = \frac{100}{4} = \underline{\underline{25 \text{ m/s}^2}}$$

$$2) s = ut + \frac{1}{2}at^2$$

$$= 5 \times 10 + \frac{1}{2} \times 25 \times 10^2$$

$$= \underline{\underline{1300 \text{ m}}}$$

2. A body of mass 15 kg falls on the ground from a height of 19.6 m. The body penetrates into the ground. Find the distance through which the body will penetrate into the ground, if the resistance by the ground to penetration is constant and equal to 4900 N. $g = 9.8 \text{ m/s}^2$.

$$m = 15 \text{ kg}, \text{ Resistance to penetration}$$

$$h = 19.6 \text{ m}$$

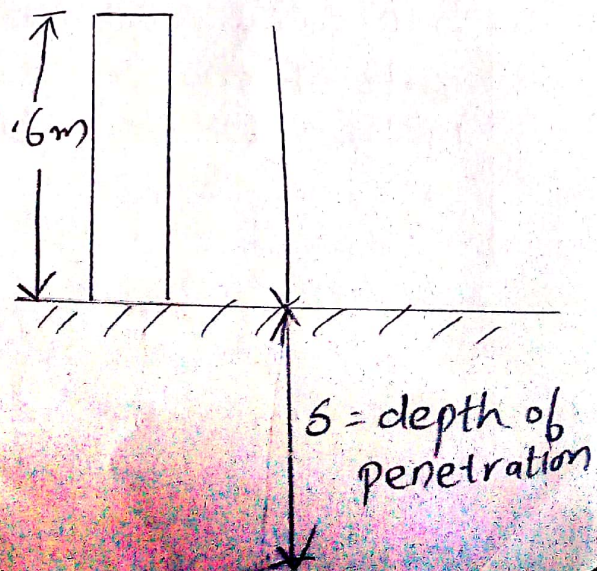
$$u = 0$$

$$F_2 = 4900 \text{ N}$$

$$v^2 - u^2 = 2gh$$

$$v^2 = 2 \times 9.8 \times 19.6$$

$$v = \underline{\underline{19.6 \text{ m/s}}}$$



Velocity with which the body strikes the ground $= 19.6 \text{ m/s}$.

$$\begin{aligned}\text{Force acting downwards} &= \text{weight of the body} \\ \text{weight of body} &= m g \\ &= 15 \times 9.8 \\ &= \underline{\underline{147 \text{ N}}}\end{aligned}$$

Since $F_2 > F_1$

$\therefore$ Net force acting on the body in the upward direction.

$$\begin{aligned}F &= F_2 - F_1 \\ &= 4900 - 147 \\ &= \underline{\underline{4753 \text{ N}}}\end{aligned}$$

This force is acting in the opposite direction of motion, so it will cause retardation.

$$F = m a$$

$$4753 = 15 \times a$$

$$a = 316.867 \text{ m/s}^2$$

Consider the motion of the body from the ground to the depth of penetration.

$$u = 19.6 \text{ m/s}$$

$$v = 0$$

$$a = -316.867 \text{ m/s}^2$$

$$v^2 - u^2 = 2 a s$$

$$0 - 19.6^2 = 2 \times -316.867 \times s$$

$$s = \underline{\underline{0.606 \text{ m}}}$$

ASE II

$$u = 300 \text{ m/s}$$

$$u = 300 \text{ m/s}$$

$$v = ?$$

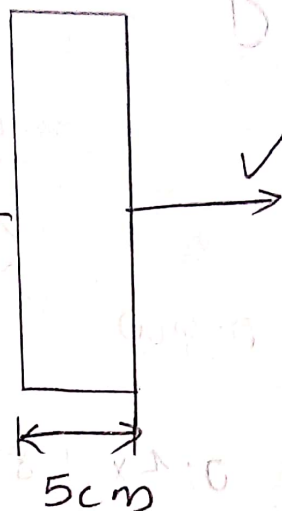
$$a = -4,50,000$$

$$s = 0.05 \text{ m}$$

$$v^2 - u^2 = 2as$$

$$v^2 - 300^2 = 2 \times -4,50,000 \times 0.05$$

$$v = \underline{\underline{212.132 \text{ m/s}}}$$



4. A car, moving on a straight road, skidded for a total distance of 60m after the brakes were applied. Determine the speed of the car, just before the brakes were applied, if the co-efficient of friction between the car tyres & the road is 0.4.

$$g = 9.8 \text{ m/s}^2$$

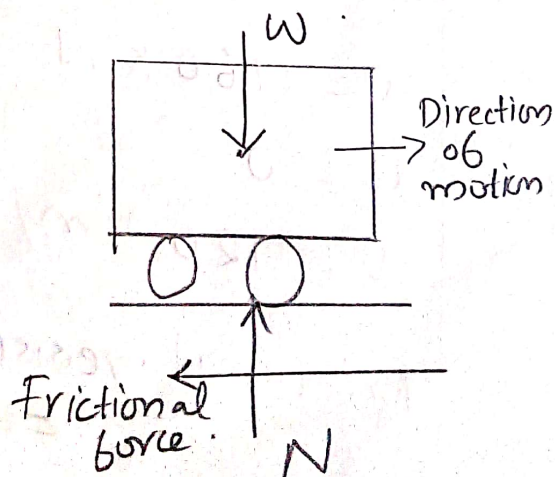
$$s = 60 \text{ m}, \mu = 0.4$$

$$v = 0$$

$$u = ?$$

$$\text{Frictional resistance} = \mu N$$

$$= 0.4 W$$



Frictional force is acting opposite to the direction of motion, hence frictional force causes retardation.

Force along the direction of motion

$$= mg$$

$$= \frac{W}{g} \times a$$

$$\frac{W}{g} a = 0.4W$$

$$a = 0.4 \times 9.8$$

$$a = -3.92 \text{ m/s}^2$$

$$v^2 - u^2 = 2as$$

$$0 - u^2 = -2 \times 3.92 \times 60$$

$$u = \underline{21.688 \text{ m/s}}$$

5. A train of weight 1960 kN starts from rest and attains a speed of 120 km/h in 5 min. If the frictional resistance of the track is 10 N per kN of train's weight, find the avg pull required.
- $g = 9.8 \text{ m/s}^2$

$$W = 1960 \text{ kN}$$

$$t = 5 \text{ min} = 5 \times 60 = 300 \text{ s}$$

$$u = 0$$

$$v = 120 \text{ km/h} = \frac{120 \times 5}{18} = 33.33 \text{ m/s}$$

Frictional resistance.

$$= 10 \text{ N / kN of train's weight}$$

$$= 10 \times 1960$$

$$= 19600 \text{ N}$$

$$v = u + at$$

$$33.33 = 0 + a \times 300$$

$$a = 0.11 \text{ m/s}^2$$

Let F_2 = Avg pull required in N

F_1 = Frictional resistance, in N

F = Net force acting on the engine in the direction of motion.

$$F = F_2 - F_1$$

$$F_2 = F + F_1$$

$$= \frac{1960 \times 1000 \times 0.11}{9.8} + 19600$$

$$= \underline{\underline{41600 \text{ N}}}$$

6. The tractive force, exerted by a railway car weighing 50 kN is 2000 N. If the frictional resistance is 5 N per kN of the railway car's weight, determine the acceleration when the railway car is moving on a level track.

$$\text{Weight of the car} = 50 \text{ kN}$$

$$= 50000 \text{ N}$$

Tractive force exerted by the car, F_2 2000 N

Frictional resistance = 5 N / kN of car's weight.

$$F_1 = 5 \times 50 \text{ N}$$

$$= 250 \text{ N}$$

$$F_1 > F_2$$

∴ Net force in the direction of motion

$$\begin{aligned} F &= F_1 - F_2 \\ &= 2000 - 250 \\ &= 1750 \text{ N} \end{aligned}$$

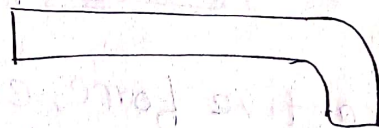
$$F = m a$$

$$1750 = \frac{30 \times 1000}{9.8} \times a$$

$$a = 0.343 \text{ m/s}^2$$

7. water drops from a pipe at the rate of 5 drops per second. Determine the vertical separation between 2 consecutive drops after the lower drop has attained a velocity of 3 m/s.

$$\begin{aligned} \text{Rate of drops} \\ &= 5 \text{ drops/s} \end{aligned}$$



∴ Drop B comes first from the pipe and drop A comes after $\frac{1}{5}$ s.

A :
B :

Consider the motion of the drop B

$$u = 0$$

$$v = 3 \text{ m/s}$$

$$g = 9.8 \text{ m/s}^2$$

$$v = u + gt$$

$$3 = 0 + 9.8 \times t$$

$$t = 0.306 \text{ s}$$

Distance travelled by drop B in 0.306 s.

$$S_B = ut + \frac{1}{2}gt^2$$

$$= 0 \times 0.306 + \frac{1}{2} \times 9.8 \times 0.306^2$$

$$= \underline{\underline{0.459 \text{ m}}}$$

Consider the motion of drop A.

$$\text{The time taken by drop A} = 0.306 - \frac{1}{5} \\ = 0.106 \text{ s.}$$

Distance travelled by drop A in 0.106 s.

$$S_A = ut + \frac{1}{2}gt^2$$

$$= 0 \times 0.106 + \frac{1}{2} \times 9.8 \times 0.106^2$$

$$= \underline{\underline{0.055 \text{ m}}}$$

Vertical separation between drops B & A

$$= 0.459 - 0.055$$

$$= \underline{\underline{0.404 \text{ m}}}$$

D'ALEMBERT'S PRINCIPLE.

It is the application of Newton's 2nd law to a moving body. A problem in dynamics can be converted into static equilibrium problem using this principle.

Newton's 2nd law of motion $F = ma$ can be written as $F - ma = 0$.

where

F is known as the effective resultant force &
 $-ma$ is known as Inertia.

Hence $F - ma = 0$ is known as De-Alembert's principle which states that the resultant of a system of forces acting on a body in motion is in dynamic equilibrium with the inertia of forces. The inertia of forces are also known as reverse effective force.

The inertia of force will act opposite to the direction of motion of body and will pass through the C.G. of the body.

ANALYSIS OF LIFT MOTION

Fig shows a lift carrying some weight and moving with a uniform acceleration.

Let w = weight carried by the lift

m = Mass carried by the lift.

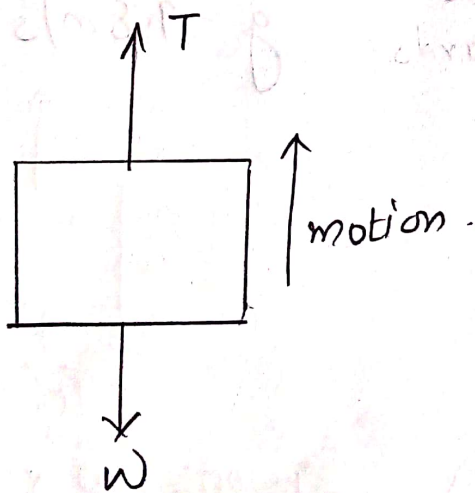
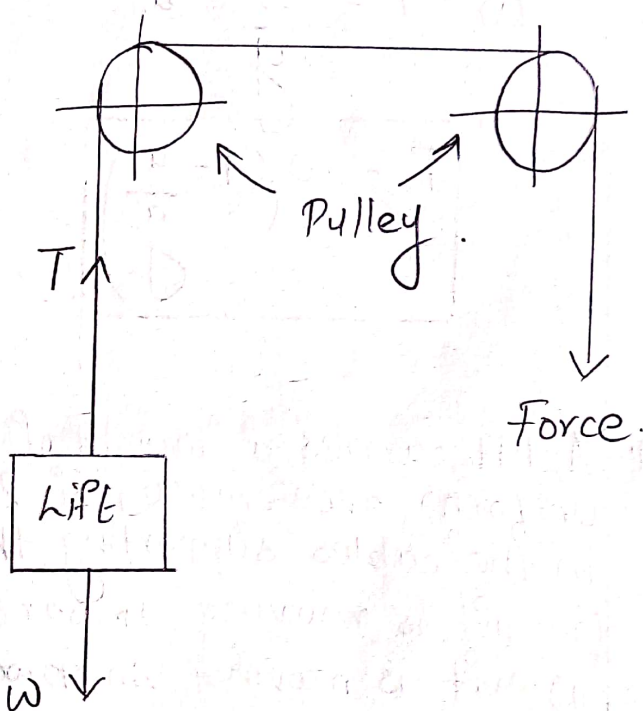
T = Tension in the cable supporting the lift.

It is also called the reaction of the lift.

a = uniform acceleration of the lift.

CASE I.

Let the lift is moving upwards. The weight carried by the lift is acting downwards while the tension in the cable is acting upwards. As the lift is moving up, the net force $= T - w$ is acting upwards.



Net force in upward direction $= T - w$
Net force produces an acceleration a .
Net force $= m a$.

$$T - w = m a$$

$$T - w = \frac{w}{g} a$$

$$T = w \left(1 + \frac{a}{g} \right)$$

CASE II

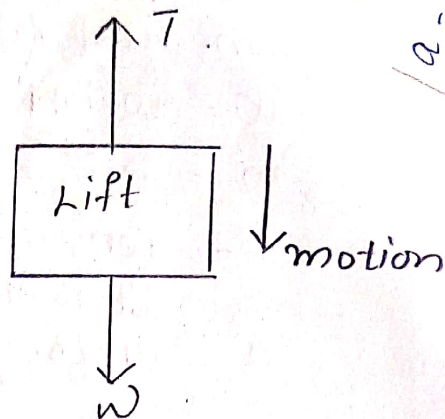
As the lift is moving downwards as in fig. The net force is acting downwards. Hence net force in downward direction = $w - T$.

Net force produces an acceleration a .

Net force = mass $\times$ acceleration

$$w - T = \frac{w}{g} \times a$$

$$T = w \left(1 - \frac{a}{g} \right)$$



lift has
a man
 $a = 1 \text{ m/s}^2$
 $w = 60$

1. A lift carries a weight of 100 N and is moving with a uniform acceleration of 2.45 m/s^2 . Determine the tension in the cables supporting the lift when.

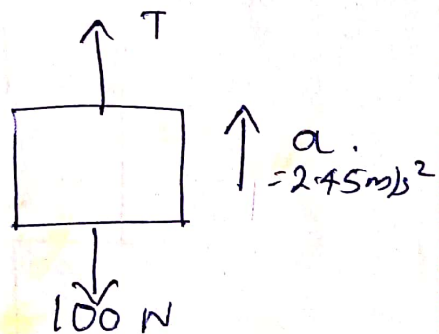
(i) lift is moving upwards.

(ii) lift is moving downwards.

$$g = 9.8 \text{ m/s}^2$$

$$w = 100 \text{ N}$$

$$a = 2.45 \text{ m/s}^2$$



$$\begin{aligned} (1) \quad T &= w \left(1 + \frac{a}{g} \right) \\ &= 100 \left(1 + \frac{2.45}{9.8} \right) \\ &= \underline{\underline{125 \text{ N}}} \end{aligned}$$

$$T - 100 = \frac{100}{9.8} \times 2.45$$

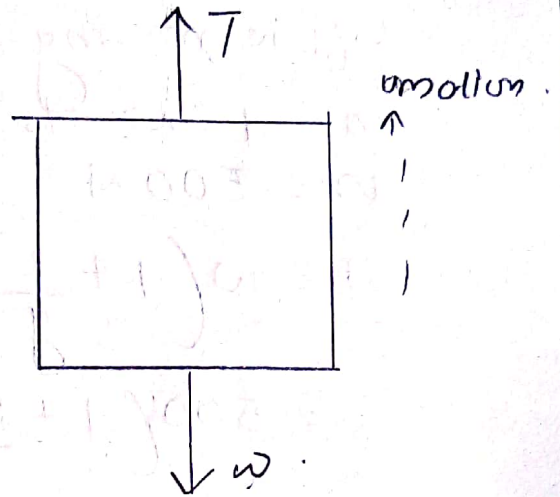
$$\begin{aligned} (2) \quad T &= w \left(1 - \frac{a}{g} \right) \\ &= 100 \left(1 - \frac{2.45}{9.8} \right) \\ &= \underline{\underline{75 \text{ N}}} \end{aligned}$$

lift has an upward acceleration of 1 m/s^2 . What pressure will a man weighing 600 N exert on the floor of the lift?

$$a = 1 \text{ m/s}^2$$

$$w = 600 \text{ N}$$

$$\begin{aligned} T &= w \left(1 + \frac{a}{g} \right) \\ &= 600 \left(1 + \frac{1}{9.8} \right) \\ &= \underline{\underline{661.22 \text{ N}}} \end{aligned}$$



Solve using D'Alembert's principle.

$$F - ma = 0$$

$$T - w - \frac{w}{g} a = 0$$

$$T = w + \frac{w}{g} a$$

$$= w \left(1 + \frac{a}{g} \right) = 600 \left(1 + \frac{1}{9.8} \right)$$

$$= \underline{\underline{661.22 \text{ N}}}$$

The tension in the cable is same as the pressure exerted by the man on the floor of the lift.

3. A lift has an upward acceleration of 1.225 m/s^2 . What pressure will a man weighing 500 N exert on the floor of the lift? What pressure would he exert if the lift had an acceleration of 1.225 m/s^2 downwards? What upward acceleration would cause his weight to exert a pressure of 600 N on the floor? $g = 9.8 \text{ m/s}^2$.

~~Case I~~

Case I.

Lift is moving upwards

$$a = 1.225 \text{ m/s}^2$$

$$W = 500 \text{ N}$$

$$T = W \left(1 + \frac{a}{g} \right)$$

$$= 500 \left(1 + \frac{1.225}{9.8} \right)$$

$$= \underline{\underline{562.5 \text{ N}}}$$

Case II

Lift is moving downwards

$$a = 1.225 \text{ m/s}^2$$

$$W = 500 \text{ N}$$

$$T = W \left(1 - \frac{a}{g} \right)$$

$$= 500 \left(1 - \frac{1.225}{9.8} \right)$$

$$= \underline{\underline{437.5 \text{ N}}}$$

Case III

$$a = ?$$

$$T = 600 \text{ N}$$

$$W = 500$$

$$T = W \left(1 + \frac{a}{g} \right)$$

$$600 = 500 \left(1 + \frac{a}{9.8} \right)$$

$$1.2 = 1 + \frac{a}{9.8}$$

$$0.2 = \frac{a}{9.8}, \quad a = \underline{\underline{1.96 \text{ m/s}^2}}$$

cage can
moving down
The cage attain
pressure exerted
 $g = 9.8$

Engineering Mechanics

(1)

Mod: 4: Dynamics

Dynamics: Study of bodies in motion.

Kinematics
~~Kinetics~~

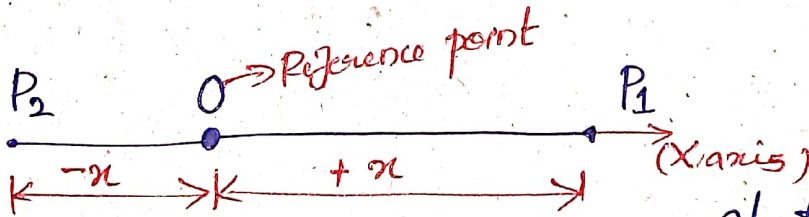
- Study of motion without considering the forces causing the motion.
- Study about position, displacement, velocity, acceleration & time.

Kinetics
~~Kinematics~~

- Study of motion ~~without~~ considering forces causing the motion.
- Study of motion, mass of the body & forces acting on it.

Rectilinear Motion

When a particle is moving along a straight line then it's said to be in rectilinear motion.



The position of a particle is specified w.r.t a reference point (O in fig.). This ref. point is called as Origin. Thus the position 'x' of particle is 've' towards the right of origin & '-ve' towards the left of origin.

Generally, the motion of particle can be defined if position of 'x' & time 't' is known $\Rightarrow x = f(t)$ $\rightarrow$ General displacement time eqn.

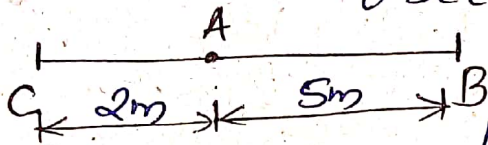
Rectangular Coordinate s/m: If a particle moves in x, y or both x, y direction its known as rectangular coordinate s/m.

Displacement:

→ vector quantity usually mentioned as

$$\Delta x = x_f - x_i$$

where $x_f \rightarrow$ x coordinate of final point
 $x_i \rightarrow$ " " " " initial point
 also depicted as x_0 i.e position at time 0 sec.



Consider a particle has moved from A to B then B to C.
 Then, the displacement = AC where as
 distance = AB + BC.

From fig:

$$\Rightarrow \text{Displacement} = -2m$$

$$\Rightarrow \text{Distance} = 5 + 2 = 7m$$

Time Interval: $\Delta t = t_f - t_i$

Average Velocity: Ratio of displacement to the time interval = $\frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

Instantaneous Velocity: or simply velocity is the limit of average velocity as the increment of time approaches zero.

$$i.e. \quad v = \lim_{t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

or $v = \frac{dx}{dt}$ i.e. velocity is defined as the rate of displacement of a particle w.r.t time
 Unit = m/s or km/hour

(3)

Average Acceleration = Ratio of ^{change in} velocity to the time interval.

$$= \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration: or simply acceleration is the rate of change of velocity or the limit of average acceleration as the increment of time approaches zero.

i.e. $a = \lim_{t \rightarrow 0} \frac{\Delta v}{\Delta t}$ or

$$\boxed{a = \frac{dv}{dt}} \quad \text{m/s}^2$$

or $a = \frac{d^2x}{dt^2}$ } in terms of displacement

Note: Motion could be a) under uniform acceleration or b) under variable acceleration. if the motion of a particle is given

as $x = f(t)$ then

$\frac{dx}{dt}$ gives eqⁿ of velocity

or $\frac{d^2x}{dt^2}$ gives eqⁿ of acceleration

Problem on Motion, Position, Time, Velocity & Acceleration.

1. The motion of a particle is defined by the relation $x = 2t^3 - 6t^2 + 15$ where x is in

(4)
metres & t is in seconds. Find the time,
position & acceleration when velocity is zero.

Solution:

Given: Motion eqn; $x = 2t^3 - 6t^2 + 15$

To find: i) time, t
ii) position, x
iii) Acceleration, a

From the motion eqn;

$$\text{Velocity, } v = \frac{dx}{dt} = 2 \times 3t^2 - 6 \times 2t + 0 \\ = 6t^2 - 12t$$

$$\therefore v = 6t^2 - 12t \quad \text{--- (1)}$$

$$\text{Acceleration, } a = \frac{dv}{dt} = 6 \times 2t - 12 \\ = 12t - 12$$

$$\therefore a = 12t - 12 \quad \text{--- (2)}$$

i) To find time t when $v=0$

Substituting eqn (1) = 0

$$\Rightarrow 6t^2 - 12t = 0$$

$$\Rightarrow t^2 - 2t = 0$$

$$\text{or } \underline{t = 2 \text{ seconds}}$$

ii) To find the position, x when $v=0$;

When $v=0$; $t=2$ sec. Sub- $t=2$ in eqn of motion

(5)

$$x = 2t^3 - 6t^2 + 15 \Rightarrow$$

$$x = 2 \times 2^3 - 6 \times 2^2 + 15 = \underline{7m}$$

iii) To find acceleration, a when $v=0$.

Substitute, $t = 2 \text{ sec}$ in eqn (2)

$$\Rightarrow a = 12 \times 2 - 12 = \underline{12 \text{ m/s}^2}$$

Result: i) time, $\underline{t = 2 \text{ sec}}$

ii) Position, $\underline{x = 7 \text{ m}}$

iii) Acceleration, $\underline{a = 12 \text{ m/s}^2}$

2. Velocity of a particle is given by
 $v = 2t^3 + 6t^2$. Find the distance travelled by
 it while its velocity increases from 8 m/s
 to 108 m/s .

Solution: Given: Velocity, $v = 2t^3 + 6t^2$
 $v_1 = 8 \text{ m/s}$; $v_2 = 108 \text{ m/s}$

To find: Distance travelled by particle, $S = x_2 - x_1$

Soln: For $v_1 = 8 \text{ m/s}$.

$$2t_1^3 + 6t_1^2 = 8$$

$$\Rightarrow t_1^3 + 3t_1^2 - 4 = 0$$

on solving; $\boxed{t_1 = 1 \text{ sec}}$

For $v_2 = 108 \text{ m/s}$

$$2t_2^3 + 6t_2^2 = 108$$

$$\Rightarrow \boxed{t_2 = 3 \text{ sec}}$$

We know that;

$$v = \frac{dx}{dt}$$

$$\Rightarrow dx = v \cdot dt \quad \text{--- (1)}$$

To find $S = x_2 - x_1$, integrate the eqn (1) from $t_1 = 1 \text{ s}$ to $t_2 = 3 \text{ s}$.

$$\Rightarrow \int_1^3 dx = \int_1^3 v \cdot dt = \int_1^3 [2t^3 + 6t^2] dt$$

$$\Rightarrow [x]_1^3 = \left[\frac{2t^4}{4} + \frac{6t^3}{3} \right]_1^3 = \left[\frac{t^4}{2} + 2t^3 \right]_1^3$$

$$\text{a. } (x_3 - x_1) = \frac{2t}{4} + \frac{6t}{3} \left[\frac{3^4}{2} + 2 \times 3^3 \right] - \left[\frac{1^4}{2} + 2 \times 1^3 \right]$$

$$\text{b. } S = \left[\frac{81}{2} + 54 - \frac{1}{2} - 2 \right]$$

$$\Rightarrow \underline{S = 92 \text{ m.}}$$

The distance travelled by particle when v increases from 8 m/s to 108 m/s is 92 m.

Practice Problems: Refer Appendix test pg 5.6 onwards

- Q. Motion of a particle is given by the eqn $x = t^3 - 12t^2 + 36t + 30$; x - m, t - sec.
Find acceleration, time & position when $v = 0$.
($t = 6$ s or 2 sec, [$a = 12$ m/s², $x = 30$ m] when $t = 6$ s
[$a = -12$ m/s², $x = 62$ m] when $t = 2$ s)

- Q. Motion of a particle is defined as
 $x = 7.5 + 22.5t - 13.5t^2 + 1.5t^3$; x - m, t - s.
Determine: a) Position when $v = 0$.
b) Position & total distance travelled when acceleration is zero

$$[t = 5 \text{ s or } 1 \text{ s. } (x = -30 \text{ m at } t = 5 \text{ s.}) \\ (x = 18 \text{ m at } t = 1 \text{ s})]$$

when $a = 0$; $t = 3$ sec

$\therefore x_0$ at $t = 0 = 7.5$ m

x_3 at $t = 3$ sec = -6 m.

$\Rightarrow S = x_3 - x_0 = \frac{-6 - 7.5}{1} = \underline{\underline{-13.5 \text{ m}}}$

- Q. A body moves along a straight line with an acceleration, $a = 3 - 4t$. After 4 seconds from the start of observation, its velocity is 45 m/s & after 6 seconds from the start of observation its position from origin is 150 m. Determine
i) a , v & x from ~~the~~ origin, when $t = 0$.
ii) The time & distance travelled when $v = 0$.

(Appendix publisher) Refer page 5.12 in Engg. Mechanics by Dr. S. Ramu Chaudhary

Motion with uniform acceleration.

Equations of motion

Consider a particle to be moving in a straight line.

$u \rightarrow$ Initial Velocity

$v \rightarrow$ Final Velocity

$s \rightarrow$ Distance Covered

$t \rightarrow$ Time

$a \rightarrow$ Acceleration

The particle is said to be in uniform acceleration when the velocity changes from u to v uniformly during the time

i) Change in Velocity = $(v - u)$ in time t

$\therefore$ Acceleration, $a = \frac{v - u}{t}$

$\Rightarrow \boxed{v = u + at}$ First eqn of motion — (1)

In differential eqn
w.k.t $a = \frac{dv}{dt} \Rightarrow dv = a \cdot dt$

Integrating on both side [from u to v on LHS
& 0 to t on RHS].

$$\int_0^t a \, dt = \int_u^v dv$$

$$\Rightarrow [v - u] = (a \cdot t - a \cdot 0) \Rightarrow \underline{v = u + at}$$

(9)

ii) Distance Travelled, s

$$s = \text{avg. velocity} \times \text{time}$$

$$= \frac{v+u}{2} \times t$$

from $\text{---} \textcircled{1}$ $v = u + at$

$$\therefore s = \left(\frac{u + at + u}{2} \right) \times t$$

$$= \frac{2ut}{2} + \frac{at^2}{2}$$

$$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

2nd eqⁿ of motion
 $\longrightarrow \textcircled{2}$

iii) from eqⁿ $\textcircled{1}$

$$v = u + at$$

$$\Rightarrow v^2 = (u + at)^2 = u^2 + 2uat + a^2t^2$$

$$= u^2 + 2a \left[ut + \frac{at^2}{2} \right]$$

$$= u^2 + 2a \times s. \quad (\because s = ut + \frac{at^2}{2} \text{ from } \textcircled{2})$$

$\therefore$ The third eqⁿ for ^{motion with} uniform acceleration

$$v^2 = u^2 + 2as$$

or $\boxed{v^2 - u^2 = 2as}$ 3rd eqⁿ of motion

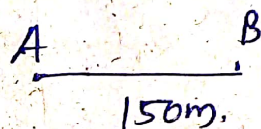
Sample problem

(10)

1. A bus is accelerated at the rate of 0.75 m/s^2 as it travels from A to B knowing that the speed of bus was $u = 27 \text{ km/hr}$ as it passed A, determine a) time req'd for bus to reach B. b) Corresponding speed as it passes B. Distance b/w A & B is 150 m .

Conversion:
 $\left[\text{km/hr} \rightarrow \text{m/s} : \times \frac{5}{18} \right]$

Soln: Given: $a = 0.75 \text{ m/s}^2$
 $u = 27 \text{ km/hr} = \frac{27 \times 1000}{3600} = 7.5 \text{ m/s}$
 $s = 150 \text{ m}$
 $\therefore$ Initial velocity, $\underline{u = 7.5 \text{ m/s}}$



a) To find, time (t).

We've

distance travelled, $s = ut + \frac{1}{2}at^2$

$$\text{i.e., } 150 = 7.5t + \frac{1}{2}(0.75)t^2$$

$$\Rightarrow t = \frac{-7.5 \pm \sqrt{7.5^2 + 4(0.375 \times 150)}}{2 \times 0.375}$$

$$\Rightarrow \underline{t = 12.36 \text{ sec.}}$$

b) To find final velocity as it passes B (v)

Conversion:

We know;

$$v = u + at$$

$$= 7.5 + 0.75 \times 12.36 = \underline{16.77 \text{ m/s}}$$

$$= 16.77 \times \frac{18}{5} \text{ km/hr} = \underline{60.37 \text{ km/hr}}$$

Velocity - Time Diagrams (10)

Graph plotted with time on x-axis & velocities on y-axis is called a Velocity-time diagram.

Note: For motion with uniform acceleration this graph consists of an inclined straight line.

Findings from V-t - Diagrams

1. Area of V-t Diagrams = Distance Travelled
2. Slope of side of V-t Diagrams = Acceleration

Consider a body in motion with uniform acceleration;

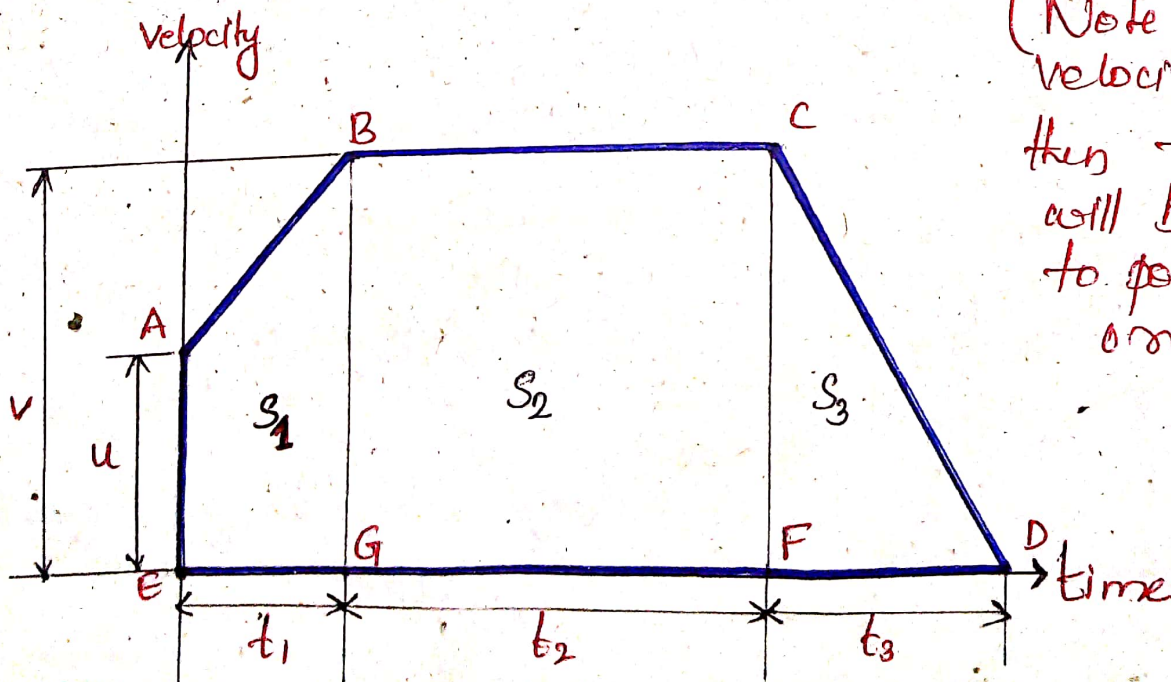
$u \rightarrow$ Initial velocity

$v \rightarrow$ Final velocity

$t \rightarrow$ time taken

$s \rightarrow$ Distance travelled

Then V-t diagrams can be constructed as



(Note: If initial velocity, $u=0$, then the line AB will be inclined to point E on origin)

From v-t graph (12)

- i) Acceleration = Slope of AB = $\frac{v-u}{t_1}$
- ii) Distance travelled, S_1 = Area of trapezium ABGE = $\frac{1}{2}(u+v)t_1$
motion from B to C is with constant velocity v then
- iii) Distance travelled, S_2 = Area of rectangle BCFG = $v \times t_2$
from C to D uniform Deceleration takes place
- iv) Deceleration = Slope of CD = $\frac{v}{t_3}$
- v) Distance travelled, S_3 = Area of triangle CDF = $\frac{1}{2}v \times t_3$
- vi) Total distance, $S = S_1 + S_2 + S_3$

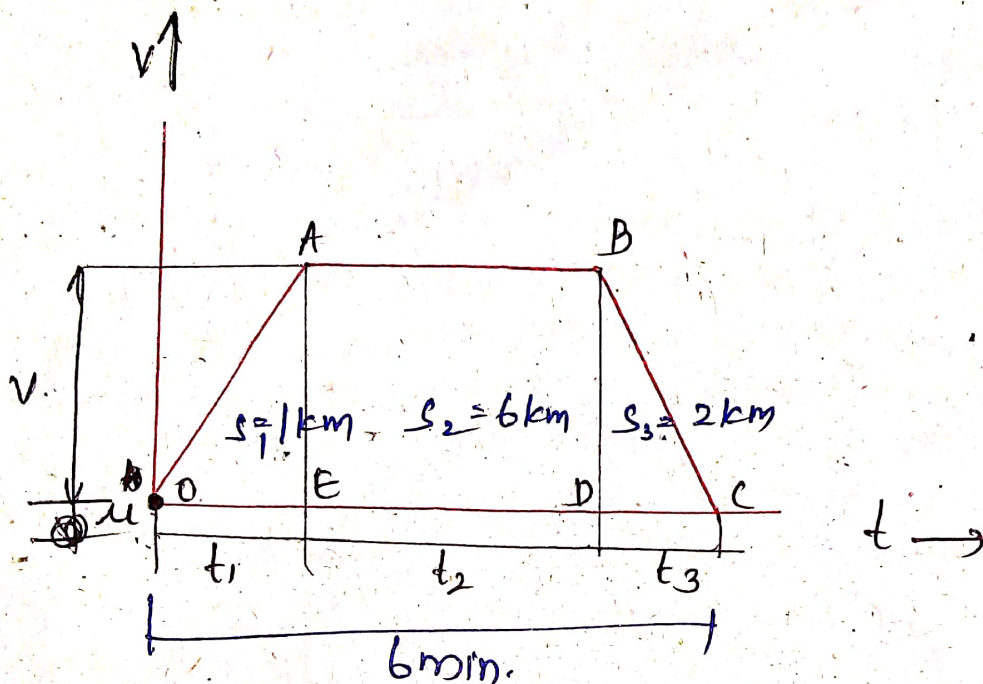
Sample problems:

Q. A train starts from rest, moves with uniform acceleration for the first km & attains a max. velocity. With this velocity it moves for the next 6 km & then comes to rest with uniform retardation. If the total journey is 9 km & it takes 6 min for the train to complete it, calculate max. velocity attained.

Soln: Given: $u = 0$;
 $S_1 = 1 \text{ km}$, $S_2 = 6 \text{ km}$
 $S = 9 \text{ km} \Rightarrow S_3 = 9 - (1 + 6) = \underline{2 \text{ km}}$
 $t = 6 \text{ min}$

To find: Max. velocity. (12)

Solution: Plot v-t diagram



Let $AE = \text{max velocity attained} = BD$.

$$*) S_1 = \text{Ar}(\triangle OAE) = \frac{1}{2} \times t_1 \times AE = 1000 \text{ m.}$$

$$\Rightarrow t_1 = \frac{1000 \times 2}{AE} = \frac{2000}{AE} \quad \text{--- (1)}$$

$$*) S_2 = \text{Ar of } \square ABDE = t_2 \times AE = 6000 \text{ m.}$$

$$\Rightarrow t_2 = \frac{6000}{AE} \quad \text{--- (2)}$$

$$*) S_3 = \text{Ar of } \triangle BDC = \frac{1}{2} t_3 \times BD$$

$$= \frac{1}{2} t_3 \times AE = 2000$$

$$\Rightarrow t_3 = \frac{4000}{AE} \quad \text{--- (3)}$$

W.K.T; $t_1 + t_2 + t_3 = 6 \text{ min} = 360 \text{ sec}$

$$\therefore \text{--- (1) + (2) + (3) = 360}$$

(14)

6;

$$\frac{2000}{AE} + \frac{6000}{AE} + \frac{4000}{AE} = 360$$

$$\Rightarrow AE = \frac{12000}{360} = \underline{\underline{33.3 \text{ m/s}}}$$

$$\therefore \text{Max Velocity} = \underline{\underline{33.3 \text{ m/s}}}$$

Free Falling Body

Consider a body allowed to fall from top of a tower. As it moves downwards its velocity increases due to gravitational acceleration. Such a body is a freely falling body & its motion is under gravity.

3 eqs of motion with uniform acceleration are applicable to motion under gravity.

i) $v = u + at$ (or) $v = u - gt$

($\because a = -g$ as it's always acting downwards)

ii) $h = ut + \frac{1}{2}at^2$ (or) $h = ut - \frac{1}{2}gt^2$

iii) $v^2 = u^2 + 2ah$ (or) $v^2 = u^2 - 2gh$

here distance h = distance travelled in

of direction & value of $g = 9.8 \text{ m/s}^2$

Sign Convention: Reference pt (P) $\rightarrow$ point from which motion starts. Distance upward R.P = +ve; Distance downward R.P is -ve.

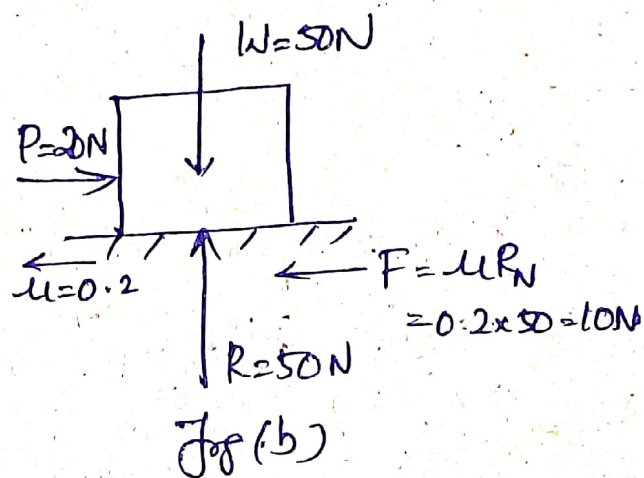
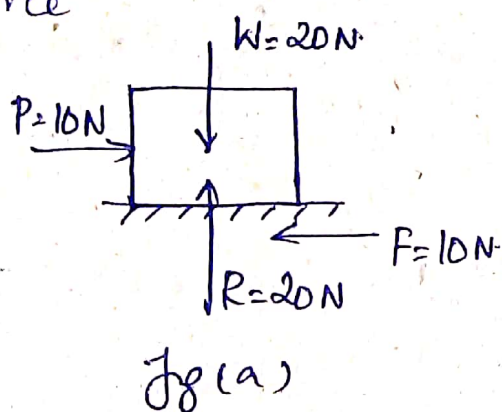
Acceleration due to gravity is acting downwards & hence treated as negative.

(15)

Rectilinear Translation.

Newton's 2nd Law:

If the resultant force acting on a particle is not zero, the particle will move with an acceleration proportional to the magnitude of resultant & in the direction of this resultant force.



In Fig (a) $R = 0$ as $\sum F_x = 0$ & $\sum F_y = 0$.

$\therefore$ The body doesn't move in any direction.

But in Fig (b) $\sum F_y = 0$ but $\sum F_x = 20 - 10 = 10\text{N}$.

Where 10N acts in the right side direction.

Hence the body will move in the right side direction due to the resultant.

Consider that a body is acted upon by force F_1 ; then the body will move with an acceleration a_1 & similarly for

force F_2, F_3 etc. the acceleration will be a_2, a_3 etc. Thus the force is proportional to acceleration i.e. $F_1 \propto a_1$; $F_2 \propto a_2$ & $F_3 \propto a_3$.

or

(16)

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \text{Constant}$$

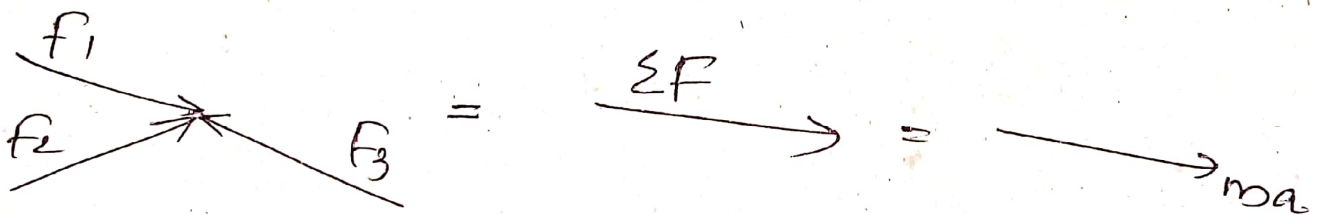
This constant of proportionality is known as 'm' of particle

$$\therefore \underline{F = m \times a}$$

When there are several forces acting on a particle then

$$\underline{\Sigma F = ma} \quad (\Sigma F = F_1 + F_2 + F_3 + \dots)$$

Diagrammatic Representation

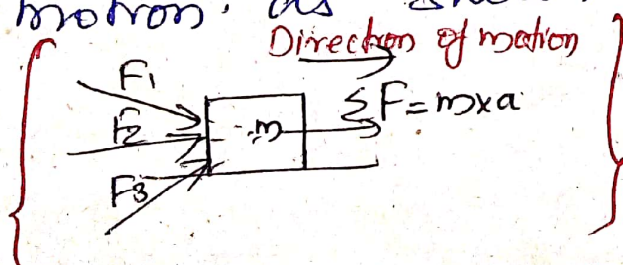


D'Alembert's Principle

We know that by Newton's 2nd Law

$$\Sigma F = ma \rightarrow \text{eqⁿ of motion}$$

a. A body having mass m acted upon by different forces sets the body in motion, as shown in fig.

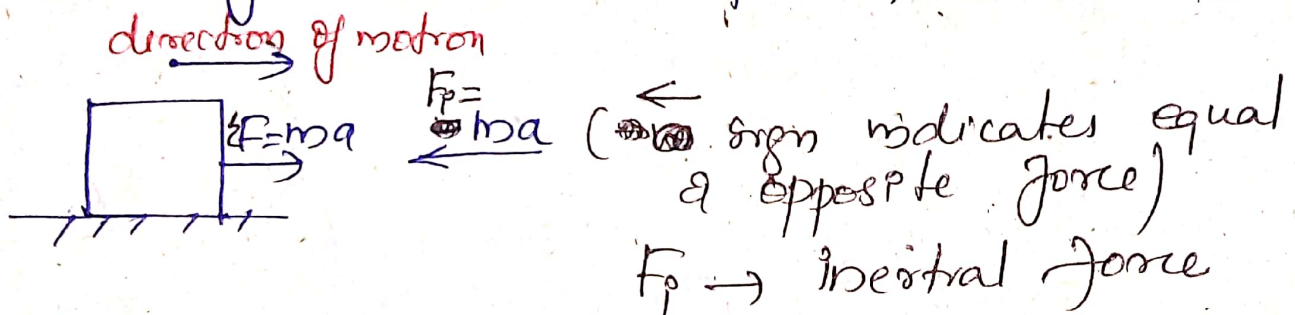


Consider this body as a system which is in motion along the direction shown in fig.

(17)

In order to bring the given system into equilibrium a force of same magnitude (i.e; ma) in direction opposite to the direction of motion. This force applied in same magnitude but in opposite direction of the motion is termed as **inertial force**.

Thus by D'Alembert's principle:



$$(i); \quad \sum F - F_i = 0$$

$$\text{or } \sum F - ma = 0$$

(Dynamic Equilibrium)

So the D'Alembert's principle can be defined as:

A system of forces acting on a body in motion is in dynamic equilibrium with inertial force.

Basically this principle is used to convert a ~~static~~ dynamically equilibrium to a statically equilibrium state.

i.e for converting a dynamic problem to a static problem.

Thus, is general: (B)

$\Sigma F = ma \rightarrow$ Equation of motion

$\Sigma F - F_f = 0 \rightarrow$ Dynamic Equilibrium

$\Sigma F = 0 \rightarrow$ Static Equilibrium
(including $-ma$)

Sample problem

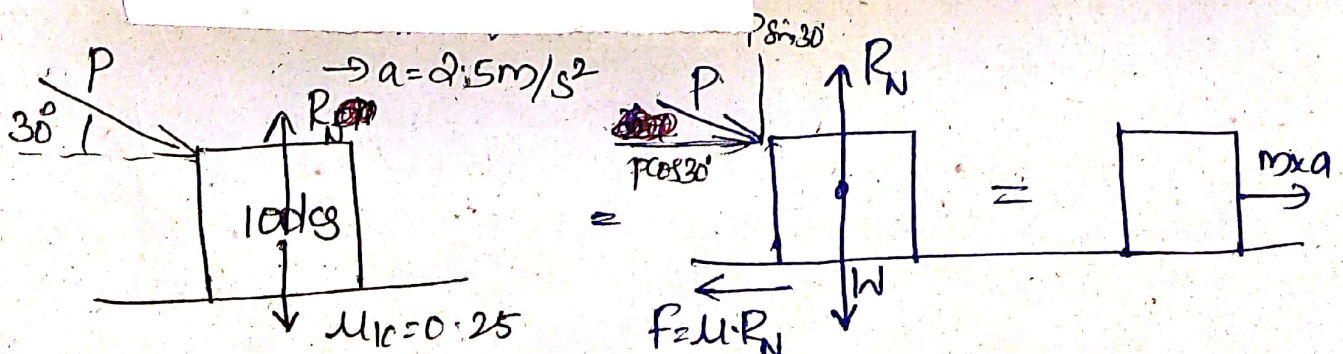
Q. A 100 kg block rests on a horizontal plane. Find the magnitude of force P req'd to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction b/w block & plane is $\mu_k = 0.25$. Refer Fig

Soln: Given: $m = 100 \text{ kg}$, $\mu_k = 0.25$,
 $a = 2.5 \text{ m/s}^2$

Weight, $W = 100 \times 9.81 = \underline{981 \text{ N}}$.

To find: Effort, or Force, $P = ?$

Soln:



(17)

$$ma_x = 100 \times 2.5 = \underline{\underline{250N}}$$

In 'x' ~~direction~~ direction:

$$\sum F_x = ma_x = \underline{\underline{250N}}$$

ie; $P \cos 30^\circ - \mu R_N = 250$ — (1)

In 'y' direction

$$R_N - W - P \sin 30^\circ = 0$$

ie $R_N = P \sin 30^\circ + W = 0.5P + 981N$ — (2)

$\rightarrow R_N$ in — (1)

$$\Rightarrow P \cos 30^\circ - 0.25(P \times 0.5 + 981) = 250$$

$$\Rightarrow 0.742P = 495.25$$

$$\Rightarrow P = \frac{495.25}{0.742} = \underline{\underline{667.5N}}$$

$P = 667.5N$

14.1. INTRODUCTION

When a particle is projected upwards at a certain angle (but not vertical), the particle traces some path in the air and falls on the ground at a point, other than the point of projection. The path traced by the particle in air is known as *trajectory* of the particle whereas the particle is called a *projectile*. The path traced by the particle is parabolic.*

14.2. TERMS USED WITH THE PROJECTILES

The following terms are generally used with the projectiles :

1. Velocity of projection.
2. Angle of projection.
3. Time of flight.
4. Horizontal range.

They are shown in Fig. 14.1.

14.2.1. Velocity of Projection. The velocity, with which a projectile is projected into space, is called the velocity of projection. This will be denoted by the symbol u .

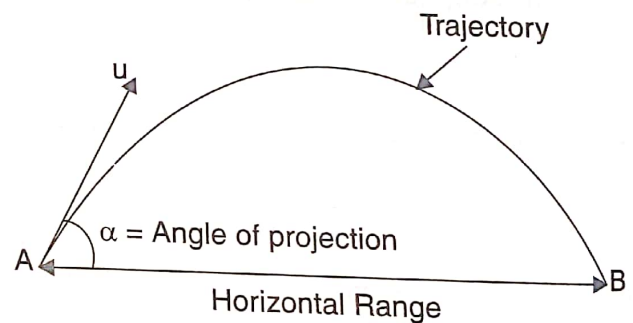


Fig. 14.1

14.2.2. Angle of Projection. This is the angle, with the horizontal, at which a projectile is projected. This will be denoted by ' α '.

14.2.3. Time of Flight. It is total time taken by a projectile for which the projectile remains in space. Or this is the interval of time since the projectile is projected and hits the ground again. This will be denoted by ' T '.

14.2.4. Horizontal Range. The horizontal distance, between the point of projection and the point where projectile strikes the ground, is called horizontal range. This will be denoted by ' R '.

14.3. EQUATION FOR THE PATH OF A PROJECTILE

Let a particle is projected upwards at an angle α with the horizontal with an initial velocity ' u ' m/s from a point A as shown in Fig. 14.2.

- $\therefore$ α = Angle of projection and
 u = Velocity of projection.

The velocity of projection (u) is resolved into horizontal component and vertical component. They are given as :

Horizontal component of velocity $= u \cos \alpha$.

Vertical component of velocity $= u \sin \alpha$.

The vertical component of the velocity is affected by gravity but the horizontal component of the velocity will remain constant (provided the air resistance is neglected).

The particle will move in air along certain path APB and will fall down at B as shown in Fig. 14.2.

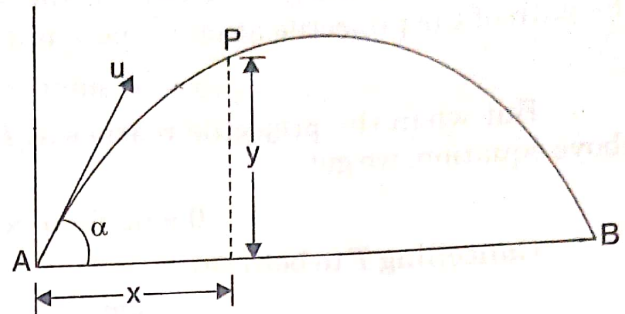


Fig. 14.2

Consider a point P on the path of the particle.

Let x and y = the co-ordinates of P with respect to A .

t = time taken by the particle to reach from A to P .

Then the horizontal and vertical distances travelled by the particle in time ' t ' are :

$$x = \text{Horizontal component of velocity} \times t \quad \dots(14.1)$$

$$= (u \cos \alpha) \times t$$

and

$$y = \text{Vertical component of velocity} \times t - \frac{1}{2} g t^2 \quad \text{(using } S = ut - \frac{1}{2} g t^2 \text{)} \quad \dots(14.2)$$

$$= (u \sin \alpha) \times t - \frac{1}{2} g t^2$$

But from equation (14.1), the value of t is given as

$$t = \frac{x}{u \cos \alpha}.$$

Substituting this value in equation (14.2), we get

$$y = (u \sin \alpha) \times \frac{x}{(u \cos \alpha)} - \frac{1}{2} g \times \left(\frac{x}{u \cos \alpha} \right)^2$$

$$= \frac{x \sin \alpha}{\cos \alpha} - \frac{g x^2}{2 u^2 \cos^2 \alpha} = x \tan \alpha - \frac{g x^2}{2 u^2 \cos^2 \alpha} \quad \dots(14.3)$$

For a given value of velocity of projection (u) and angle of projection (α), equation (14.3) gives the variation of y with the square of x , which means the equation (14.3) is the equation of a parabola. Hence the path travelled by the particle in air is parabolic. Equation (14.3) is also known as equation of trajectory.

14.3.1. Maximum Height Attained by the Projectile. The projectile will reach the maximum height, when the vertical component of velocity of projection (i.e., $u \sin \alpha$) becomes zero.

$\therefore$ Initial velocity in vertical direction $= u \sin \alpha$

Final velocity in vertical direction $= 0$.

Let $h_{\max}$ is the maximum height attained by the projectile. Then using

$$(\text{Final velocity})^2 - (\text{Initial velocity})^2 = 2aS$$

or

$$0^2 - (u \sin \alpha)^2 = -2 \times g \times h_{\max} \quad (\because a = -g \text{ and } S = h_{\max})$$

or

$$-u^2 \sin^2 \alpha = -2g \times h_{\max}$$

$\therefore$

$$h_{\max} = \frac{u^2 \sin^2 \alpha}{2g} \quad \dots(14.4)$$

14.3.2. Time of Flight. It is the time taken by the projectile in reaching from point A to point B as shown in Fig. 14.2. Let T is the time of flight. The y co-ordinate of any point lying on the path of the projectile after a time ' t ' is given by equation (14.2) as

$$y = (u \sin \alpha) \times t - \frac{1}{2} g t^2.$$

But when the projectile reaches at B, $y = 0$ and $t = T$. Substituting these values in the above equation, we get

$$0 = (u \sin \alpha) \times T - \frac{1}{2} g T^2 \quad \text{or} \quad (u \sin \alpha) \times T = \frac{1}{2} g T^2$$

Cancelling T to both sides,

$$u \sin \alpha = \frac{gT}{2}$$

$$\therefore T = \frac{2u \sin \alpha}{g} \quad \dots(14.5)$$

14.3.3. Horizontal Range of the Projectile (R). It is the horizontal distance between the point of projection and the point, where the projectile strikes back to the earth i.e., the horizontal distance AB in Fig. 14.2 is called the horizontal range of the projectile. Let this horizontal range is denoted by R .

Then

$$\begin{aligned} R &= \text{Velocity component in horizontal direction} \\ &\quad \times \text{Time taken by the projectile to reach from A to B} \\ &= (u \cos \alpha) \times \text{Time of flight} \end{aligned}$$

$$\begin{aligned} &= (u \cos \alpha) \times \frac{2u \sin \alpha}{g} \quad \left\{ \because \text{Time of flight} = \frac{2u \sin \alpha}{g} \right\} \\ &= \frac{u^2}{g} \times 2 \sin \alpha \cos \alpha \quad \dots(14.6) \quad (\because 2 \sin \alpha \cos \alpha = \sin 2\alpha) \end{aligned}$$

$$\therefore R = \frac{u^2}{g} \times \sin 2\alpha$$

14.3.4. Value of Angle of Projection (α) for Maximum Horizontal Range. Equation (14.6) gives the horizontal range of the projectile. The horizontal range (R) will be maximum for a given velocity of projection, when $\sin 2\alpha$ is maximum. But $\sin 2\alpha$ will be maximum when

$$\begin{aligned} \sin (2\alpha) &= 1 & \text{or} & \sin (2\alpha) = \sin 90^\circ \\ \text{or} & & 2\alpha &= 90^\circ & \alpha &= \frac{90}{2} = 45^\circ \end{aligned} \quad \dots(14.7)$$

Then the maximum horizontal range ($R_{\max}$) will be obtained by substituting $\alpha = 45^\circ$ in equation (14.6).

$$R_{\max} = \frac{u^2}{g} \sin (2 \times 45^\circ) = \frac{u^2}{g} \quad \dots(14.8)$$

14.3.5. Time to Reach the Highest Point. When the projectile reaches the highest point, the final velocity in the vertical direction is zero. Let T' is the time taken by the projectile to reach the highest point. Using the equation,

$$\begin{aligned} \text{Final velocity} &= \text{Initial velocity} + \text{Acceleration} \times \text{Time} \\ \text{or} \quad 0 &= u \sin \alpha - g \times T \end{aligned} \quad (\because V = u + at)$$

$$\{\because \text{Acceleration} = -g\}$$

$$\therefore T' = \frac{u \sin \alpha}{g} \quad \dots(14.9)$$

From equation (14.5) it is clear that $T' =$ half the time of flight.

Problem 14.1. A particle is projected at an angle of 60° with the horizontal. The horizontal range of the particle is 5 kilometres. Find : (i) the velocity of projection, and (ii) the maximum height attained by the projectile.

Sol. (i) Given :

Angle of projection, $\alpha = 60^\circ$

Horizontal range, $R = 5 \text{ kilometres} = 5 \times 1000 = 5000 \text{ m}$

Let $u =$ Velocity of projection

Using equation (14.6) for horizontal range,

$$R = \frac{u^2}{g} \times \sin 2\alpha \quad \text{or} \quad 500 = \frac{u^2}{9.81} \times \sin (2 \times 60)$$

$$\text{or} \quad u^2 = \frac{5000 \times 9.81}{\sin 120^\circ} = \frac{5000 \times 9.81}{0.866} = 56639.7$$

$$\therefore u = \sqrt{56639.7} = 237.99 \text{ m/s. Ans.}$$

(ii) **Maximum height attained by the projectile**

Let $h_{\max}$ = Maximum height attained by projectile. Using equation (14.4) for maximum height,

$$\begin{aligned} h_{\max} &= \frac{u^2 \sin^2 \alpha}{2g} = \frac{237.99^2 \times \sin^2 60^\circ}{2 \times 9.81} \\ &= \frac{237.99^2 \times (0.866)^2}{2 \times 9.81} = 2164.98 \text{ m. Ans.} \end{aligned}$$

Problem 14.2. A particle is projected in air with a velocity 100 m/s and at an angle of 30° with the horizontal. Find :

(i) The horizontal range,

(ii) The maximum height by the particle, and

(iii) The time of flight.

Sol. Given :

Velocity of projection, $u = 100 \text{ m/s}$

Angle of projection, $\alpha = 30^\circ$

Let

$R =$ Horizontal range

$h_{\max} =$ Maximum height attained by the particle, and

$T =$ Time of flight.

(i) Using equation (14.6) for horizontal range,

$$\begin{aligned} R &= \frac{u^2 \times \sin 2\alpha}{g} = \frac{100^2 \times \sin (2 \times 30^\circ)}{9.81} \\ &= \frac{100.00 \times \sin 60}{9.81} = 882.77 \text{ m. Ans.} \end{aligned}$$

(ii) Using equation (14.4) for maximum height,

$$h_{\max} = \frac{u^2 \sin^2 \alpha}{2g} = \frac{100^2 \times \sin^2 (30^\circ)}{2 \times 9.81}$$

$$= \frac{100^2 \times (0.5)^2}{2 \times 9.81} \quad (\because \sin 30^\circ = 0.5)$$

$$= 127.42 \text{ m. Ans.}$$

(iii) Using equation (14.5) for time of flight,

$$T = \frac{2u \sin \alpha}{9.81} = \frac{2 \times 100 \times \sin 30^\circ}{9.81} = \frac{2 \times 100 \times 0.5}{9.81} = 10.19 \text{ s. Ans.}$$

Problem 14.3. A particle is projected at such an angle with the horizontal that the horizontal range is four times the greatest height attained by the particle. Find the angle of projection.

Sol. Given :

Horizontal range = 4 times the greatest height

$$\text{or} \quad R = 4 \times h_{\max} \quad \dots(i)$$

where R = Horizontal range, and

$h_{\max}$ = Maximum height attained

Let α = Angle of projection.

$$\text{But from equation (14.6), } R = \frac{u^2 \sin 2\alpha}{g}$$

$$\text{and from equation (14.4), } h_{\max} = \frac{u^2 \sin^2 \alpha}{2g}$$

Substituting these values in equation (i), we get

$$\frac{u^2 \sin 2\alpha}{g} = \frac{4 \times u^2 \times \sin^2 \alpha}{2 \times g}$$

$$\text{or} \quad \sin 2\alpha = 2 \sin^2 \alpha$$

$$\text{or} \quad 2 \sin \alpha \cos \alpha = 2 \sin^2 \alpha$$

$$\text{or} \quad \cos \alpha = \sin \alpha$$

$$\text{or} \quad \frac{\sin \alpha}{\cos \alpha} = 1 \quad \text{or} \quad \tan \alpha = 1.0$$

$$\therefore \alpha = \tan^{-1}(1.0) = 45^\circ. \text{ Ans.}$$

Problem 14.4. A particle is projected with a velocity of 20 m/s in air at angle ' α ' with the horizontal. The x and y co-ordinates of a point lying on the trajectory* of the particle with respect to point of projection are 20 m and 8 m respectively. Find the angle of projection of the particle.

Sol. Given :

Velocity of projection, $u = 20 \text{ m/s}$

Angle of projection = α

x co-ordinate = 20 m

y co-ordinate = 8 m.

*Trajectory is the path traced by a particle when projected in air at certain angle (but not vertical) with horizontal.

The equation of the path traced by a projectile is given by (14.3). Hence using equation (14.3), we have

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

or

$$8 = 20 \tan \alpha - \frac{9.81 \times 20^2}{2 \times 20^2 \cos^2 \alpha}$$

$$= 20 \tan \alpha - \frac{4.905}{\cos^2 \alpha}$$

$$= 20 \tan \alpha - 4.905 \sec^2 \alpha$$

$$\left(\because \frac{1}{\cos^2 \alpha} = \sec^2 \alpha \right)$$

$$= 20 \tan \alpha - 4.905 [1 + \tan^2 \alpha]$$

$$(\because \sec^2 \alpha = 1 + \tan^2 \alpha)$$

$$= 20 \tan \alpha - 4.905 - 4.905 \tan^2 \alpha$$

$$\text{or} \quad 4.905 \tan^2 \alpha - 20 \tan \alpha + 4.905 + 8 = 0$$

$$\text{or} \quad 4.905 \tan^2 \alpha - 20 \tan \alpha + 12.905 = 0$$

The above equation is a quadratic equation in $\tan \alpha$. Hence its two roots are given by

$$\tan \alpha = \frac{20 \pm \sqrt{20^2 - 4 \times 4.905 \times 12.905}}{2 \times 4.905} = \frac{20 \pm \sqrt{400 - 253.19}}{9.81} = \frac{20 \pm \sqrt{146.81}}{9.81}$$

$$= \frac{20 + 12.116}{9.81} = \frac{32.116}{9.81} \quad \text{and} \quad \frac{20 - 12.116}{9.81} = 3.273 \quad \text{and} \quad 0.8037$$

If $\tan \alpha = 3.273$ then $\alpha = 73^\circ 0.8'$. **Ans.**

If $\tan \alpha = 0.8036$ then $\alpha = 38^\circ 47'$. **Ans.**

Hence for the given values there will be two angles of projection

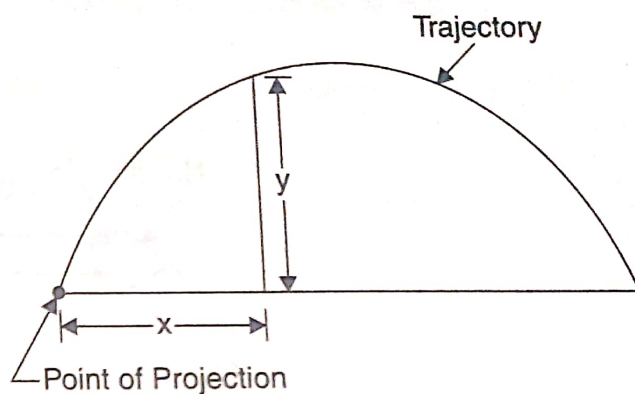


Fig. 14.3

Kinetics of Rigid Bodies and Laws of Motion

15.1. INTRODUCTION

Kinetics is that branch of Engineering Mechanics which deals with the force system which produces acceleration and resulting motion of bodies. In the previous chapter *i.e.*, chapter of kinematics of Rigid bodies, we have dealt with the motion of the bodies (*i.e.*, displacement, velocity and acceleration of rigid bodies) without consideration of forces which produce these motion.

When a body is at rest or moving in a straight line or rotating about an axis, the body obeys certain laws of motion. These laws are called Newton's laws of motion. There are three laws of motion. These laws for linear motions and angular motions are given in the next articles.

15.2. NEWTON'S LAWS FOR LINEAR MOTION

First Law. It states that a body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external force to change that state.

Second Law. It states that the rate of change of momentum of a body is proportional to the external force applied on the body and takes place in the direction of the force.

Third Law. It states that to every action, there is always an equal and opposite reaction.

15.7. MOTION ON AN INCLINED SMOOTH SURFACE

Fig. 15.2 shows a body of weight W , sliding down on a smooth inclined plane.

Let θ = Angle made by inclined plane with horizontal

W = Weight of the body

a = Acceleration of the body

m = Mass of the body

$$= \frac{W}{g}.$$

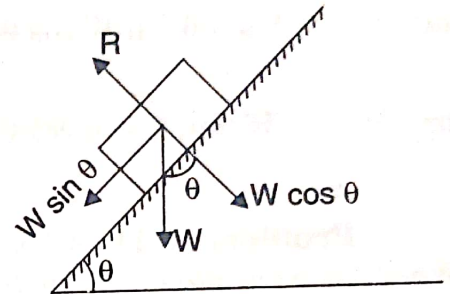


Fig. 15.2

As the surface of the plane is smooth, hence the frictional force will be zero. Hence the forces acting on the body are its own weight W and reaction R of the plane. The resolved part of W perpendicular to the plane is $W \cos \theta$, which is balanced by R , while the resolved part parallel to the plane is $W \sin \theta$, which produces acceleration down the plane. This force is responsible for the movement of the body down the plane.

$\therefore$ Net force acting on the body down the plane is

$$F = W \sin \theta$$

Now using the equation (15.2), we have

$$F = m \times a.$$

Substituting the values of F and m in the above equation, we get

$$W \sin \theta = \frac{W}{g} \times a$$

$$\therefore a = g \sin \theta \quad \dots(15.3)$$

If the body is moving up the plane, the corresponding acceleration will be $-g \sin \theta$.

15.8. MOTION ON AN INCLINED ROUGH SURFACE

Fig. 15.3 shows a body of weight W , sliding down the rough inclined surface.

Let a = Acceleration of the body

$$m = \text{Mass of the body} = \frac{W}{g}$$

θ = Inclination of the plane with horizontal

μ = Co-efficient of friction

F_1 = Force of friction.

As the body is moving down the plane, the force of friction will be acting up the plane as shown in Fig. 15.3.

Now force of friction,

$$F_1 = \mu R$$

$$= \mu \times W \cos \theta \quad (\because R = W \cos \theta)$$

Force acting down the plane,

$$F_2 = W \sin \theta$$

$\therefore$ Net force acting on the body down the plane,

$$F = F_2 - F_1 = W \sin \theta - \mu W \cos \theta.$$

Now using the equation (15.2),

$$F = m \times a$$

$$\text{or} \quad (W \sin \theta - \mu W \cos \theta) = \frac{W}{g} \times a$$

$$\text{or} \quad W (\sin \theta - \mu \cos \theta) = \frac{W}{g} \times a \quad \text{or} \quad a = W (\sin \theta - \mu \cos \theta) \times \frac{g}{W}$$

$$\left(\because m = \frac{W}{g} \right)$$

$$\therefore a = g (\sin \theta - \mu \cos \theta).$$

...(15.4)

Problem 15.11. A body of weight 200 N is initially stationary on a 45° inclined plane. What distance along the inclined plane must the body slide, before it reaches a speed of 2 m/s. The co-efficient of friction between the body and the plane = 0.1.

Sol. Given :

Weight of body, $W = 200 \text{ N}$

$$\therefore \text{Mass of body, } m = \frac{W}{g} = \frac{200}{9.81} \text{ kg}$$

Angle of plane, $\theta = 45^\circ$

Initial velocity, $u = 0$

Final velocity, $v = 2 \text{ m/s}$

Co-efficient of friction, $\mu = 0.1$.

The acceleration of the body is given by equation (15.4) as

$$a = g [\sin \theta - \mu \cos \theta] = 9.81 [\sin 45^\circ - 0.1 \cos 45^\circ]$$

$$= 9.81 [0.707 - 0.1 \times 0.707] = 6.242 \text{ m/s}^2.$$

Now using the relation,

$$v^2 - u^2 = 2as \quad \text{or} \quad 2^2 - 0^2 = 2 \times 6.242 \times s$$

$$\therefore s = \frac{2 \times 2}{2 \times 6.242} = 0.32 \text{ m} = 32 \text{ cm. Ans.}$$

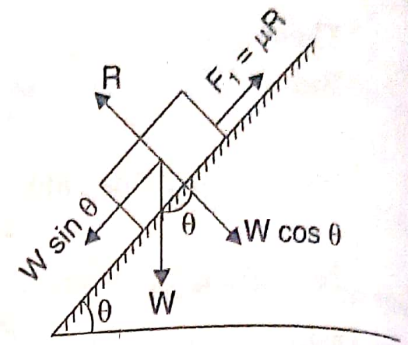


Fig. 15.3

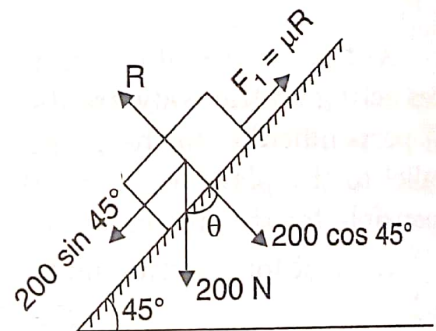


Fig. 15.4

Problem 15.12. Two bodies directly in line and 10 m apart are held stationary on an inclined plane having inclination of 20° . The co-efficient of friction between the plane and lower body is 0.08 and that between the plane and the upper body is 0.05. If both the bodies are set in motion at the same instant, calculate the distance through which each body travels before they meet together.

Sol. Given :

Initial velocity, $u = 0$

Angle of inclined plane, $\theta = 20^\circ$

Co-efficient of friction between the plane and lower body, $\mu_1 = 0.08$.

Co-efficient of friction between the plane and upper body, $\mu_2 = 0.05$.

Distance between two bodies, $d = 10 \text{ m}$.

Let the two bodies meet after ' t ' seconds.

If s_1 = Distance travelled by lower body in time ' t ', and

s_2 = Distance travelled by upper body in time ' t '.

Then $s_2 = s_1 + 10$

...(i)

Let a_1 = Acceleration of the lower body down the plane, and

a_2 = Acceleration of the upper body down the plane.

But the acceleration of a body moving down a rough plane is given by equation (15.4).

$$a = g(\sin \theta - \mu \cos \theta).$$

$\therefore$ Acceleration of lower body,

$$\begin{aligned} a_1 &= g(\sin \theta - \mu_1 \cos \theta) \\ &= 9.81(\sin 20^\circ - .08 \times \cos 20^\circ) \quad (\because \mu_1 = .08, \theta = 20^\circ) \\ &= 9.81(0.342 - .08 \times 0.9397) \\ &= 9.81(0.342 - 0.0752) = 2.617 \text{ m/s}^2. \end{aligned}$$

Acceleration of upper body,

$$\begin{aligned} a_2 &= g(\sin \theta - \mu_2 \cos \theta) \\ &= 9.81(\sin 20^\circ - .05 \cos 20^\circ) \quad (\because \mu_2 = .05) \\ &= 9.81(0.342 - .05 \times 0.9397) \\ &= 9.81(0.342 - 0.04698) = 2.894 \text{ m/s}^2. \end{aligned}$$

The distance travelled by the body is given by

$$s = ut + \frac{1}{2}at^2$$

$\therefore$ Distance travelled by the lower body is given by

$$\begin{aligned} s_1 &= ut + \frac{1}{2} a_1 t^2 \\ &= 0 \times t + \frac{1}{2} \times 2.617 \times t^2 \quad (\because u = 0 \text{ and } a_1 = 2.617) \\ &= \frac{1}{2} \times 2.617 \times t^2 = 1.3085 t^2 \end{aligned}$$

...(ii)

Distance travelled by the upper body is given by

$$\begin{aligned} s_2 &= ut + \frac{1}{2} a_2 t^2 = 0 \times t + \frac{1}{2} \times 2.894 \times t^2 \\ &= \frac{1}{2} \times 2.894 \times t^2 = 1.447 t^2 \end{aligned}$$

...(iii)

Substituting the values of s_1 and s_2 in equation (i), we get

$$1.447 t^2 = 1.3085 t^2 + 10$$

$$\text{or } 1.447 t^2 - 1.3085 t^2 = 10 \quad \text{or } 0.1385 t^2 = 10$$

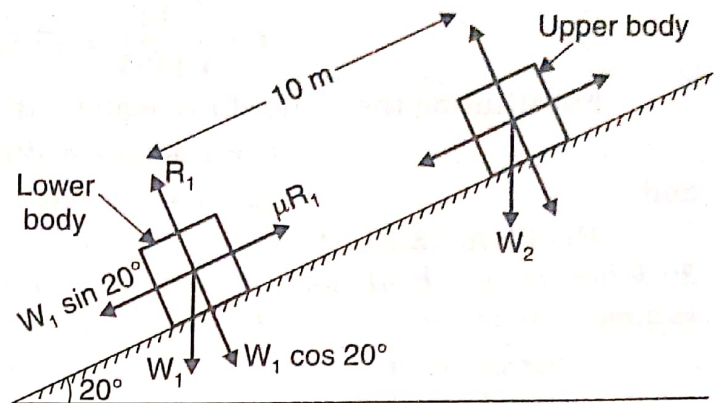


Fig. 15.5

$$t = \sqrt{\frac{10}{.1385}} = \sqrt{72.202} = 8.497 \text{ s.}$$

Substituting the value of t in equations (ii) and (iii), we get

$$s_1 = 1.3085 \times 8.497^2 = 94.476 \text{ m. Ans.}$$

$$s_2 = 1.447 \times 8.497^2 = 104.476 \text{ m. Ans.}$$

and

Problem 15.13. A truck weighing 6 kN just moves freely (engine is not running) at 36 kilometre per hour down a slope of 1 in 40, the road resistance at this speed just being sufficient to prevent any acceleration. Find the road resistance per kN weight of truck.

What power will the engine have to exert to run up the same slope at double the speed when the road resistance remains the same?

Sol. Given :

Weight of truck, $W = 6 \text{ kN} = 6 \times 1000 \text{ N}$
 $= 6000 \text{ N}$

Speed of truck, $u = 36 \text{ km/hr}$
 $= \frac{36 \times 1000}{60 \times 60} = 10 \text{ m/s}$

Slope of the road $= 1 \text{ in } 40$

$$\therefore \sin \theta = \frac{1}{40}$$

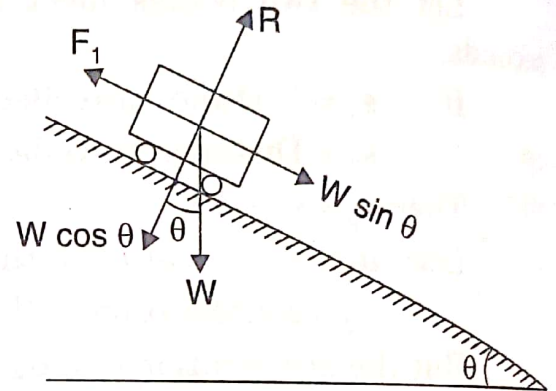


Fig. 15.6

Fig. 15.6 shows the position of the truck, when it is moving down the plane. The road resistance (F_1) is acting in the opposite direction of the motion of the truck. The truck is not having any acceleration and hence it is moving with a constant velocity of 10 m/s. Hence the net force on the truck in the direction of motion should be zero. But net force on the truck in the direction of motion (See Fig. 15.6).

$$= W \sin \theta - F_1$$

$$= 6000 \times \frac{1}{40} - F_1$$

$$\left\{ \because W = 6000 \text{ N, } \sin \theta = \frac{1}{40} \right\}$$

$\therefore$ According to the given condition,

$$6000 \times \frac{1}{40} - F_1 = 0$$

$$\therefore F_1 = 6000 \times \frac{1}{40} = \frac{6000}{40} = 150 \text{ N}$$

$\therefore$ Frictional force per tonne weight of truck

$$= \frac{F_1}{\text{Weight of truck in kN}} = \frac{150}{6} = 25 \text{ N. Ans.}$$

2nd Case. Truck is moving up an inclined plane of slope 1 in 40 with double speed. Road resistance is same.

$$\therefore \text{Speed of truck} = 2u = 2 \times 10 = 20 \text{ m/s}$$

$$\sin \theta = \frac{1}{40}$$

$$\text{Road resistance} = F_1 = 150 \text{ N.}$$

The truck is moving at constant speed of 20 m/s up the plane.

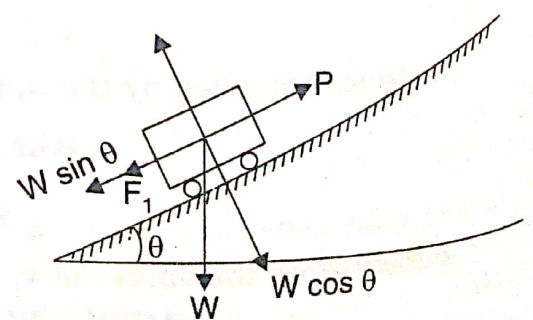


Fig. 15.7

Let P = force exerted by engine up the plane.

As the truck is moving with uniform speed, the net force on the truck along the plane should be zero (See Fig. 15.7)

or $P - W \sin \theta - F_1 = 0$

$$\therefore P = W \sin \theta + F_1 = 6000 \times \frac{1}{40} + 150 = 150 + 150 = 300 \text{ N}$$

*Power exerted by engine

$$= \frac{(\text{Force exerted by engine in N}) \times \text{Speed of engine in m/s}}{1000}$$

$$= \frac{(300 \text{ N}) \times 20 \text{ m/s}}{1000} = \frac{300 \times 20}{1000} = 6 \text{ kW. Ans.}$$

Problem 15.14. A train weighing 200 kN moves down a slope of 1 in 150 at 18 km/hour and engine develops 3.5 kW. What power will be required to pull the train up the slope at the same speed?

Sol. Given :

Weight of train, $W = 200 \text{ kN} = 200 \times 1000 \text{ N}$

Slope of the track = 1 in 150

$$\therefore \sin \theta = \frac{1}{150}$$

Speed of train, $u = 18 \text{ km/hr} = \frac{18 \times 1000}{60 \times 60} \text{ m/s} = 5 \text{ m/s}$

Power developed by engine = 3.5 kW.

1st Case. Train is moving down the plane as shown in Fig. 15.8 with a constant velocity and hence it will not have any acceleration. So the net force acting on the train in the direction of motion should be zero. But the forces acting on the train in the direction of motion are :

- (i) $W \sin \theta$ in the direction of motion
- (ii) Force of friction F in the opposite direction of motion
- (iii) Force (P) exerted by engine* in the direction of motion.

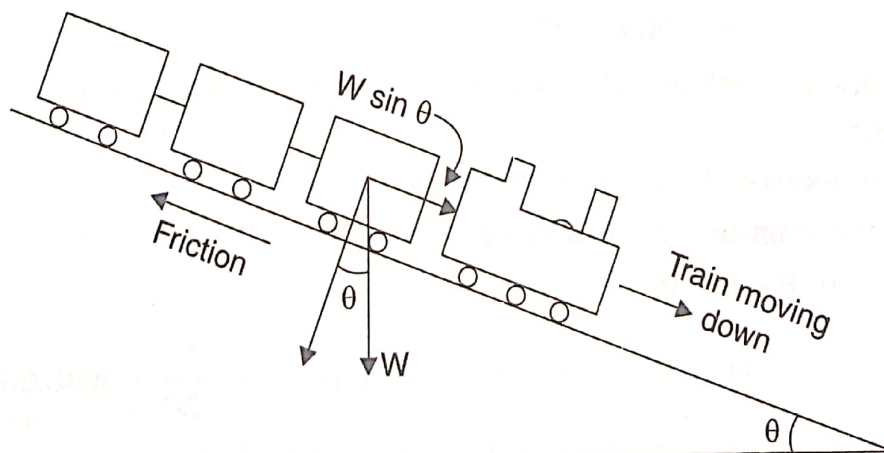


Fig. 15.8

$$*\text{Power in kW} = \frac{\text{Force in N} \times \text{Speed in m/s}}{1000}$$

Now the net force acting on engine in the direction of motion
 $= W \sin \theta - F + P$

$$\therefore W \sin \theta - F + P = 0$$

$$\text{or } (200 \times 1000) \times \frac{1}{150} - F + P = 0 \quad \text{or } 1333.33 - F + P = 0 \quad \dots(i)$$

But power developed by engine

$$= \frac{(\text{Force exerted by engine in N}) \times u}{1000} \quad \text{or } 3.5 = \frac{P \times 5}{1000}$$

$$\therefore P = \frac{3.5 \times 1000}{5} = 750 \text{ N.}$$

Substituting the value of P in equation (i), we get

$$1333.33 - F + 750 = 0$$

$$\text{or } F = 1333.33 + 750 = 2083.33 \text{ N.}$$

2nd Case. Train is moving up the same plane at the same speed as shown in Fig. 15.9. As the plane is same and hence frictional force will remain same.

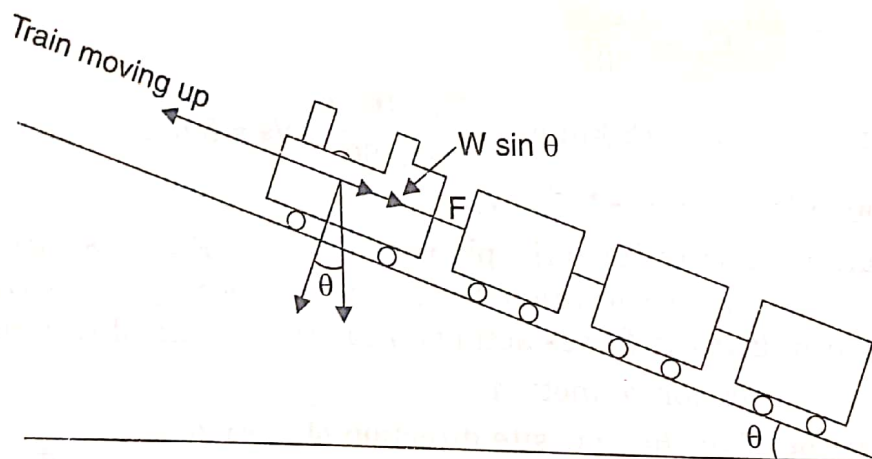


Fig. 15.9

$$\therefore F = 2083.33 \text{ N.}$$

The train is moving with constant speed and hence acceleration is zero. And net force on engine should be zero.

Let P^* = Force exerted by engine while moving up.

$$\text{Net force on the engine} = P^* - W \sin \theta - F.$$

$$\therefore P^* - W \sin \theta - F = 0$$

or

$$P^* = W \sin \theta + F = (200 \times 1000) \times \frac{1}{150} + 2083.33 \\ = 1333.33 + 2083.33 = 3416.66 \text{ N.}$$

$$\therefore \text{Power developed by engine} = \frac{P^* \times u}{1000} = \frac{3416.66 \times 5}{1000} = 17.083 \text{ kW. Ans.}$$

*In Problem 15.13, when the truck was moving down the plane engine was not running and hence force exerted by engine was zero. In this case engine is working and hence force will be exerted by engine.

Problems using D'Alembert's Principle.

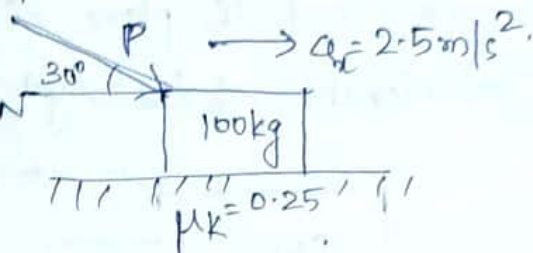
1. A 100 kg block rests on a horizontal plane. Find the magnitude of the force P required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction b/w the block & the plane is $\mu_k = 0.25$.

Ans Given.

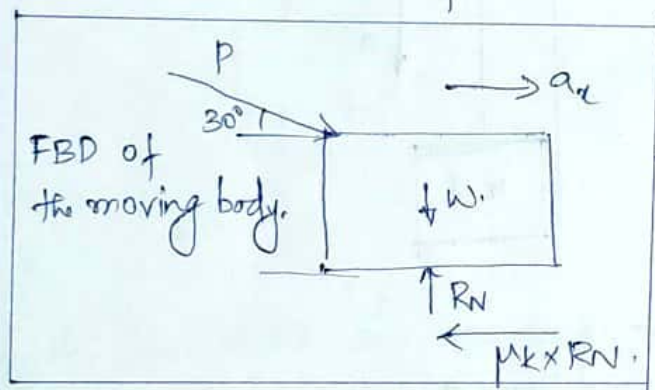
$$m = 100 \text{ kg}, W = 100 \times 9.81 \text{ N}$$

$$a = 2.5 \text{ m/s}^2$$

$$\mu_k = 0.25$$



Since when the force P is given, the body moves, the problem can be treated as from kinetics. FBD will be.



By D'Alembert's principle, (to transform it to static)
 $\sum F - ma = 0$.

[OR]

In problems we can take as the forces in the direction of motion as positive, others -ve.

In x direction,

Net force in the direction of motion = ma_x .

$$\therefore P \cos 30 - \mu_k \times R_N = 100 \times 2.5$$

In y direction

$$R_N - W - P \sin 30 = 0 \Rightarrow R_N = 981 + \frac{P}{2}$$

Substituting,

$$P \cos 30 - 0.25 \times \left(981 + \frac{P}{2} \right) = 100 \times 2.5$$

$$\frac{\sqrt{3}P}{2} - 0.25 \times \left(981 + \frac{P}{2} \right) = 100 \times 2.5$$

$$\Rightarrow P = \underline{\underline{667.5 \text{ N}}}$$

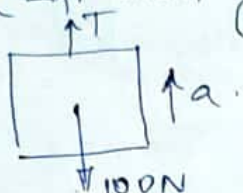
Problems from Lift.

- Q. A lift carries a weight of 100N and is moving with a uniform acceleration of 2.45 m/s^2 . Determine the tension in this cables supporting the lift, when.

- (a) Lift is moving upward
(b) " " " downward.

ANS

Case I (Lift moving upward)



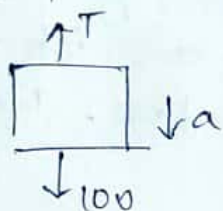
By D'Alembert's $\Rightarrow T - W = m a$

$$\Rightarrow T - W = \frac{W}{g} \times a$$

$$\Rightarrow T = W \left(1 + \frac{a}{g} \right)$$

$$= 100 \left(1 + \frac{2.45}{9.81} \right) = \underline{\underline{125 \text{ N}}}$$

Case II (Lift moving downward)



~~100N~~ $W - T = \frac{W}{g} a$

$$T = W \left(1 - \frac{a}{g} \right)$$

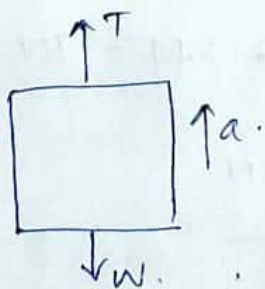
$$= 100 \left(1 - \frac{2.45}{9.81} \right) = \underline{\underline{75 \text{ N}}}$$

- Q. A lift has an upward acceleration of 1.225 m/s^2 .

- (a) What force will a man weighing 500N exert on the floor of the lift? (b) What force would he exert if the lift had an acc of 1.225 m/s^2 downwards (c) what upward acceleration would cause his weight to exert a force of 600N on the floor?

ANS

(a)



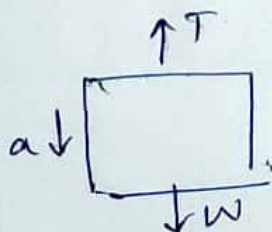
$$T - W = m a$$

$$T - 500 = \frac{500}{9.81} \times 1.225$$

$$T = \underline{\underline{562.4 \text{ N}}}$$

$\therefore$ Force exerted by man = $\underline{\underline{562.4 \text{ N}}}$

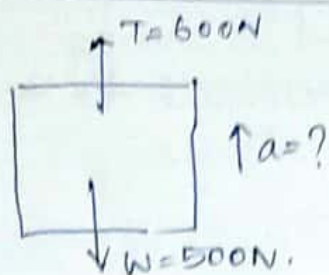
(b)



$$T = W \left(1 - \frac{a}{g} \right)$$

$$= 500 \left(1 - \frac{1.225}{9.81} \right) = 437.5 \text{ N}$$

(c)



Lift moving with upward acceleration exerting force of 600N i.e. $T = 600\text{N}$.

$$T = W(1 + a/g)$$

$$600 = 500(1 + a/9.81)$$

$$\frac{600}{500} = 1 + a/9.81$$

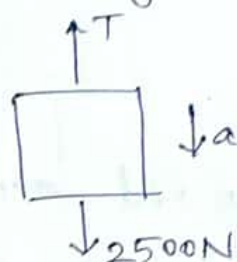
$$a = \underline{\underline{1.962\text{ m/s}^2}}$$

- Q An elevator weighs 2500N and is moving vertically downwards with a constant acceleration. Write the equation for the elevator cable tension. Starting from rest it travels a distance of 25m during an interval of 15 seconds. Find the cable tension during this time. Neglect all other resistance to motion.

$$W - T = ma$$

$$T = W - \frac{W}{g}a$$

$$\boxed{T = W(1 - a/g)}$$



From rest, $\therefore u = 0$, $s = 25\text{m}$, $t = 15\text{s}$

$$\therefore s = ut + \frac{1}{2}at^2$$

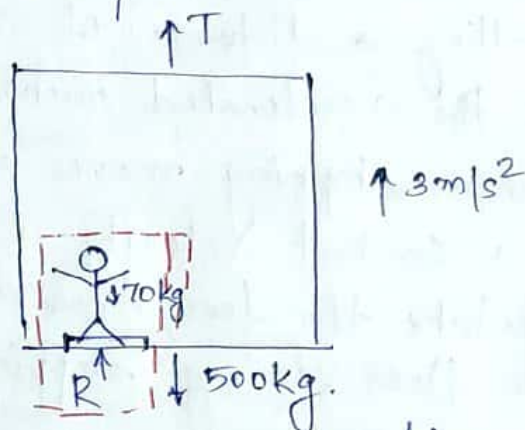
$$25 = \frac{1}{2} \times a \times 15^2 \Rightarrow a = \underline{\underline{0.222\text{ m/s}^2}}$$

$$\therefore T = 2500 \left(1 - \frac{0.222}{9.81}\right) = \underline{\underline{2443.37\text{N}}}$$

Tension in cable $T = 2443.37\text{N}$

- (Q) An elevator of mass 500 kg is ascending with an acceleration of 3 m/s^2 . During this ascent, its operator whose mass is 70 kg is standing on the scale placed on the floor. What is the scale reading? What will be the total tension in the cables of the elevator during this motion.

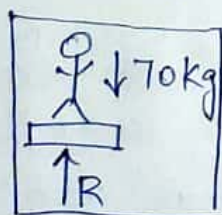
Ans In earlier problems, the weight of the lift was not given, Hence, the force exerted by the person would be equal to the tension of the lift cable. However in this case, since the weight of lift is given we analyse the problem as shown.



Here since the man is standing on the scale there is a reaction R on the weight of the man, and this reaction, R will be the scale reading

(a) To find scale reading.

Consider the man & the weight as a s/m. (system)



$\uparrow 3\text{ m/s}^2 \rightarrow$ By D'Alembert's
 $\Sigma F = ma$

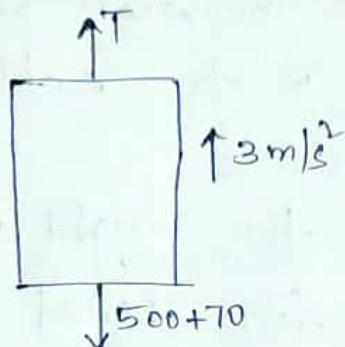
$$R - 70 \times 9.81 = 70 \times 3$$

$$R = 70 [3 + 9.81] \\ = \underline{\underline{896.7\text{ N}}}$$

(b) To find Tension in cable

Take the whole lift into consideration

$$T - (\text{Total weight}) = (\text{Total mass}) \times a.$$



$$T = (500+70) [a \uparrow + 9.8] = \underline{7301.7 \text{ N}}$$

(Q) An elevator of gross mass 500 kg starts moving upwards with a constant acceleration and acquires a velocity of 2 m/s, after travelling a distance of 3 m. Find the pull in the cables during the accelerated motion.

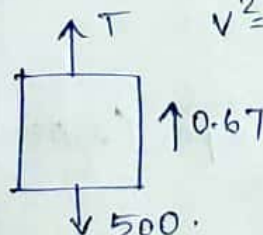
If the elevator when stopping moves with a constant deceleration from a constant velocity of 2 m/s and comes to rest in 2 s, calculate the force transmitted by a man of mass 75 kg the floor during stopping.

Ans Given. Case I
 $m = 500 \text{ kg}$, $s = 3 \text{ m}$, $v = 2 \text{ m/s}$, $T = ?$

Let $a_1 =$ constant acceleration

$$v^2 = u^2 + 2a_1 s \Rightarrow 2^2 = 0^2 + 2a_1 \times 3$$

$$4 = 6a_1 \Rightarrow a_1 = 2/3 = \underline{0.67 \text{ m/s}^2}$$



$$T - 500 \times 9.8 = 500 \times 0.67$$

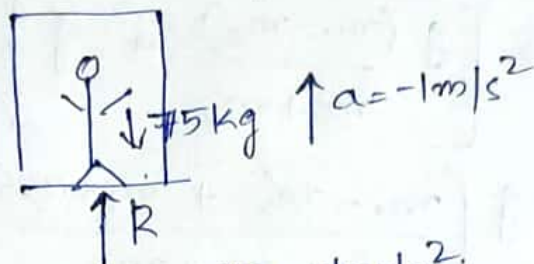
$$T = 500 [9.8 + 0.67] = \underline{5235 \text{ N}}$$

Now considering deceleration. In this case, $u = 2 \text{ m/s}$

$v = 0$ [comes to rest] $t = 2 \text{ s}$ & mass of man $m = 75 \text{ kg}$

$a_2 =$ deceleration $v = u + at \Rightarrow 0 = 2 + 2a_2 \Rightarrow a_2 = -1 \text{ m/s}^2$

For the man, (Considering only man & floor)



$$R - 75 \times 9.8 = 75 \times -1 \text{ m/s}^2$$

$$R = 75 [9.8 - 1]$$

$$= 75 \times 8.8 = \underline{\underline{660 \text{ N}}}$$

Motion of 2 bodies connected by a string and passing over a smooth pulley.

If the string is light and inextensible, the tension will be same throughout its length.

• Since the string is light and inextensible, tension = T

• If $m_1 > m_2$, m_1 moves downwards with acceleration

• Since string is inextensible, upward acceleration of m_2 = downward acceleration of m_1

• For $m_1 \Rightarrow$ By D'Alembert's principle,

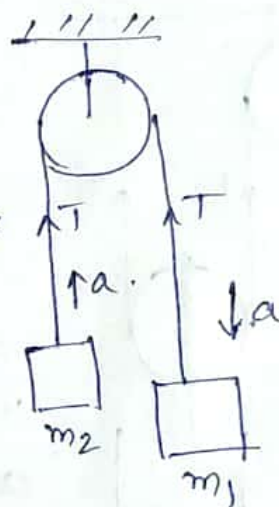
$$m_1 g - T = m_1 a \rightarrow (1)$$

• For $m_2 \Rightarrow$ By D'Alembert's

$$T - m_2 g = m_2 a \rightarrow (2)$$

$$(1) + (2) \Rightarrow g(m_1 - m_2) = (m_1 + m_2) a$$

$$a = g \frac{(m_1 - m_2)}{m_1 + m_2} \rightarrow (3)$$



From (2)

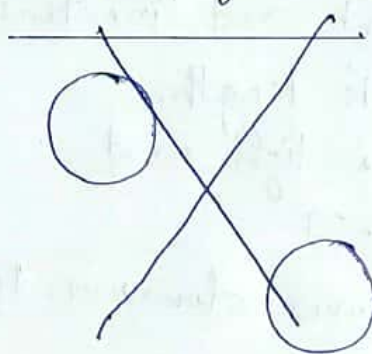
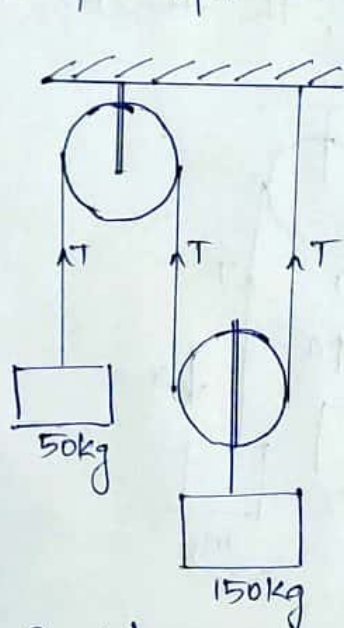
$$T = m_2 a + m_2 g$$

$$= m_2 \left[g \frac{(m_1 - m_2)}{m_1 + m_2} + g \right]$$

$$= m_2 g \left[\frac{m_1 - m_2 + m_1 + m_2}{m_1 + m_2} \right] = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

$$\therefore T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

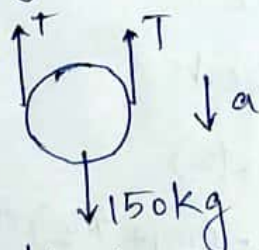
Q. Determine the tension in the strings and accelerations of two blocks of mass 150 kg and 50 kg connected by a string and a frictionless and weightless pulley as shown.



Given
 $m_1 = 150 \text{ kg}$, $m_2 = 50 \text{ kg}$

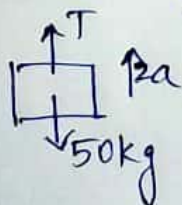
Since $150 > 50$,
 Motion of 150 kg block
 will be downwards,
 and motion of 50 kg block will be
 upwards. Let there be a constant
 acceleration = ' a ' m/s^2

Considering 150 kg block, pulley



$$150 \times 9.8 - 2T = 150 \times a \quad \text{--- (1)}$$

Consider 50 kg block. A little consideration shows that acceleration of 50 kg block = $2a$.

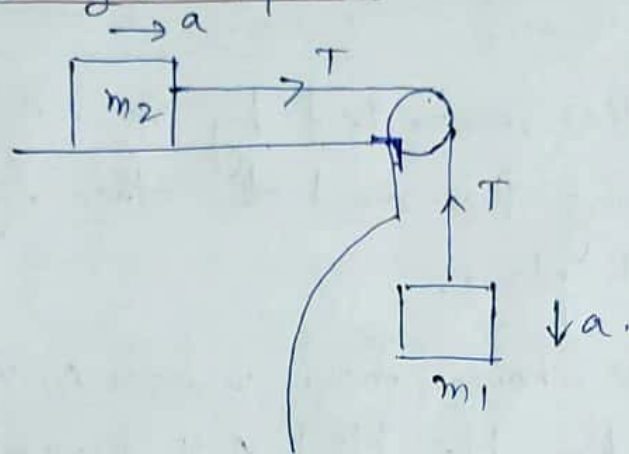


$$T - 50 \times 9.8 = 50 \times 2a$$

$$\text{Solving, } a = 1.4 \text{ m/s}^2, 2a = 2.8 \text{ m/s}^2$$

$$T = 630 \text{ N}$$

Motion of 2 bodies connected by a string, one of which is hanging free and the other lying on a smooth horizontal plane.



For body m_1 ,

$$m_1 g - T = m_1 a.$$

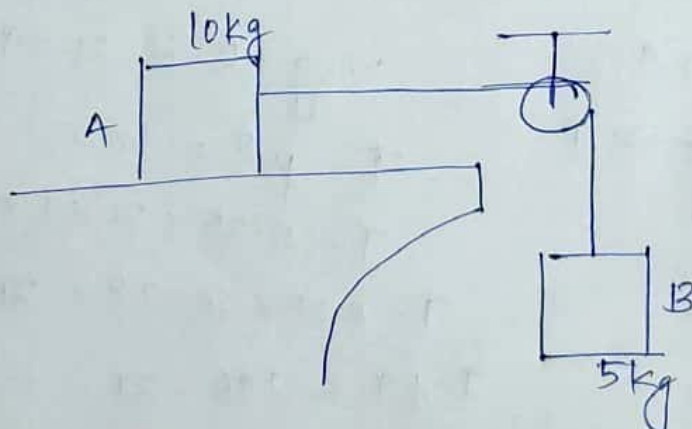
For body m_2

$$T = m_2 a \Rightarrow T \propto m_2$$

$$m_1 g - m_2 a = m_1 a$$

$$\boxed{\begin{aligned} a &= \frac{m_1 g}{m_1 + m_2} \\ T &= \frac{m_1 m_2 g}{m_1 + m_2} \end{aligned}}$$

Q Find the acceleration of a solid body A of mass 10 kg, when it is being pulled by another body B of mass 5 kg along a smooth horizontal plane. as shown.



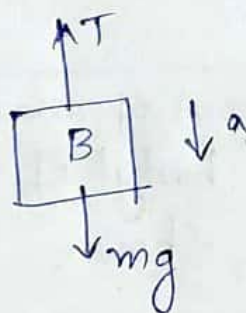
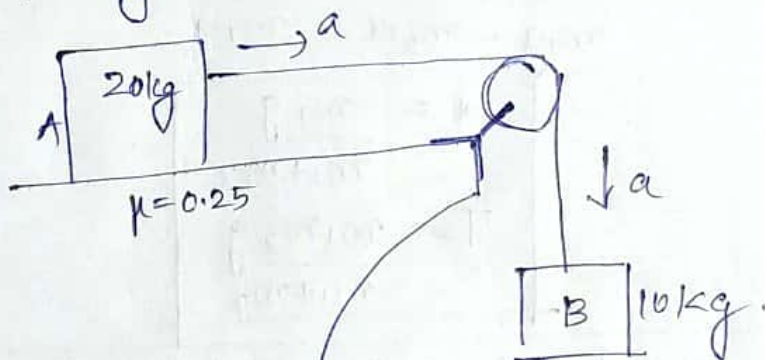
Also find tension in string, assume string inextensible
 $g = 9.8 \text{ m/s}^2$

$$\text{Acceleration, } a = \frac{m_1 g}{m_1 + m_2} = \frac{5 \times 9.8}{5 + 10} = \underline{\underline{3.27 \text{ m/s}^2}}$$

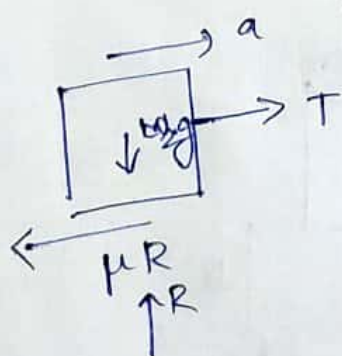
$$\text{Tension, } T = \frac{m_1 m_2 g}{m_1 + m_2} = \frac{5 \times 10 \times 9.8}{5 + 10} = \underline{\underline{32.7 \text{ N}}}$$

Motion of 2 bodies connected by a string, one of which is hanging free and the other lying on a rough horizontal plane.

- Q Two blocks as shown, have masses $A = 20 \text{ kg}$ & $B = 10 \text{ kg}$ and coeff of friction b/w block A & horizontal plane, $\mu = 0.25$. If the system is released from rest and block B falls through a vertical distance of 1 m , what is the velocity acquired by it? Neglect friction in pulley & extension of string.



$$\Rightarrow (10 \times 9.8) - T = 10a \quad \rightarrow (1)$$



$$m_2 g = R \Rightarrow 20 \times 9.8 = R$$

$$T - \mu R = ma$$

$$T - 0.25 \times 20 \times 9.8 = 20 \times a \rightarrow (2)$$

$$T - 0.25 \times 20 \times 9.8 = 20 \times \frac{(10 \times 9.8) - T}{10}$$

$$T - 49 = 196 - 2T$$

$$3T = 245 \Rightarrow T = \underline{\underline{81.67 \text{ N}}}$$

$$a = \underline{\underline{1.63 \text{ m/s}^2}}$$

$$v^2 = u^2 + 2as. \quad u = 0, s = 1\text{m}$$

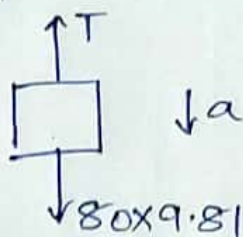
$$v^2 = 0 + 2 \times 1.63 \times 1 \Rightarrow v = \underline{1.807\text{ m/s}}$$

- Q A body of mass 150 kg, rests on a rough plane inclined at 10° to the horizontal. It is pulled up the plane, from rest, by means of a light inextensible string running parallel to the plane. The portion of the rope, beyond the pulley hangs vertically down and carries a mass of 80 kg at the end. If the coefficient of friction for the plane and the body is 0.2, find
- (a) the tension in rope (b) acceleration in m/s^2 with which the body moves up the plane (c) the distance in metres moved by the body in 4 seconds starting from the rest.

Ans



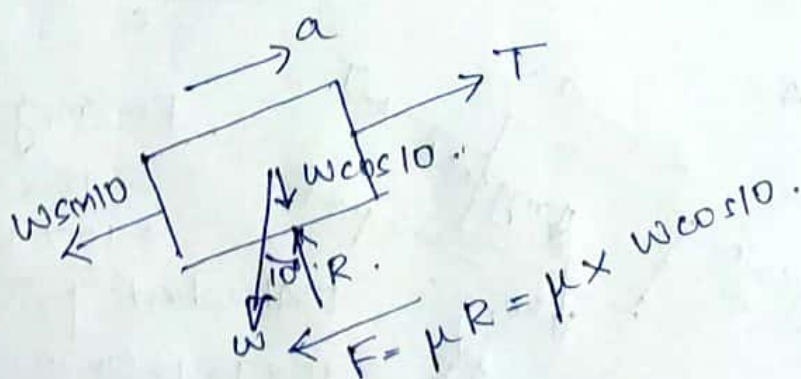
Block B



$$\Rightarrow \boxed{80 \times 9.81 - T = 80 \times a.}$$

$$\Rightarrow a = \frac{80 \times 9.81 - T}{80} = 9.81 - \frac{T}{80}$$

Block A



$$T - W \sin 10 - \mu W \cos 10 = m a$$

$$T - 150 \times 9.81 \times \sin 10 - 0.2 \times 150 \times 9.81 \times \cos 10 = 150 \times a$$

$$T - 255.523 - 289.829 = 150 a$$

$$\boxed{T - 545.351 = 150 a}$$

$$T - 545.351 = 150 \times \left[9.81 - \frac{T}{80} \right]$$

$$T - 545.351 = 1471.5 - 1.875 T$$

$$2.875 T = 2016.851$$

$$\boxed{T = 701.51 \text{ N}} \text{ Ans (a)}$$

$$\boxed{a = 1.04 \text{ m/s}^2} \text{ Ans (b)}$$

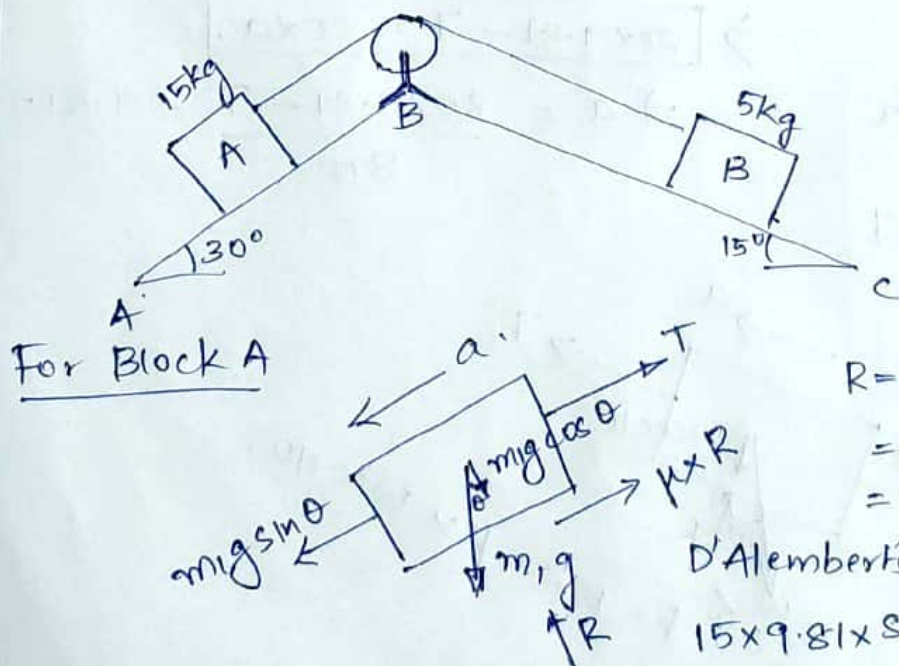
(c)

$$u = 0, t = 4 \text{ s}, a = 1.04 \text{ m/s}^2$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = 0 + \frac{1}{2} \times 1.04 \times 4^2$$

$$\underline{\underline{s = 8.32 \text{ m}}}$$

Q. Two rough planes inclined at 30° and 15° to the horizontal and of the same height are placed back to back. Two bodies of masses 15 kg and 5 kg are placed on the faces and connected by a string over the top of the planes. If $\mu = 0.3$, find from fundamentals the resulting acceleration.



$$R = m_1 g \cos \theta$$

$$= 15 \times 9.81 \times \cos 30$$

$$= 127.4356 \text{ N}$$

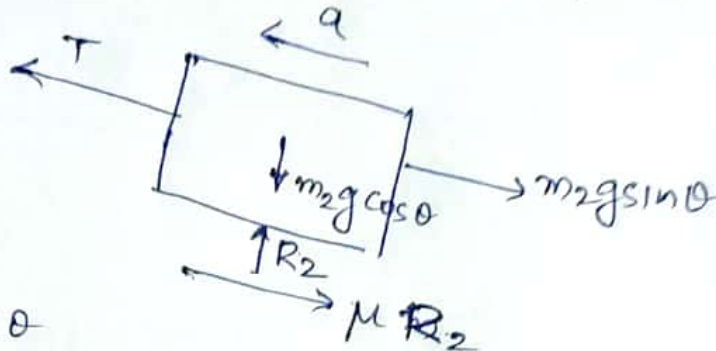
D'Alembert's principle

$$15 \times 9.81 \times \sin 30 - T - 0.3 \times 127.4356 = 15 \times a$$

$$73.575 - 38.23 - T = 15a$$

$$\boxed{35.344 - T = 15a} \longrightarrow (1) \Rightarrow \underline{\underline{T = 35.344 - 15a}}$$

Block B



$$R_2 = m_2 g \cos \theta$$

$$= 5 \times 9.81 \times \cos 15 = \underline{\underline{47.378 \text{ N}}}$$

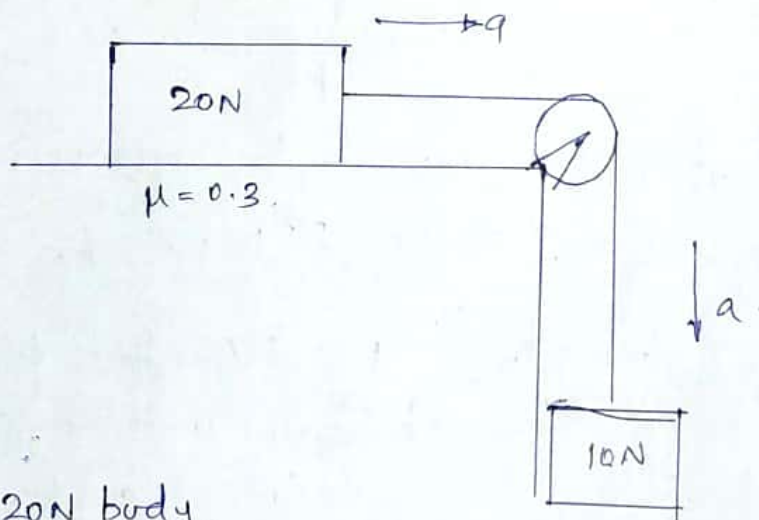
$$T - 0.3 \times 47.378 - 5 \times 9.81 \times \sin 15 = 5 \times a$$

$$\boxed{T - 26.9084 = 5a} \longrightarrow (2)$$

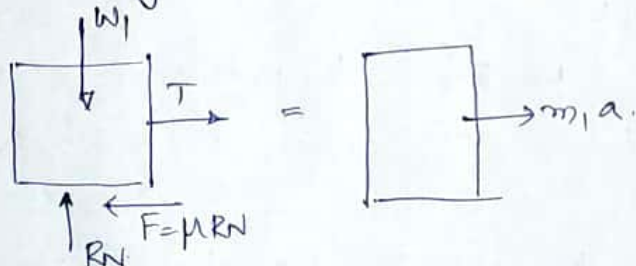
$$35.34 - 15a - 26.90 = 5a$$

$$8.44 = 20a \Rightarrow \underline{\underline{a = 0.42 \text{ m/s}^2}}$$

2. Two bodies of weight 20N and 10N are connected to the 2 ends of a light inextensible string, passing over a smooth pulley. The weight of 20N is placed on a horizontal surface while the weight of 10N is hanging free in air. The horizontal surface is a rough one, having coeff of friction between the weight 20N and the plane surface equal to 0.3, determine.
- (1) The acceleration of the system (2) The tension in the string



FBD of 20N body.

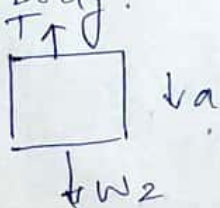


$$\sum F_y = 0 = R_N - W_1 \Rightarrow R_N = 20N$$

$$\sum F_x = m a \quad \left[\text{D'Alembert's} \right]$$

$$T - \mu R_N = m_1 a \Rightarrow \boxed{T = \frac{20}{9.81} a + 0.3 \times 20} \rightarrow (1)$$

FBD of 10N body.



$$\text{D'Alembert's} \Rightarrow W_2 - T = m_2 a \rightarrow (2)$$

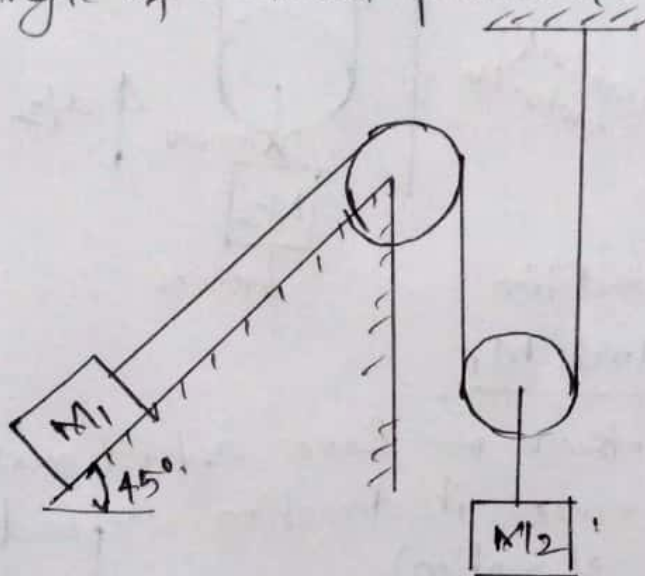
$$10 - T = \frac{10}{9.81} a \rightarrow (2)$$

$$\therefore \Rightarrow 10 - \frac{20a}{9.81} - 20 \times 0.3 = \frac{10}{9.81} a$$

$$\Rightarrow a = 1.308 \text{ m/s}^2$$

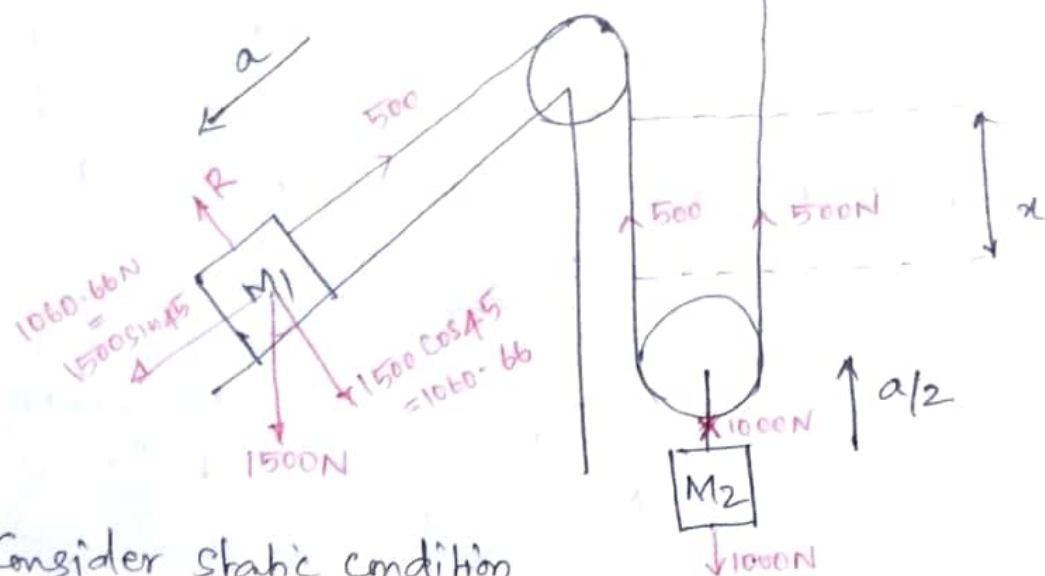
Sub a in (2) $\boxed{T = 8.67N}$

Q A block of mass M_1 , resting on an inclined plane is connected by a string and pulleys to another block of mass M_2 as shown in Fig. Find the tension in the string and acceleration of the blocks. Assume the coefficient of friction between the blocks M_1 and the plane to be 0.2. $M_1 = 1500\text{ N}$, $M_2 = 1000\text{ N}$. Angle of inclined plane $= 45^\circ$.



$$\begin{aligned} M_1 &= 1500\text{ N} \\ M_2 &= 1000\text{ N} \\ \mu &= 0.2 \end{aligned}$$

To find the direction of motion



Consider static condition

Consider block M_1

Here we know that we have a frictional force, whose direction is unknown (since its direction depends on the direction of motion)

Case I (Friction towards downward)

Net force to the upward direction
= 500 N

Net force to the downward direction

$$\begin{aligned}
 &= 1500 \sin 45 + F_r \text{ (Friction)} \\
 &= 1500 \sin 45 + 0.2 \times 1500 \cos 45 \\
 &= \underline{\underline{1272.792 \text{ N}}}
 \end{aligned}$$

$\therefore$ downward > upward
(M_1 moves downward)

Case II (Friction upward)

$$\begin{aligned}
 \text{Net force to upward direction} &= \\
 &500 + 0.2 \times 1500 \cos 45 \\
 &= \underline{\underline{712.132 \text{ N}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Net force to downward direction} &= 1500 \sin 45 \\
 &= 1272.792
 \end{aligned}$$

down > Upward.

(M_1 moves downward)

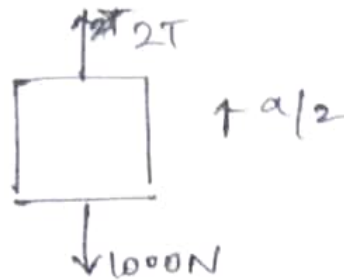
$\therefore$ from both case, M_1 moves downward & M_2 upward

To find accelerations

Little considerations show that M_1 moves downward with a , acceler, M_2 moves up with $a/2$ acceleration

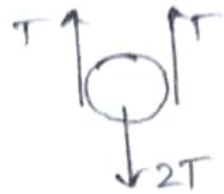
To find unknowns

For M_2



$$2T - 1000 \text{ N} = \frac{1000}{9.81} \times \frac{a}{2}$$

For pulley



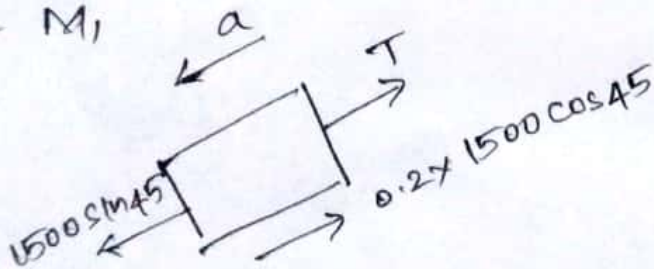
~~For pulley~~



~~$2T - 1000 \text{ N} = 101.9$~~

$$\boxed{2T - 1000 = 101.936 \frac{a}{2} \rightarrow 0),}$$

For M_1



$$1500 \sin 45 - 0.2 \times 1500 \cos 45 - T = \frac{1500}{9.81} \times a$$

$$\boxed{848.528 - T = 152.905 a}$$

$$2 [848.528 - 152.905 a] - 1000 = 101.936 \frac{a}{2}$$

$$\boxed{\begin{aligned} a &= 1.954 \text{ m/s}^2 \\ a/2 &= 0.977 \text{ m/s}^2 \\ T &= 549.75 \text{ N} \end{aligned}}$$

Problems using D'Alembert's Principle.

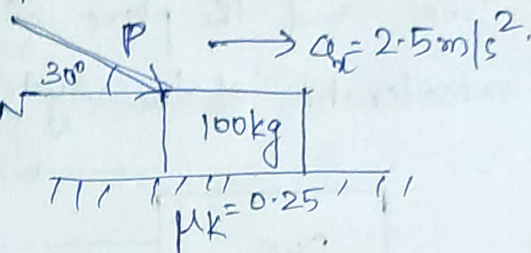
1. A 100 kg block rests on a horizontal plane. Find the magnitude of the force P required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction b/w the block & the plane is $\mu_k = 0.25$.

Ans Given.

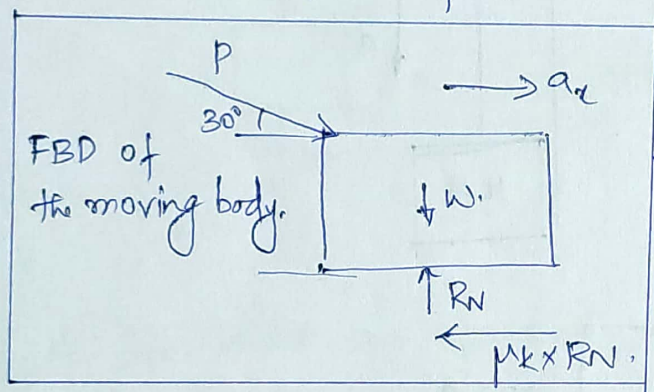
$$m = 100 \text{ kg}, W = 100 \times 9.81 \text{ N}$$

$$a = 2.5 \text{ m/s}^2$$

$$\mu_k = 0.25$$



Since when the force P is given, the body moves, the problem can be treated as from kinetics. FBD will be.



By D'Alembert's principle, (to transform it to static)

$$\sum F - ma = 0.$$

[OR]

In problems we can take as the forces in the direction of motion as positive, others -ve.

In x direction,

Net force in the direction of motion = ma_x .

$$\therefore P \cos 30 - \mu_k \times R_N = 100 \times 2.5$$

In y direction

$$R_N - W - P \sin 30 = 0. \Rightarrow R_N = 981 + \frac{P}{2}$$

Substituting,

$$P \cos 30 - 0.25 \times \left(981 + \frac{P}{2} \right) = 100 \times 2.5$$

$$\frac{\sqrt{3}P}{2} - 0.25 \times \left(981 + \frac{P}{2} \right) = 100 \times 2.5$$

$$\Rightarrow P = \underline{\underline{667.5 \text{ N}}}$$

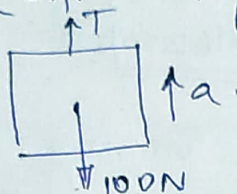
Problems from Lift.

Q. A lift carries a weight of 100N and is moving with a uniform acceleration of 2.45 m/s^2 . Determine the tension in this cables supporting the lift, when.

- (a) Lift is moving upward
(b) " " " downward.

ANS

Case I (Lift moving upward)



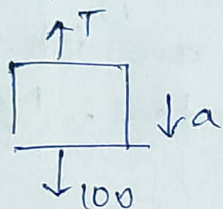
By D'Alembert's $\Rightarrow T - W = ma$

$$\Rightarrow T - W = \frac{W}{g} \times a$$

$$\Rightarrow T = W \left(1 + \frac{a}{g} \right)$$

$$= 100 \left(1 + \frac{2.45}{9.81} \right) = \underline{\underline{125 \text{ N}}}$$

Case II (Lift moving downward)



~~100N~~ $W - T = \frac{W}{g} a$

$$T = W \left(1 - \frac{a}{g} \right)$$

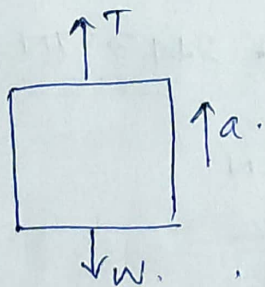
$$= 100 \left(1 - \frac{2.45}{9.81} \right) = \underline{\underline{75 \text{ N}}}$$

Q. A lift has an upward acceleration of 1.225 m/s^2 .

- (a) What force will a man weighing 500N exert on the floor of the lift? (b) What force would he exert if the lift had an acc of 1.225 m/s^2 downwards (c) What upward acceleration would cause his weight to exert a force of 600N on the floor?

ANS

(a)



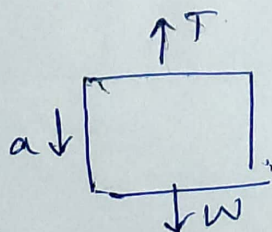
$$T - W = ma$$

$$T - 500 = \frac{500}{9.81} \times 1.225$$

$$T = \underline{\underline{562.4 \text{ N}}}$$

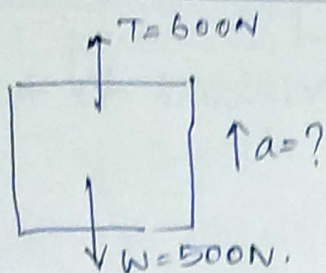
$\therefore$ Force exerted by man = $\underline{\underline{562.4 \text{ N}}}$

(b)



$$T = W \left(1 - \frac{a}{g} \right) = 500 \left(1 - \frac{1.225}{9.81} \right) = \underline{\underline{437.5 \text{ N}}}$$

(c)



Lift moving with upward acceleration exerting force of 600 N i.e. $T = 600 \text{ N}$.

$$T = W(1 + a/g)$$

$$600 = 500(1 + a/9.81)$$

$$\frac{600}{500} = 1 + a/9.81$$

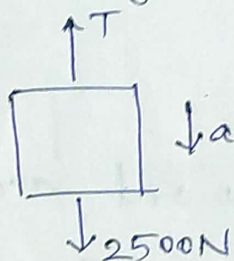
$$a = \underline{\underline{1.962 \text{ m/s}^2}}$$

- Q An elevator weighs 2500 N and is moving vertically downwards with a constant acceleration. Write the equation for the elevator cable tension. Starting from rest it travels a distance of 25 m during an interval of 15 seconds. Find the cable tension during this time. Neglect all other resistance to motion.

$$W - T = ma$$

$$T = W - \frac{W}{g}a$$

$$\boxed{T = W(1 - a/g)}$$



From rest, $\therefore u = 0$, $s = 25 \text{ m}$, $t = 15 \text{ s}$

$$\therefore s = ut + \frac{1}{2}at^2$$

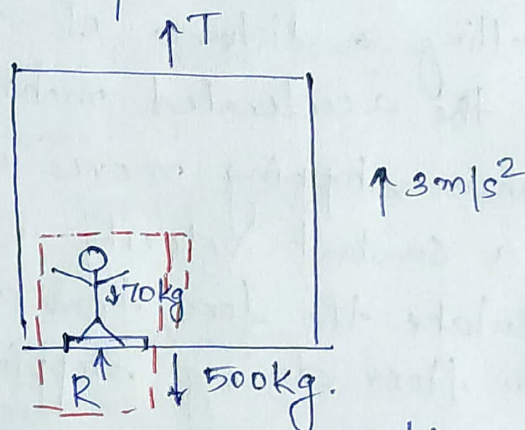
$$25 = \frac{1}{2} \times a \times 15^2 \Rightarrow a = \underline{\underline{0.222 \text{ m/s}^2}}$$

$$\therefore T = 2500 \left(1 - \frac{0.222}{9.81}\right) = \underline{\underline{2443.37 \text{ N}}}$$

Tension in cable $T = 2443.37 \text{ N}$

- (Q) An elevator of mass 500 kg is ascending with an acceleration of 3 m/s^2 . During this ascent, its operator whose mass is 70 kg is standing on the scale placed on the floor. What is the scale reading? What will be the total tension in the cables of the elevator during this motion.

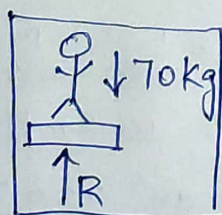
Ans In earlier problems, the weight of the lift was not given, Hence, the force exerted by the person would be equal to the tension of the lift cable. However in this case, since the weight of lift is given we analyse the problem as shown.



Here since the man is standing on the scale there is a reaction R on the weight of the man, and this reaction, R will be the scale reading

(a) To find scale reading.

Consider the man & the weight as a s/m. (system)



$\uparrow 3\text{ m/s}^2 \rightarrow$ By D'Alembert's $\Sigma F = ma$.

$$R - 70 \times 9.81 = 70 \times 3.$$

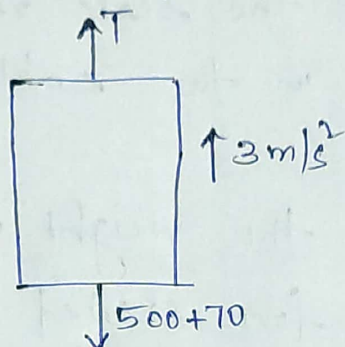
$$R = 70 [3 + 9.81]$$

$$= \underline{\underline{896.7\text{ N}}}$$

(b) To find Tension in cable

Take the whole lift into consideration

$$T - (\text{Total weight}) = (\text{Total mass}) \times a.$$



$$T = (500+70) [a \uparrow 3 + 9.8] = \underline{7301.7 \text{ N}}$$

(Q) An elevator of gross mass 500 kg starts moving upwards with a constant acceleration and acquires a velocity of 2 m/s, after travelling a distance of 3 m. Find the pull in the cables during the accelerated motion.

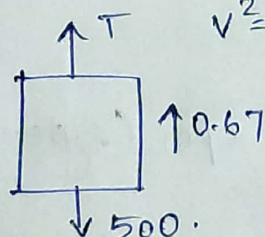
If the elevator when stopping moves with a constant deceleration from a constant velocity of 2 m/s and comes to rest in 2 s, calculate the force transmitted by a man of mass 75 kg the floor during stopping.

Ans Given. Case I
 $m = 500 \text{ kg}$, $s = 3 \text{ m}$, $v = 2 \text{ m/s}$, $T = ?$

Let $a_1 =$ constant acceleration

$$v^2 = u^2 + 2a_1 s \Rightarrow 2^2 = 0^2 + 2a_1 \times 3$$

$$4 = 6a_1 \Rightarrow a_1 = 2/3 = \underline{0.67 \text{ m/s}^2}$$



$$T - 500 \times 9.8 = 500 \times 0.67$$

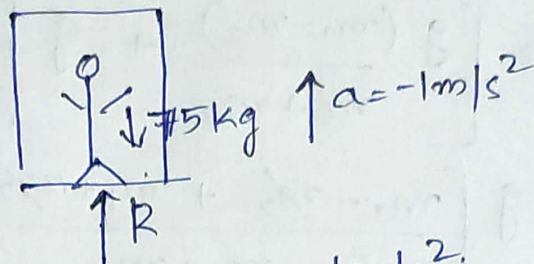
$$T = 500 [9.8 + 0.67] = \underline{5235 \text{ N}}$$

Now considering deceleration. In this case, $u = 2 \text{ m/s}$

$v = 0$ [comes to rest] $t = 2 \text{ s}$ & mass of man $m = 75 \text{ kg}$

$$a_2 = \text{deceleration} \quad v = u + at \Rightarrow 0 = 2 + 2a_2 \Rightarrow a_2 = -1 \text{ m/s}^2$$

For the man, (Considering only man & floor)



$$R - 75 \times 9.8 = 75 \times -1 \text{ m/s}^2$$

$$R = 75 [9.8 - 1]$$

$$= 75 \times 8.8 = \underline{\underline{660 \text{ N}}}$$

Motion of 2 bodies connected by a string and passing over a smooth pulley.

If the string is light and inextensible, the tension will be same throughout its length.

• Since the string is light and inextensible, tension = T

• If $m_1 > m_2$, m_1 moves downwards with acceleration

• Since string is inextensible, upward acceleration of m_2 = downward acceleration of m_1

• For $m_1 \Rightarrow$ By D'Alembert's principle,

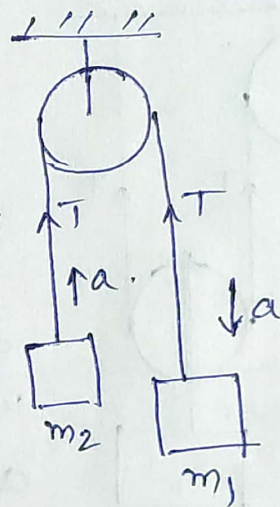
$$m_1 g - T = m_1 a \rightarrow (1)$$

• For $m_2 \Rightarrow$ By D'Alembert's

$$T - m_2 g = m_2 a \rightarrow (2)$$

$$(1) + (2) \Rightarrow g(m_1 - m_2) = (m_1 + m_2) a$$

$$a = g \frac{(m_1 - m_2)}{m_1 + m_2} \rightarrow (3)$$



From (2)

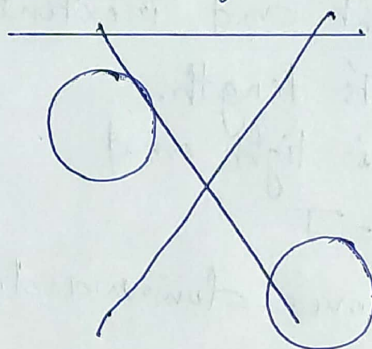
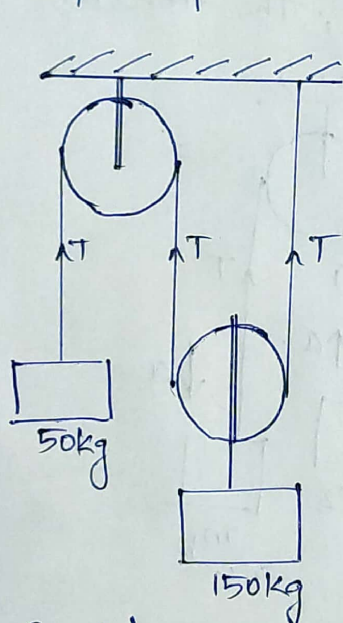
$$T = m_2 a + m_2 g$$

$$= m_2 \left[g \frac{(m_1 - m_2)}{m_1 + m_2} + g \right]$$

$$= m_2 g \left[\frac{m_1 - m_2 + m_1 + m_2}{m_1 + m_2} \right] = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

$$\therefore T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

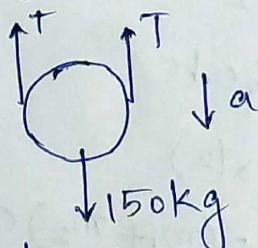
Q. Determine the tension in the strings and accelerations of two blocks of mass 150 kg and 50 kg connected by a string and a frictionless and weightless pulley as shown.



Given
 $m_1 = 150 \text{ kg}$, $m_2 = 50 \text{ kg}$

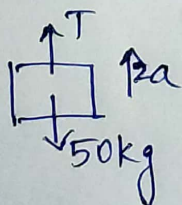
Since $150 > 50$,
 Motion of 150 kg block will be downwards, and motion of 50 kg block will be upwards. Let there be a constant acceleration = ' a ' m/s^2

Considering 150 kg block, pulley



$$150 \times 9.8 - 2T = 150 \times a \quad \text{--- (1)}$$

Consider 50 kg block. A little consideration shows that acceleration of 50 kg block = $2a$.

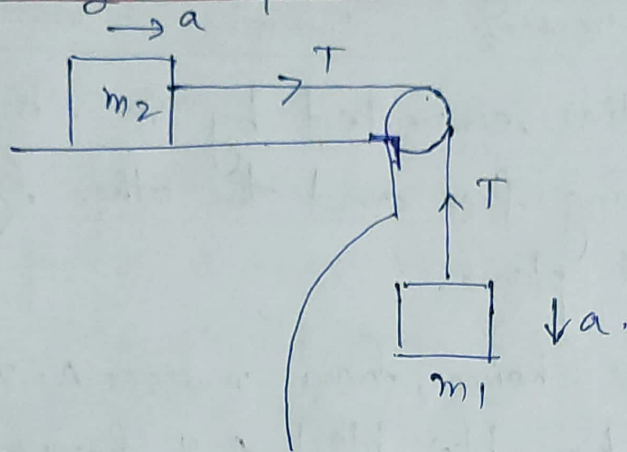


$$T - 50 \times 9.8 = 50 \times 2a$$

$$\text{Solving, } a = 1.4 \text{ m/s}^2, 2a = 2.8 \text{ m/s}^2$$

$$T = 630 \text{ N}$$

Motion of 2 bodies connected by a string, one of which is hanging free and the other lying on a smooth horizontal plane.



For body m_1 ,

$$m_1 g - T = m_1 a.$$

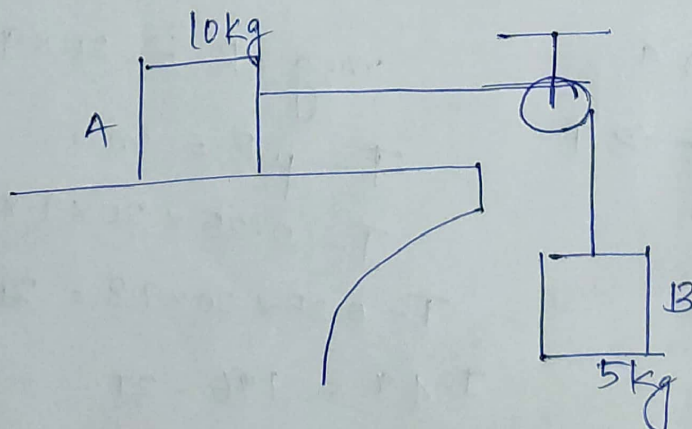
For body m_2

$$T = m_2 a \Rightarrow T \propto m_2$$

$$m_1 g - m_2 a = m_1 a$$

$$\boxed{\begin{aligned} a &= \frac{m_1 g}{m_1 + m_2} \\ T &= \frac{m_1 m_2 g}{m_1 + m_2} \end{aligned}}$$

Q Find the acceleration of a solid body A of mass 10 kg, when it is being pulled by another body B of mass 5 kg along a smooth horizontal plane. as shown.



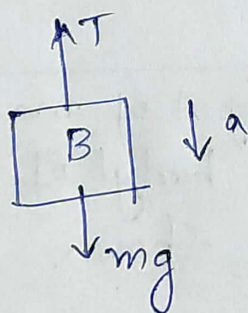
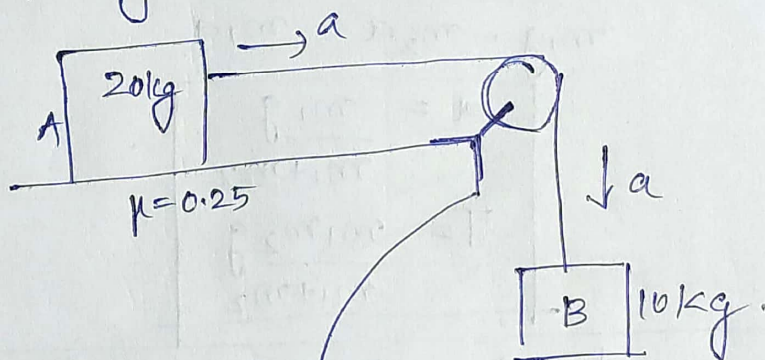
Also find tension in string, assume string inextensible
 $g = 9.8 \text{ m/s}^2$

$$\text{Acceleration, } a = \frac{m_1 g}{m_1 + m_2} = \frac{5 \times 9.8}{5 + 10} = \underline{\underline{3.27 \text{ m/s}^2}}$$

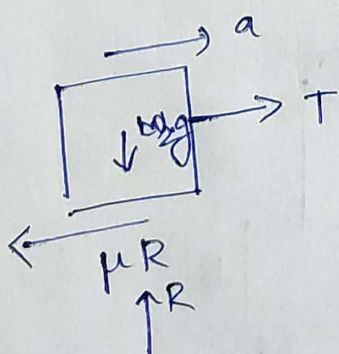
$$\text{Tension, } T = \frac{m_1 m_2 g}{m_1 + m_2} = \frac{5 \times 10 \times 9.8}{5 + 10} = \underline{\underline{32.7 \text{ N}}}$$

Motion of 2 bodies connected by a string, one of which is hanging free and the other lying on a rough horizontal plane.

- Q Two blocks as shown, have masses $A = 20 \text{ kg}$ & $B = 10 \text{ kg}$ and coeff of friction b/w block A & horizontal plane, $\mu = 0.25$. If the system is released from rest and block B falls through a vertical distance of 1 m , what is the velocity acquired by it? Neglect friction in pulley & extension of string.



$$\Rightarrow (10 \times 9.8) - T = 10 \times a \quad \rightarrow (1)$$



$$m_2 g = R \Rightarrow 20 \times 9.8 = R$$

$$T - \mu R = ma$$

$$T - 0.25 \times 20 \times 9.8 = 20 \times a \rightarrow (2)$$

$$T - 0.25 \times 20 \times 9.8 = 20 \times \frac{(10 \times 9.8) - T}{10}$$

$$T - 49 = 196 - 2T$$

$$3T = 245 \Rightarrow T = 81.67 \text{ N}$$

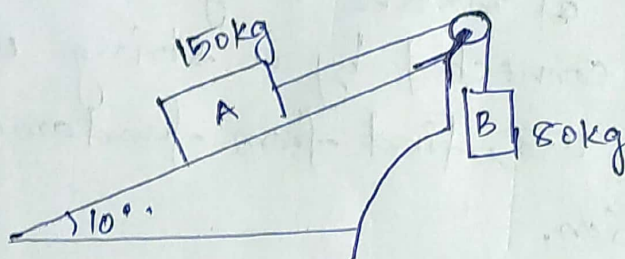
$$a = 1.63 \text{ m/s}^2 //$$

$$v^2 = u^2 + 2as. \quad u = 0, s = 1\text{m}$$

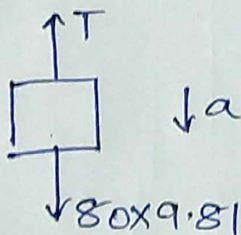
$$v^2 = 0 + 2 \times 1.63 \times 1 \Rightarrow v = \underline{1.807\text{m/s}}$$

- Q A body of mass 150 kg, rests on a rough plane inclined at 10° to the horizontal. It is pulled up the plane, from rest, by means of a light inextensible string running parallel to the plane. The portion of the rope, beyond the pulley hangs vertically down and carries a mass of 80 kg at the end. If the coefficient of friction for the plane and the body is 0.2, find
- (a) the tension in rope (b) acceleration in m/s^2 with which the body moves up the plane (c) the distance in metres moved by the body in 4 seconds starting from the rest.

Ans



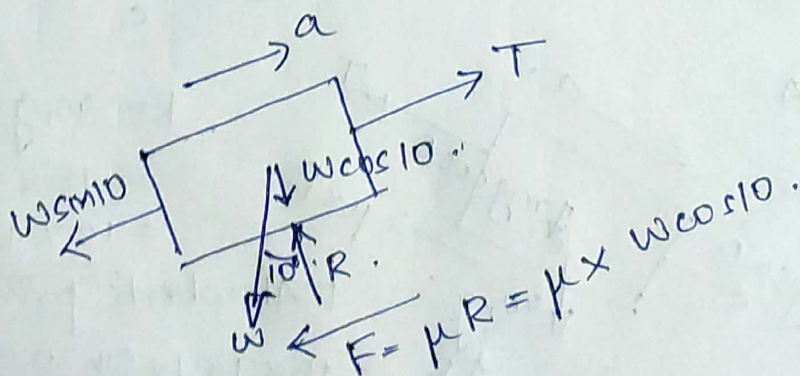
Block B



$$\Rightarrow 80 \times 9.81 - T = 80 \times a$$

$$\Rightarrow a = \frac{80 \times 9.81 - T}{80} = 9.81 - \frac{T}{80}$$

Block A



$$T - W \sin 10 - \mu W \cos 10 = m a.$$

$$T - 150 \times 9.81 \times \sin 10 - 0.2 \times 150 \times 9.81 \times \cos 10 = 150 \times a$$

$$T - 255.523 - 289.829 = 150 a.$$

$$\boxed{T - 545.351 = 150 a.}$$

$$T - 545.351 = 150 \times \left[9.81 - \frac{T}{80} \right]$$

$$T - 545.351 = 1471.5 - 1.875 T$$

$$2.875 T = 2016.851$$

$$\boxed{T = 701.51 \text{ N}} \text{ Ans (a)}$$

$$\boxed{a = 1.04 \text{ m/s}^2} \text{ Ans (b)}$$

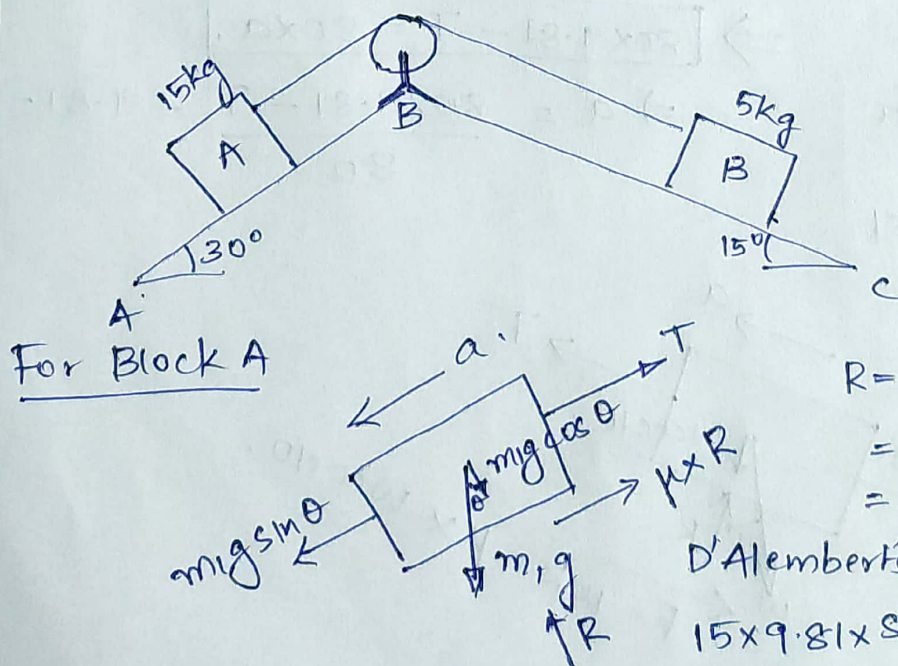
(c).

$$u = 0, t = 4 \text{ s}, a = 1.04 \text{ m/s}^2$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = 0 + \frac{1}{2} \times 1.04 \times 4^2$$

$$\underline{\underline{s = 8.32 \text{ m}}}$$

Q. Two rough planes inclined at 30° and 15° to the horizontal and of the same height are placed back to back. Two bodies of masses 15 kg and 5 kg are placed on the faces and connected by a string over the top of the planes. If $\mu = 0.3$, find from fundamentals the resulting acceleration.



For Block A

$$R = m_1 g \cos \theta$$

$$= 15 \times 9.81 \times \cos 30$$

$$= 127.4356 \text{ N}$$

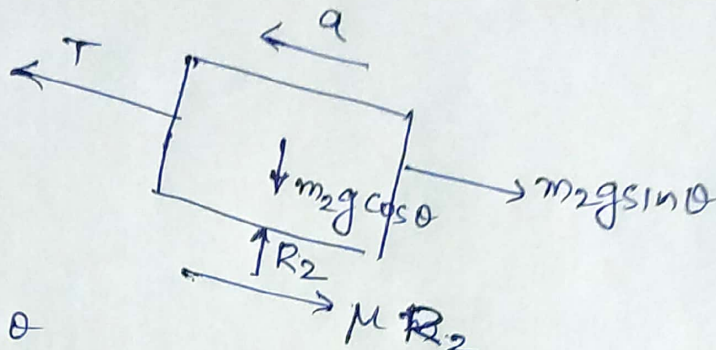
D'Alembert's principle.

$$15 \times 9.81 \times \sin 30 - T - 0.3 \times 127.4356 = 15 \times a.$$

$$73.575 - 38.23 - T = 15a.$$

$$\boxed{35.344 - T = 15a} \longrightarrow (1) \Rightarrow \underline{\underline{T = 35.344 - 15a}}$$

Block B



$$R_2 = m_2 g \cos \theta$$

$$= 5 \times 9.81 \times \cos 15 = \underline{\underline{47.378 \text{ N}}}$$

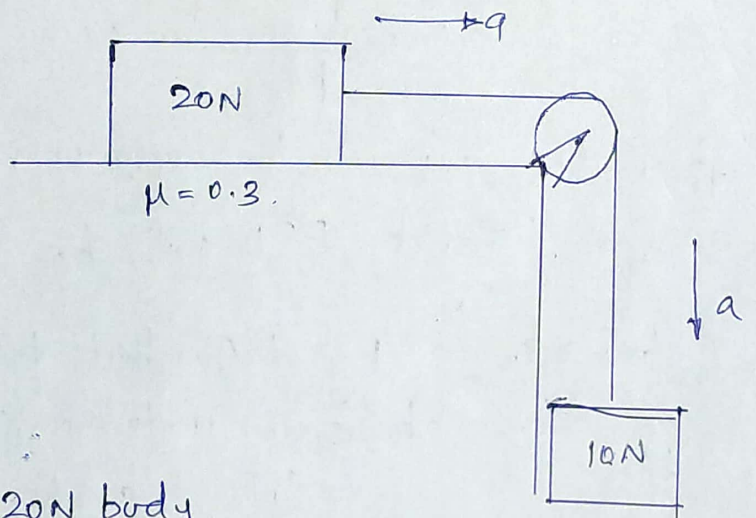
$$T - 0.3 \times 47.378 - 5 \times 9.81 \times \sin 15 = 5 \times a$$

$$\boxed{T - 26.9084 = 5a} \longrightarrow (2)$$

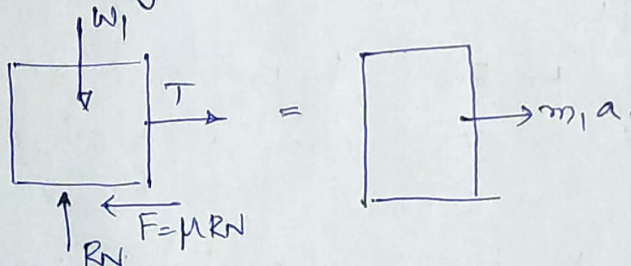
$$35.34 - 15a - 26.90 = 5a$$

$$8.44 = 20a \Rightarrow \underline{\underline{a = 0.42 \text{ m/s}^2}}$$

2. Two bodies of weight 20N and 10N are connected to the 2 ends of a light inextensible string, passing over a smooth pulley. The weight of 20N is placed on a horizontal surface while the weight of 10N is hanging free in air. The horizontal surface is a rough one, having coeff of friction between the weight 20N and the plane surface equal to 0.3, determine.
- (1) The acceleration of the system (2) The tension in the string



FBD of 20N body.

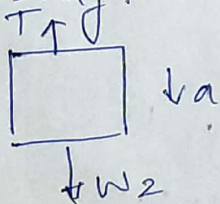


$$\sum F_y = 0 = R_N - W_1 \Rightarrow R_N = 20N$$

$$\sum F_x = m a \quad [\text{D'Alembert's}]$$

$$T - \mu R_N = m_1 a \Rightarrow \boxed{T = \frac{20}{9.81} a + 0.3 \times 20} \rightarrow (1)$$

FBD of 10N body.



$$\text{D'Alembert's} \Rightarrow W_2 - T = m_2 a \rightarrow (2)$$

$$10 - T = \frac{10}{9.81} a \rightarrow (2)$$

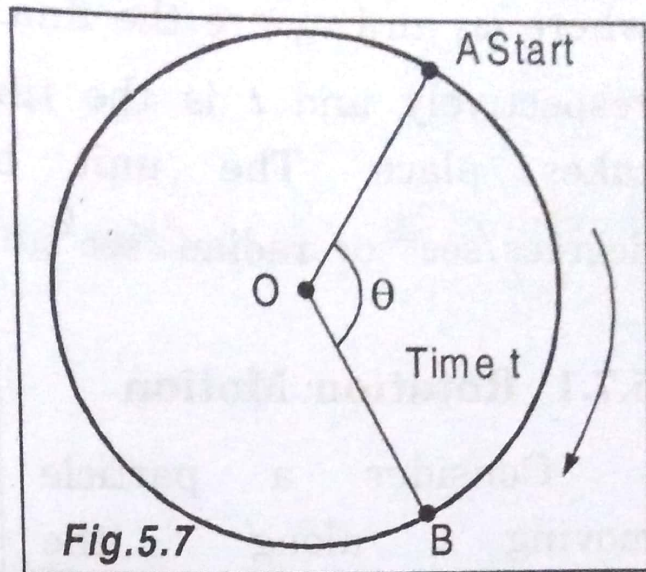
$$\therefore \Rightarrow 10 - \frac{20a}{9.81} - 20 \times 0.3 = \frac{10}{9.81} a$$

$$\Rightarrow a = 1.308 \text{ m/s}^2$$

$$\text{Sub } a \text{ in (2)} \quad \boxed{T = 8.67N}$$

5.7 CYLINDRICAL CO-ORDINATE SYSTEM

(i) **Angular displacement (θ):** (Fig. 5.7) The angle subtended at the center by the path (A to B) travelled by the particle is its angular displacement (θ). The unit of angular displacement is degree or radian or revolution. It is denoted by θ if measured in degrees or in radians and by N if measured in revolutions.



(ii) **Angular velocity (ω):** The angular displacement of a body per unit time is its angular velocity. It is denoted by (omega) ω .

$$\omega = \frac{\text{Angular displacement}}{\text{Time interval}} = \frac{\theta}{t}$$

The unit of ω is degrees per second (or) radians per second. The unit of N is revolutions per second (rps) or **revolutions per minute (rpm)**. Now θ can be written as $\theta = \omega t$.

(iii) **Angular acceleration (α):** The change in angular velocity per unit time is the angular acceleration. It is denoted by α .

$$\alpha = \frac{\text{Change in angular velocity}}{\text{Time interval}}$$

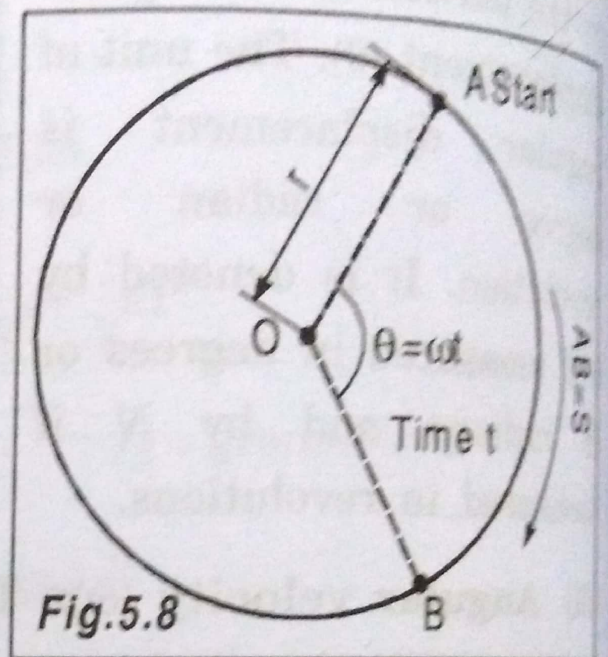
$$= \frac{\omega_1 - \omega_0}{t}$$

where ω_1 and ω_0 are the final and initial angular velocities respectively and t is the time during which this change takes place. The unit of angular acceleration is degrees/sec² or radian/sec².

5.7.1 Rotation Motion

Consider a particle moving along the circumference of a circle of radius r with constant angular velocity ω rad/s.

Let the particle at A (Refer **Fig 5.8**) moves to B in t seconds.



It moves through an angle θ radians.

$\therefore$ The angular displacement $\theta = \omega t$

The linear distance travelled in t seconds along the circumference is equal to the length of arc $AB = s$; θ in radians = $\frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$

$$\text{So } s = r\theta \quad \dots (i)$$

If the distance is covered in t seconds,

$$\therefore \text{ Tangential velocity (or) Linear velocity } v = \frac{s}{t} = \frac{r\theta}{t}$$

substitute $\frac{\theta}{t} = \omega$, we get

$$v = r\omega \quad \dots(ii)$$

We can also write $\omega = \frac{v}{r}$

If the body is moving with a constant angular acceleration α ;

$$\alpha = \frac{\omega_1 - \omega_0}{t}$$

But from (ii) $\omega_1 = \frac{v}{r}$ and $\omega_0 = \frac{u}{r}$

$$\therefore \alpha = \frac{v - u}{r \times t} = \frac{a}{r} \quad \left(\because \frac{v - u}{t} = a \right)$$

$$\therefore a = r\alpha \quad \dots (iii)$$

In all these equations from (i) to (iii) the value of θ , ω and α must necessarily be in radian units. Then the terms in translation motion s , v and a will have the units corresponding to the unit of r . Thus if θ is in radians and r is in metres, then the distance s obtained by the formula $s = r\theta$ is in metres. The same procedure applies to the units of linear acceleration.

5.7.2 Equations of rotational motion:

For the rotational motion with uniform angular acceleration, there are three equations similar to the three equations of translation motion with uniform acceleration.

Consider a body moving along the circumference of a circle. It has ω_0 as its initial angular velocity and α the uniform angular acceleration. After time t , the angular

velocity is ω_1 called the final angular velocity and θ is the angular displacement.

Change in angular velocity = $\omega_1 - \omega_0$

$$\therefore \alpha = \frac{\omega_1 - \omega_0}{t}$$

$$\therefore \omega_1 = \omega_0 + \alpha t \quad \dots (i)$$

(This equation is similar to $v = u + at$)

Angular displacement θ

θ = Average angular velocity $\times$ Time

$$= \frac{(\omega_0 + \omega_1)}{2} \times t \quad \dots (ii)$$

Substitute $\omega_1 = \omega_0 + \alpha t$ in eqn. (ii), we get

$$\therefore \theta = \frac{(\omega_0 + \omega_0 + \alpha t)}{2} \times t$$

$$\therefore \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \dots (iii)$$

$\left(\text{This is similar to } s = ut + \frac{1}{2} at^2 \right)$

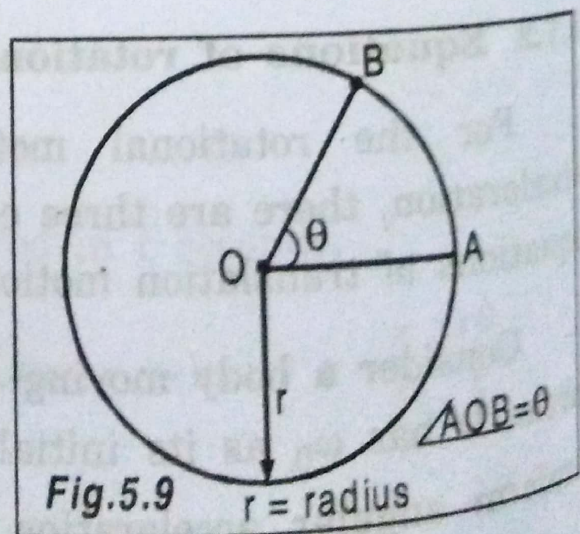
From equation (i)

$$\omega_1 = \omega_0 + \alpha t$$

Squaring on both sides,
we get

$$\omega_1^2 = (\omega_0 + \alpha t)^2$$

$$\omega_1^2 = \omega_0^2 + 2\omega_0 \alpha t + \alpha^2 t^2$$



$$= \omega_0^2 + 2\alpha(\omega_0 t + \frac{1}{2} \alpha t^2)$$

$$\left[\because \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \right]$$

$$\therefore \omega_1^2 = \omega_0^2 + 2\alpha\theta$$

...(iv)

(This is similar to $v^2 = u^2 + 2as$)

The three equations of angular motion (rotational motion) with uniform angular acceleration are

$$1. \omega_1 = \omega_0 + \alpha t$$

$$2. \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$3. \omega_1^2 = \omega_0^2 + 2\alpha\theta$$

Circular motion or motion of rotation may be explained in a different manner in the following pages.

The motion of a body along a circular path is known as circular motion. In circular motion, the centre of rotation remains fixed. The examples of the bodies moving in a circular path are: Shafts, Flywheels, Pulleys, etc.

Angular Velocity ω

It is defined as the rate of change of angular displacement of a body.

$$\omega = \frac{\text{Angular displacement}}{\text{Time}}$$

$$= \frac{d\theta}{dt} \text{ in rad/sec.}$$

$$\text{Linear velocity } v = \frac{\text{Linear displacement}}{\text{Time}}$$

$$= \frac{dx}{dt} \text{ in m/sec}$$

Relation between v and ω is $v = r\omega$

Angular acceleration α

It is defined as the rate of change of angular velocity.

$$\alpha = \frac{d\omega}{dt}$$

$$\text{where } a = \frac{dv}{dt}$$

$$\alpha = \omega \frac{d\omega}{d\theta}$$

$$\text{where } a = v \frac{dv}{dx}$$

Relation between ' a ' and ' α ' is $a = r\alpha$

5.8 SUMMARY OF TRANSLATION AND ROTATIONAL MOTION

Equations of Translation and Rotational Motion with Uniform acceleration

Circular (Rotation)

$$1. \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$2. \omega_1 = \omega_0 + \alpha t$$

$$3. \omega_1^2 = \omega_0^2 + 2\alpha\theta$$

N = revolution
per minute;

Rotation motion with
constant angular velocity

Linear (Translation)

$$\text{where } s = ut + \frac{1}{2} at^2$$

$$\text{where } v = u + at$$

$$\text{where } v^2 = u^2 + 2as \text{ and}$$

$$s = x - x_0$$

$$\omega = \frac{2\pi N}{60}$$

$$\omega_1 = \omega_0$$

$$\text{So } \theta = \omega_0 t = \omega_1 t$$

$$\alpha = 0$$

Total number of
revolutions = $\frac{\theta}{2\pi}$ for any
type of motion.

Problem 5.15: The angle of rotation of a body is given by the equation $\theta = 2t^3 - 5t^2 + 8t + 6$. Determine

1. the angular velocity and

2. angular acceleration of the body when $t = 0$ and when $t = 4$ seconds.

(Oct. 2003 - AU)

Solution:

Angular displacement $\theta = 2t^3 - 5t^2 + 8t + 6$

Angular velocity $\omega = \frac{d\theta}{dt} = 6t^2 - 10t + 8$

Angular acceleration $\alpha = \frac{d\omega}{dt} = 12t - 10$

when $t = 0$

$$\omega = 8 \text{ rad/s} \quad [\because \omega = 6t^2 - 10t + 8]$$

$$\alpha = -10 \text{ rad/s}^2 \quad [\because \alpha = 12t - 10]$$

when $t = 4 \text{ sec}$

$$\begin{aligned}\omega &= 6(4)^2 - 10(4) + 8 \\ &= 96 - 40 + 8 = 64 \text{ rad/s}\end{aligned}$$

$$\omega = 64 \text{ rad/s}$$

$$\alpha = 12(4) - 10 = 38 \text{ rad/sec}^2$$

$$\alpha = 38 \text{ rad/s}^2.$$

Problem 5.16: A body is rotating with an angular velocity of 5 rad/s. After 4 sec the angular velocity of body becomes 13 rad/s. Determine the angular acceleration of the body.

(Apr 2006 - AU)

Solution:

Given

$$\omega_0 = 5 \text{ rad/s}; t = 4 \text{ sec}; \omega_1 = 13 \text{ rad/s}$$

$$\alpha = \text{Angular acceleration} = ?$$

$$\omega_1 = \omega_0 + \alpha t \quad \text{i.e. } 13 = 5 + \alpha(4)$$

$$\alpha = \frac{8}{4} = 2 \text{ rad/sec}^2$$

$$\text{Angular acceleration } \alpha = 2 \text{ rad/s}^2$$

Problem 5.17: The rotor of an electric motor is rotating at a speed of 720 rpm. When the steam supply is suddenly cut off, it is observed that 5 mins are required for the rotor to come to rest. Assuming uniformly accelerated motion determine

- (a) the angular acceleration and
- (b) the total number of revolutions.

(KTU - June 2011)(Cochin University - May 2014)

Solution:

$$1 \text{ revolution} = 2\pi \text{ rad}$$

$$N_0 = 720 \text{ rpm}; \omega_0 = \frac{2\pi N_0}{60}$$

$$= \frac{2\pi \times 720}{60} = 75.4 \text{ rad/s}$$

$t = 5 \text{ min} = 300 \text{ sec}; \omega_1 = 0$ [∵ finally the rotor comes to rest]

$$\omega_1 = \omega_0 + \alpha t$$

$$0 = 75.4 + \alpha(300)$$

$$\alpha = \frac{-75.4}{300} = -0.251 \text{ rad/s}^2$$

Angular acceleration (deceleration) = 0.251 rad/s^2

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$= 75.4(300) + \frac{1}{2}(-0.251)300^2 = 11325$$

$$\theta = 11325 \text{ rad}$$

2π radian 1 revolution

$$11325 \text{ radian} = \frac{1}{2\pi} \times 11325 \text{ revolutions} = 1802.43 \text{ revolutions}$$

Total no. of revolutions = 1802.43 revolutions

Problem 5.18: A flywheel rotating at 1500 rpm comes to rest with constant angular deceleration in 100 seconds owing to friction in the bearings. If the moment of inertia of the flywheel with respect to its axis of rotation $I = 12.5 \text{ kg.m}^2$, determine friction couple that produces this angular deceleration.

(Cochin University, June 2014)

Solution:

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 1500}{60} = 157.08 \text{ rad/s}$$

$$\omega_2 = 0$$

$$\omega_2 = \omega_1 + \alpha t$$

$$0 = 157.08 + \alpha (100)$$

$$\alpha = \frac{-157.08}{100} = -1.571 \text{ rad/s}^2$$

[- sign indicates deceleration]

To find

Frictional couple (or) Frictional Torque T

$$T = I \alpha$$

$$= 12.5 \times (-1.571) = 19.63 \text{ N-m}$$

$$\mathbf{T = -19.63 \text{ N-m}}$$

Problem 5.19: The rotor of flywheel is governed by the equation $\omega = 2t^2 - 2t + 2$ where ω is rad/s and t is in second. After 1 second from start, the angular displacement was 4 radian. Determine the angular displacement, angular velocity and angular acceleration of the fly wheel when $t = 3$ second.

(Cochin University, May 2014)

Solution:

$$\text{Angular velocity, } \omega = \frac{d\theta}{dt} = 2t^2 - 2t + 2$$

$$\int_{\theta_0}^{\theta} d\theta = \int_{t_0}^t (2t^2 - 2t + 2) dt$$

Angular Displacement,

$$\theta - \theta_0 = \left[\frac{2t^3}{3} - \frac{2t^2}{2} + 2t \right]_0^t \quad \left| \begin{array}{l} \text{when } t = 1s, \\ \theta = 4 \text{ rad} \end{array} \right.$$

$$\theta - \theta_0 = 0.67t^3 - t^2 + 2t$$

$$\theta - \theta_0 = 0.67 - 1 + 2 = 1.67 \text{ rad}$$

$$\theta_0 = 4 - 1.67 = 2.33 \text{ rad}$$

$$\text{Hence } \theta = 0.67t^3 - t^2 + 2t + 2.33$$

$$\text{Now } \alpha = \frac{d^2\theta}{dt^2} = 4t - 2$$

$$\text{Angular acceleration } \alpha = 4t - 2$$

$$\text{When } t = 3 \text{ sec: } \theta = 0.67t^3 - t^2 + 2t + 2.33$$

$$\theta = 0.67(3^3) - 3^2 + 2(3) + 2.33 = 17.42 \text{ rad/s}$$

$$\boxed{\theta = 17.42 \text{ rad/s}}$$

$$\omega = 2t^2 - 2t + 2 = 2(3^2) - 2(3) = 12 \text{ rad/s}$$

$$\boxed{\omega = 12 \text{ rad/s}}$$

$$\alpha = 4t - 2 = 4(3) - 2 = 10 \text{ rad/s}^2$$

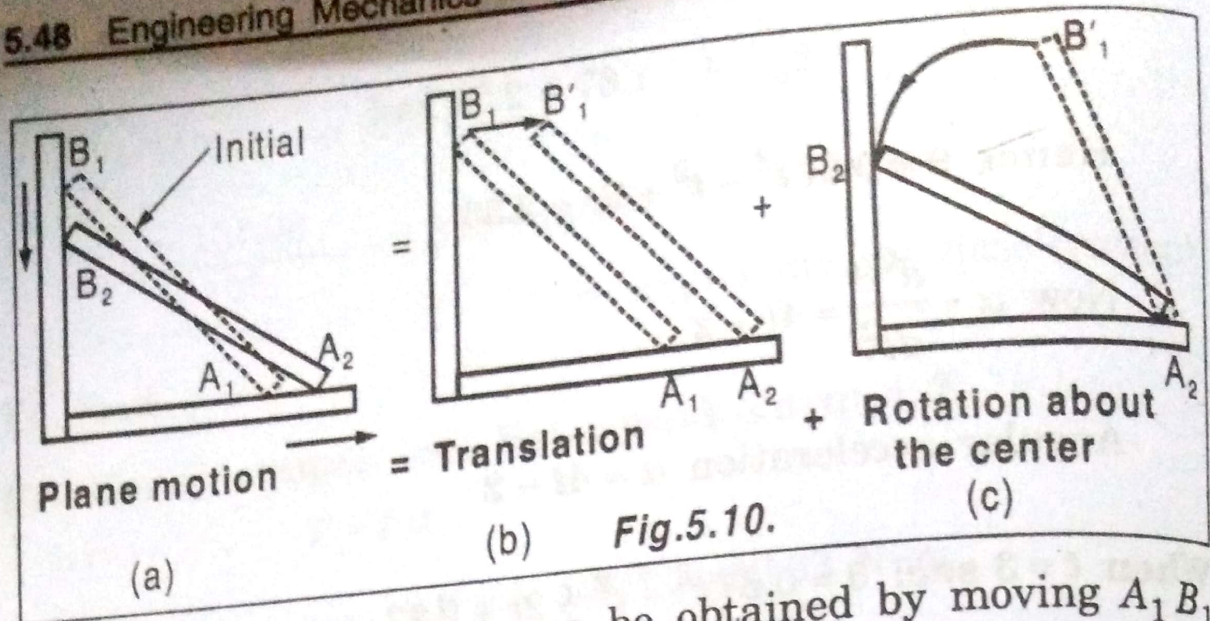
$$\boxed{\alpha = 12 \text{ rad/s}^2}$$

5.9 COMBINED MOTION OF ROTATION AND TRANSLATION [GENERAL PLANE MOTION]

General Plane motion is considered as the sum of a translation motion and a rotational motion.

Example 1

- (i) Consider a rod at its initial position $A_1 B_1$ (Dotted line). The rod moves to new position $A_2 B_2$ as shown in Fig 5.10 (a). This movement is called General plane motion.

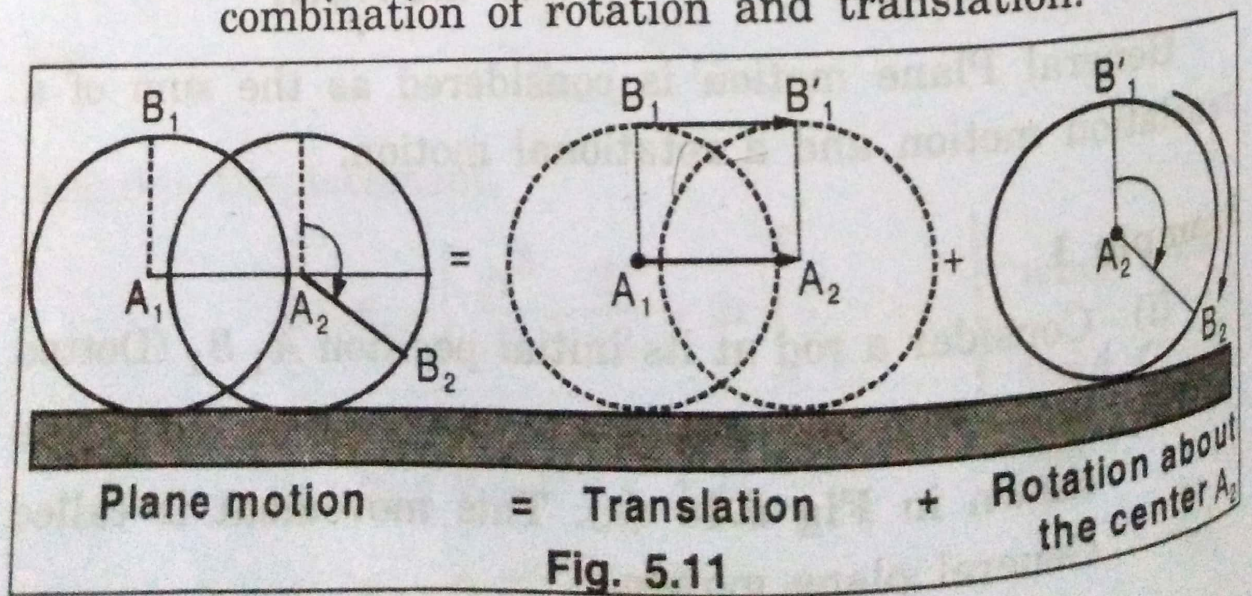


- (ii) The movement can be obtained by moving A_1B_1 horizontally to A_2B_1' (Translation) as in Fig. 5.10 (b) and then
- (iii) Rotating A_2B_1' about A_2 so that B_1' rotates to B_2 anticlockwise (Rotation) as shown in Fig. 5.10(c).

So General plane motion $\left\{ \right. = \text{Translation} + \text{Rotation about end } A_2$

Similarly consider example 2 (Fig. 5.11)

- (i) A wheel is rolling on road. After some time, the point A_1 and B_1 moves to new position A_2 and B_2 . This motion is general plane motion, i.e. combination of rotation and translation.



The same can be obtained by,

- (ii) Moving A_1 and B_1 horizontally into A_2, B_1' respectively (Translation motion) and then
- (iii) Rotating $A_2 B_1'$ about A_2 so that B_1' rotates to B_2 clockwise. (Rotation motion).

So General plane motion = Translation + Rotation about the center.

By using above principle, the absolute velocity of B can found out by the following formulae.

v_B	=	v_A	+	$v_{B/A}$
(Plane motion)		Translation		(Rotation of AB about A)

where v_B = Absolute velocity of B

v_A = Absolute velocity of A

$v_{B/A}$ = Relative velocity of B with respect to A

= The length AB $\times$ angular velocity of the rod AB

= $r\omega$

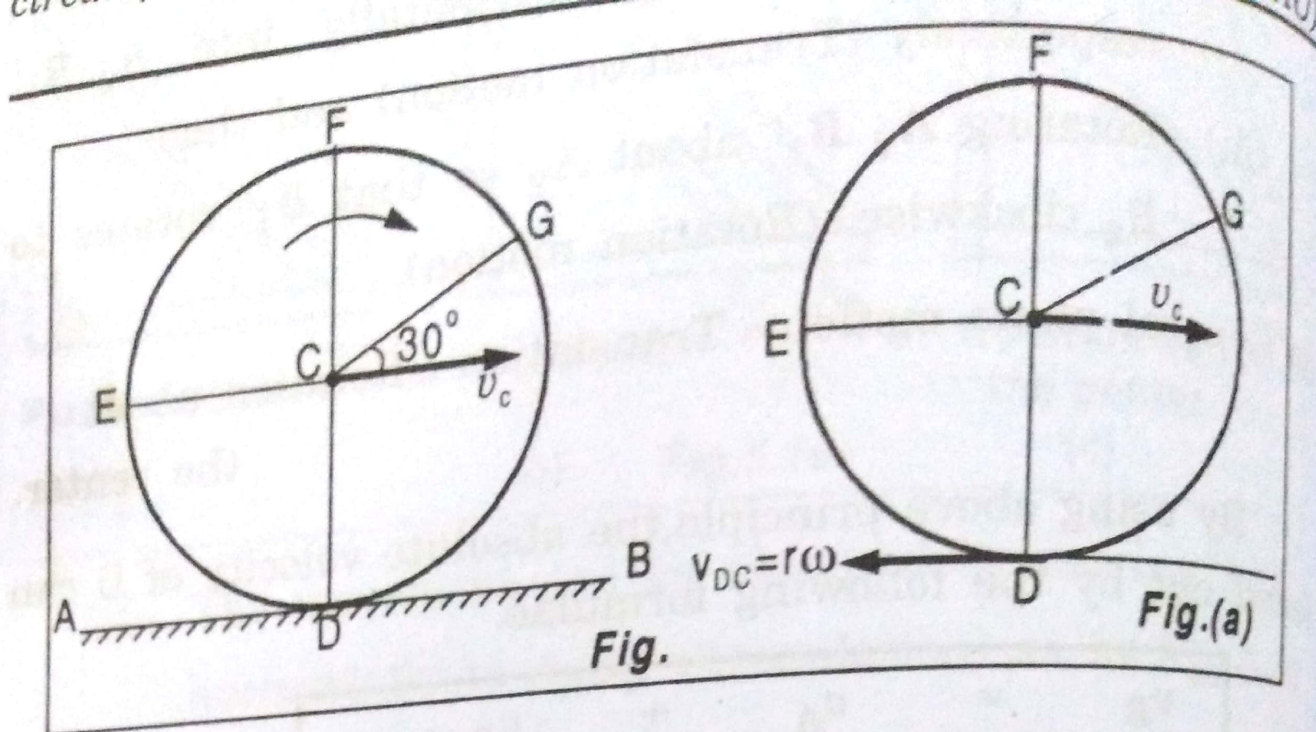
The formula can also be written as $v_A = v_B + v_{A/B}$

Similarly, to find acceleration of plane motion,

$$a_A = a_B + a_{A/B}$$

$$a_B = a_A + a_{B/A}$$

Problem 5.20: A cylinder of radius 1 m rolls without slipping along a horizontal plane. Its centre has a uniform velocity of 20 m/s. Find the velocity of the points E, F and G on the



Solution:

At D , velocity of D is zero. [Since D is in contact with road]

The motion of the center ' C ' is known - ie.
 $v_C = 20 \text{ m/s}$

To find ω

Plane motion = Translation + Rotation about the center

$$v_D = \vec{v}_C + \vec{v}_{D/C} \quad (\text{Vector sum}) \quad \text{and} \quad v_{D/C} = r\omega = 1 \times \omega$$

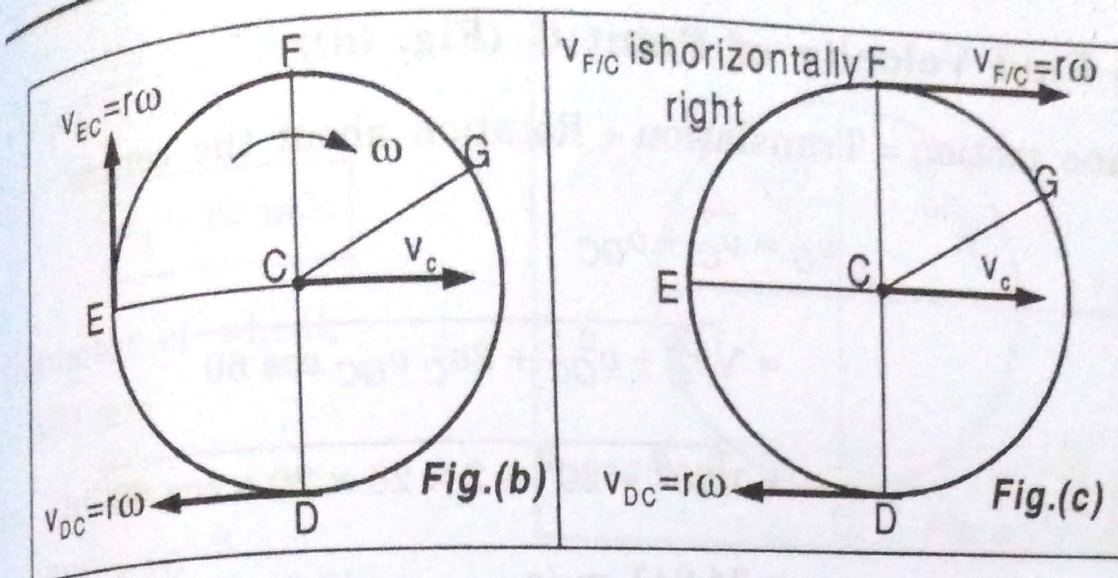
$$0 = 20 - 1 \times \omega \quad [\text{Since } v_{D/C} \text{ is in leftside direction, it is negative}]$$

$$\left. \begin{array}{l} \text{Angular velocity} \\ \text{of cylinder} \end{array} \right\} \omega = \frac{20}{1} = 20 \text{ rad/s}$$

Velocity of point E (Fig. (b))

$v_{E/C}$ is vertically upward

$$v_E = \vec{v}_C + v_{E/C} \uparrow \quad (\text{vector sum})$$



$$v_E = \sqrt{v_C^2 + v_{E/C}^2}$$

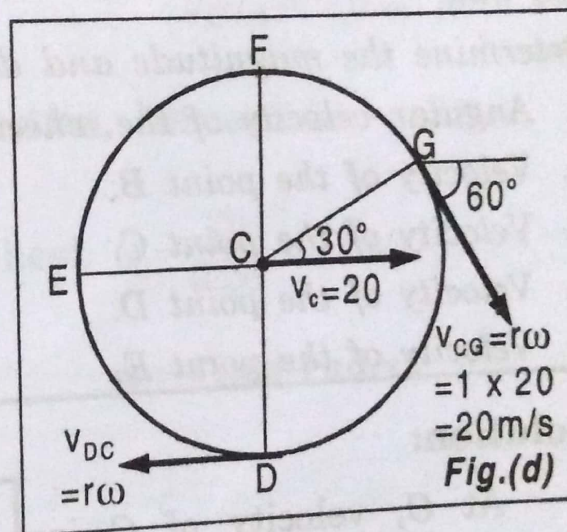
$$[\text{And } v_{E/C} = r\omega = 1 \times 20 = 20 \text{ rad/s}]$$

$$= \sqrt{20^2 + 20^2} = 28.28 \text{ m/s}$$

$$\tan \theta = \frac{v_{E/C}}{v_C} = \frac{20}{20} = 1; \theta = 45^\circ$$

To find Velocity of point F (Fig. (c))

$v_{F/C}$ is horizontally right.



Plane motion = Translation + Rotation about the center

$$\vec{v}_F = \vec{v}_C + \vec{v}_{F/C}$$

$$\vec{v}_F = \vec{20} + \vec{v}_{F/C} \quad \left[\vec{v}_{F/C} = r\omega = 1 \times 20 = 20 \text{ rad/s} \right]$$

$$\vec{v}_F = \vec{20} + \vec{20} = 40 \text{ m/s}$$

To Find Velocity of Point G (Fig. (d))

Plane motion = Translation + Rotation about the centre.

$$\vec{v}_G = \vec{v}_C + \vec{v}_{GC}$$

$$= \sqrt{v_C^2 + v_{GC}^2 + 2v_C v_{GC} \cos 60}$$

$$= \sqrt{20^2 + 20^2 + 2 \times 20 \times 20 \times \cos 60}$$

$$= 34.641 \text{ m/s}$$

$$v_G = 34.641 \text{ m/s}$$

Problem 5.21: An automobile travels to the right at a constant speed of 72 km/h. The diameter of the wheel is 560 mm.

Determine the magnitude and direction of the following:

1. Angular velocity of the wheel.
2. Velocity of the point B.
3. Velocity of the point C.
4. Velocity of the point D.
5. Velocity of the point E.

(May/June 2012 AU)

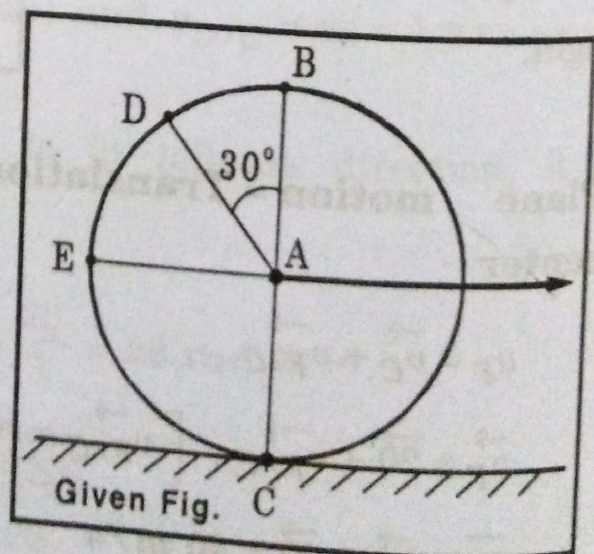
Solution:

At C, velocity of C is zero. [$\because$ C is in contact with road]

$$\text{i.e. } v_C = 0$$

The motion of the center 'A' is known

$$\text{i.e. } v_A = 72 \text{ km/h}$$



$$\Rightarrow v_A = \frac{72 \times 1000}{3600}$$

$$v_A = 20 \text{ m/s}$$

Diameter of wheel,
= 560 mm

$\Rightarrow$ radius,

$$r = \frac{560 \times 10^{-3}}{2} = 0.28 \text{ m}$$

Planemotion = Translation + Rotation about the center

$$v_C = \vec{v}_A + \vec{v}_{C/A}$$

$$\therefore v_{C/A} = r\omega$$

$$0 = \vec{20} - 0.28 \times \omega$$

$$v_{C/A} = -0.28 \omega$$

[Here $v_{C/A}$ is in left side direction, hence it is - ve]

$$\Rightarrow \text{Angular velocity of wheel, } \omega = \frac{20}{0.28} = 71.42$$

$$\omega = 71.42 \text{ rad/s}$$

2. Velocity of the point E

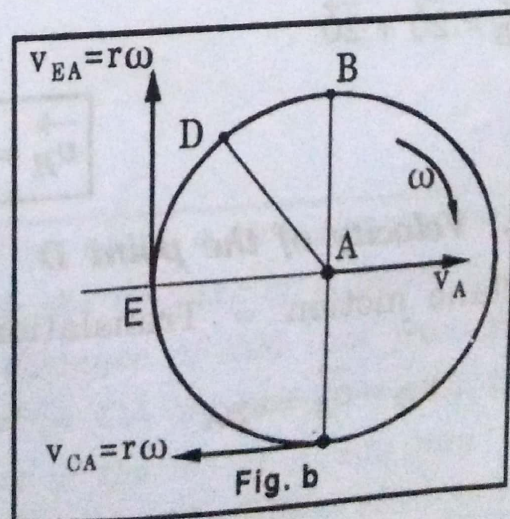
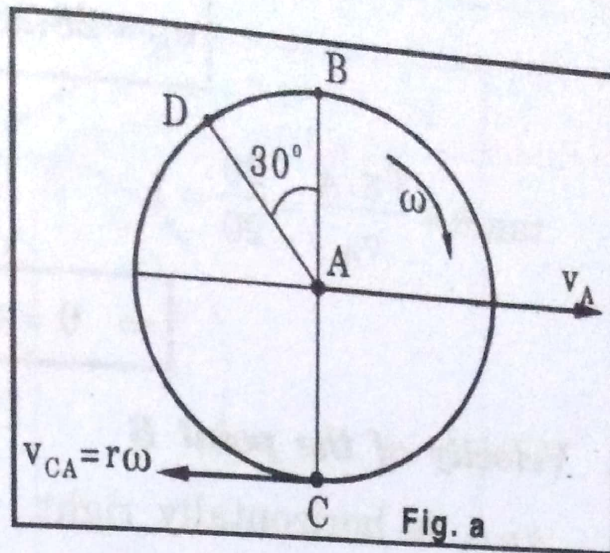
$v_{E/A}$ is vertically upward.

$$v_E = \vec{v}_A + v_{E/A} \uparrow$$

$$v_E = \sqrt{v_A^2 + v_{E/A}^2}$$

$$\therefore v_{E/A} = r\omega = 0.28 \times 71.42 = 20 \text{ rad/s}$$

$$= \sqrt{(20)^2 + (20)^2}$$



$$v_E = 28.28 \text{ m/s}$$

$$\tan \theta = \frac{v_{E/A}}{v_A} = \frac{20}{20} = 1$$

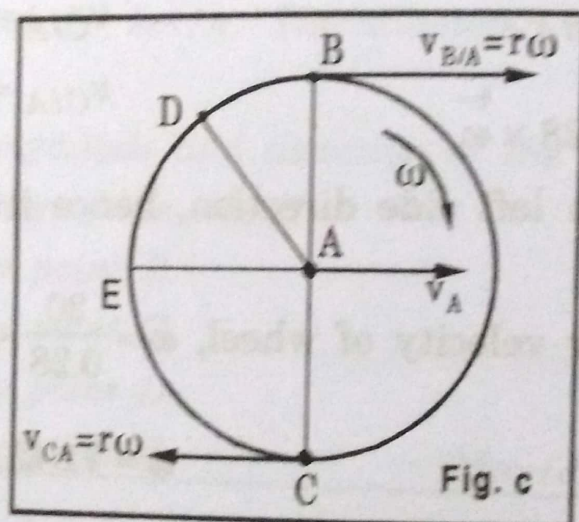
$$\Rightarrow \theta = 45^\circ$$

3. Velocity of the point B

$v_{B/A}$ is horizontally right.

Plane motion = Translation + rotation about the center

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$



$$\vec{v}_B = \vec{20} + \vec{v}_{B/A}$$

$$[v_{B/A} = r\omega = 0.28 \times 71.42 = 20 \text{ rad/s}]$$

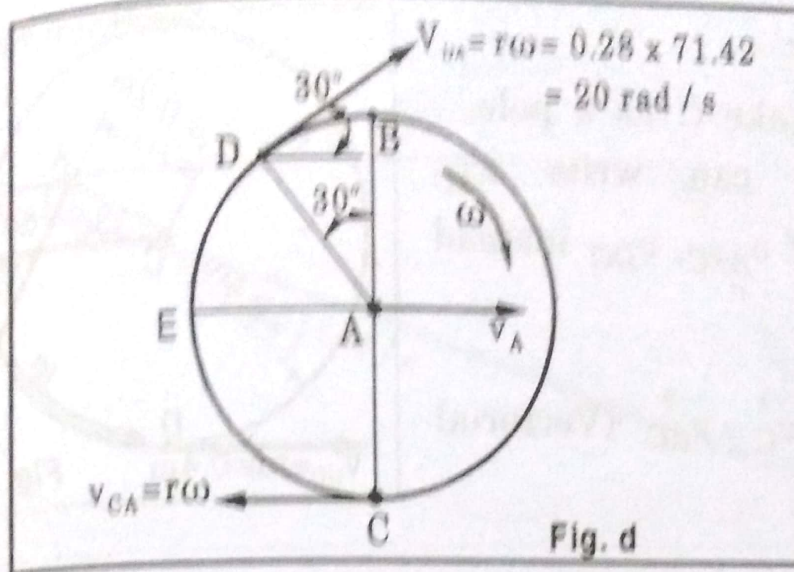
$$\vec{v}_B = \vec{20} + \vec{20}$$

$$\vec{v}_B = 40 \text{ m/s}$$

4. Velocity of the point D

Plane motion = Translation + Rotation about the centre

$$\vec{v}_D = \vec{v}_A + \vec{v}_{DA}$$



$$= \sqrt{v_A^2 + v_{DA}^2 + 2v_A v_{DA} \cos 30^\circ}$$

$$= \sqrt{(20)^2 + (20)^2 + (2 \times 20 \times 20 \times \cos 30^\circ)}$$

$$v_D = 38.64 \text{ m/s}$$

Result

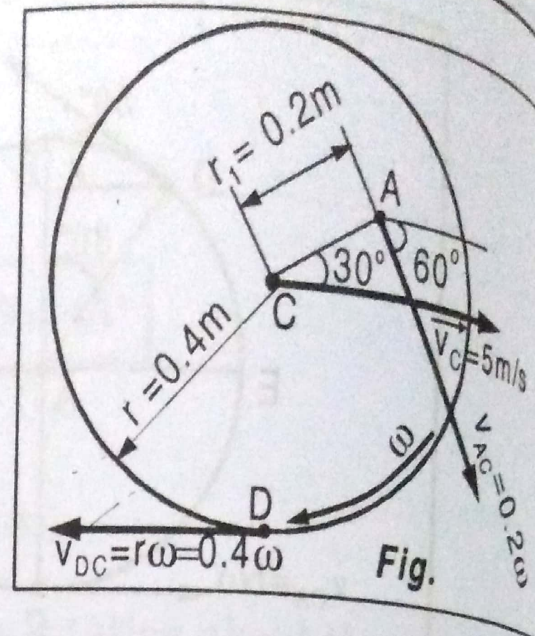
1. $\omega = 71.42 \text{ rad/s}$
2. $v_B = 40 \text{ m/s}$
3. $v_C = \text{zero}$ [At C, Velocity of C is zero]
4. $v_D = 38.64 \text{ m/s}$
5. $v_E = 28.28 \text{ m/s}$

Problem 5.22: A cycle is travelling along a straight road with a velocity of 5 m/sec . Determine the velocity of a point A on the wheel situated at a radial distance of 200 mm from the center C of the front wheel, when CA makes 30° with the horizontal through C . The radius of the wheel is 400 mm
(May/June 2010-AU)

Solution:

We take C as a pole.
(and we can write v_{AC}
instead of $v_{A/C}$, v_{DC} instead
of $v_{D/C}$)

$$\vec{v}_A = \vec{v}_C + \vec{v}_{AC} \quad (\text{Vectorial addition})$$



where $v_{AC} = r_1 \omega = 0.2 \omega$ [$\because CA = r_1 = 0.2 \text{ m}$]

To find ω

The velocity of point 'D' which is having contact with the plane must be zero, i.e. $v_D = 0$

$$\text{But } \vec{v}_D = \vec{v}_C + \vec{v}_{DC} \quad (\text{Vectorial addition})$$

$$0 = 5 - 0.4 \omega$$

[v_{DC} is leftside direction, so it is negative]

$$\omega = \frac{5}{0.4} = 12.5 \text{ rad/sec}$$

$$\omega = 12.5 \text{ rad/sec}$$

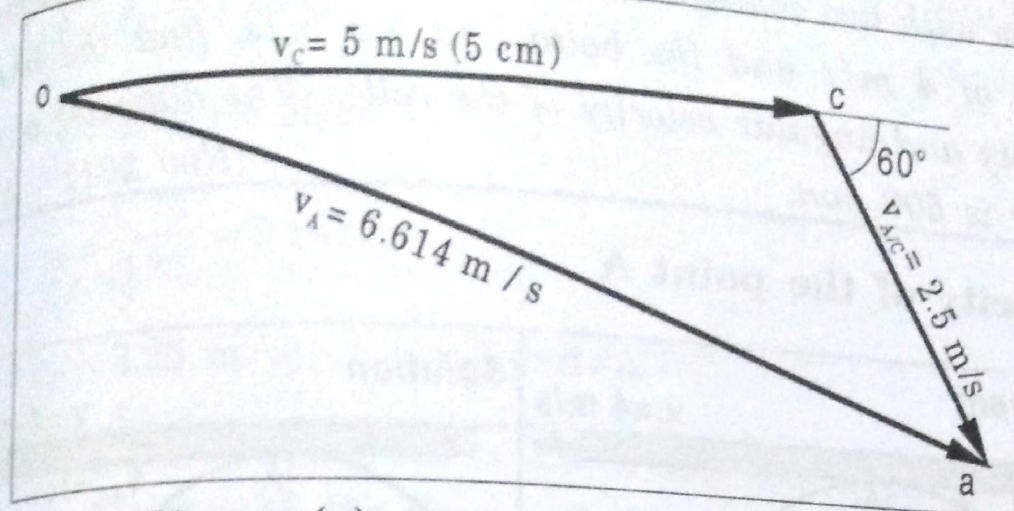
(So $v_{AC} = r_1 \omega = 0.2 \times 12.5 = 2.5 \text{ m/s}$)

Now $\vec{v}_A = \vec{v}_C + \vec{v}_{AC}$ (Vectorial addition)

$$= 5 + (0.2 \times 12.5) \quad (\text{Vectorial addition})$$

By drawing the velocity diagram, we can easily get the solution

Graphical method



Refer the Figure (a)

1. Take a point 'o'
2. From 'o' draw a line horizontally parallel to v_C and equal to 5 cm and mark the point c.
(Take 1 cm = 1 m/sec)
3. Draw a line parallel to v_{AC} inclined at 60° to horizontal from c and length equal to 2.5 cm (equal to 2.5 m/s) and mark a. (v_{AC} direction is 60° inclined to horizontal)
4. Now join oa which is equal to 6.614 cm. Then length oa is equal to v_A i.e. $v_A = 6.614 \text{ m/s}$.

Analytically

$$\vec{v}_A = \vec{v}_C + \vec{v}_{AC} \text{ (vectorial addition)}$$

$$v_A = \sqrt{v_C^2 + (v_{A/C})^2 + (2v_C (v_{A/C}) \cos 60^\circ)}$$

$$= \sqrt{5^2 + 2.5^2 + (2 \times 5 \times 2.5 \times \cos 60^\circ)}$$

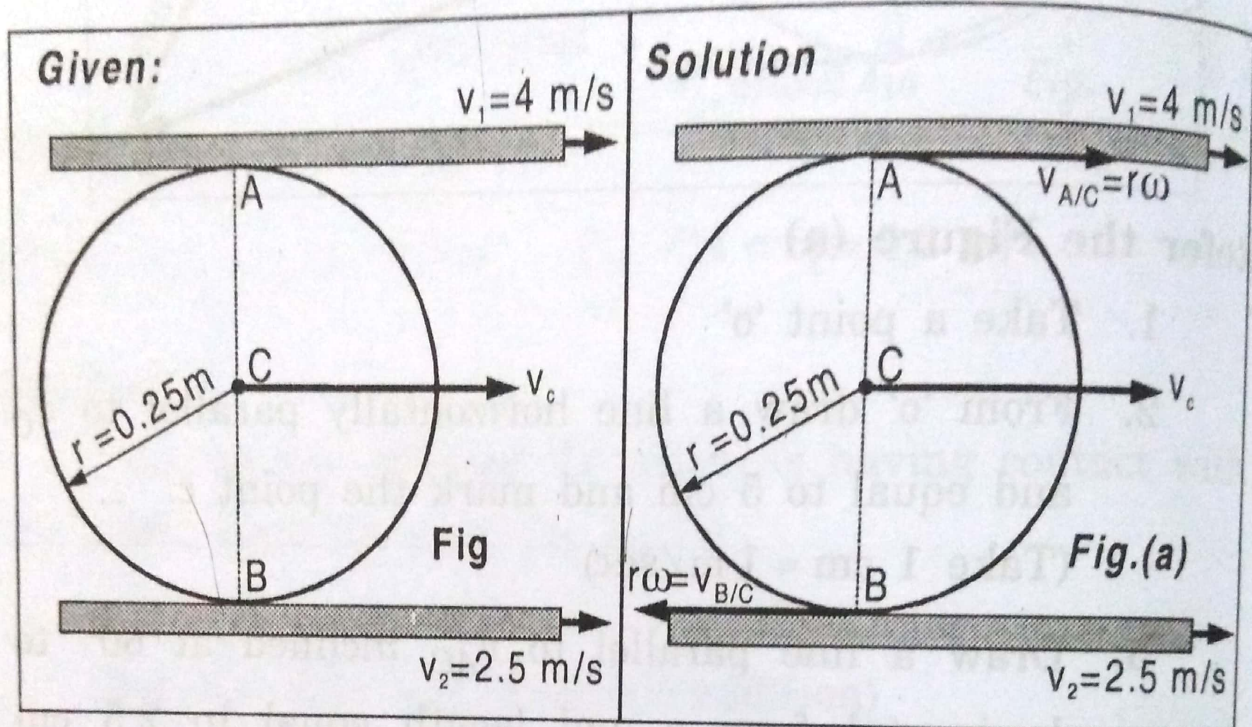
$$= \sqrt{25 + 6.25 + 12.5} = 6.614 \text{ m/sec}$$

$$v_A = 6.614 \text{ m/sec}$$

v_A is same in both methods.

Problem 5.23: A cylindrical roller is in contact at its top and bottom with two conveyor belts. If the top belts runs at uniform speed of 4 m/s and the bottom at 2.5 m/s, find the linear velocity and angular velocity of the roller. The diameter of the roller is 500 mm. (Nov 2011 - AU)

Velocity of the point A



$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}; \text{ substitute } \vec{v}_A = \vec{v}_1 = 4 \text{ m/s}$$

$$4 = \vec{v}_C + r\omega \quad \dots (1)$$

Velocity of the point B

$$\vec{v}_B = \vec{v}_C - \vec{v}_{B/C}; [\vec{v}_B = v_2 = 2.5 \text{ m/s}]$$

$$2.5 = \vec{v}_C - r\omega \quad \dots (2)$$

Add 1 & 2

$$6.5 = 2v_C + 0$$

$$v_C = \frac{6.5}{2} = 3.25 \text{ m/s}$$

Substitute $v_C = 3.25$ in equation (1), we get

$$\vec{v}_4 = \vec{v}_C + 0.25 \omega$$

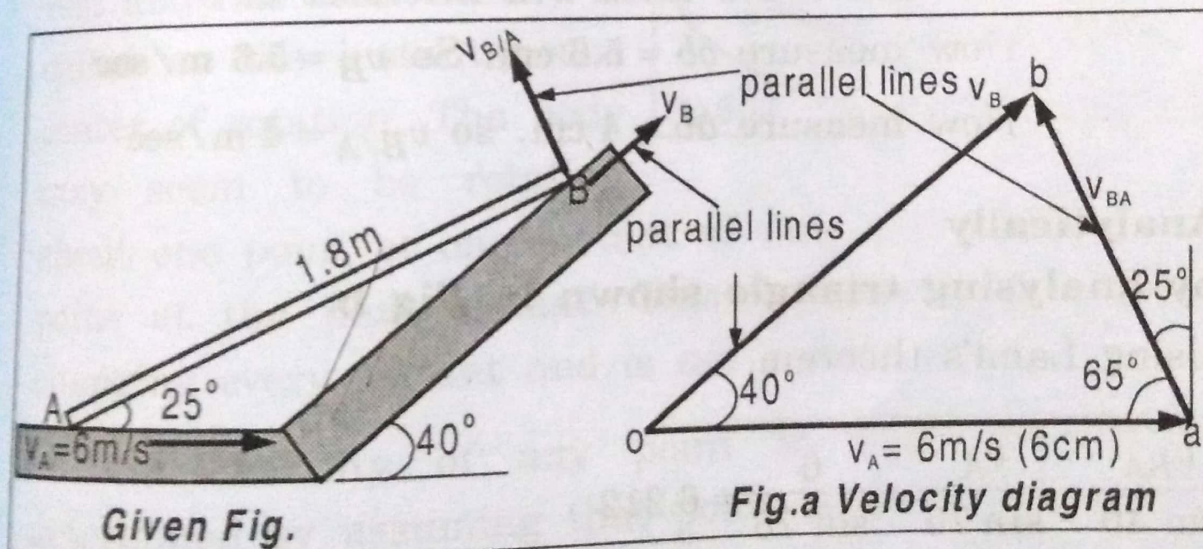
$$\vec{v}_4 = 3.25 + 0.25 \omega$$

$$\omega = \frac{0.75}{0.25} = 3 \text{ rad/s}$$

$$v_C = 3.25 \text{ m/s}; \quad \omega = 3 \text{ rad/s.}$$

Problem 5.24: The rod AB is 1.8 m long and slides with its ends in contact with the floor and the inclined plane. End A moves with a constant velocity of 6 m/s to the right. At the instant when $\theta = 25^\circ$, determine (a) the angular velocity of the rod (b) the velocity of the end B (Refer the fig.)

(Nov/Dec 2007-AU)



Solution:

The velocity of the end 'A' is known, So we choose 'A' as the pole. Now apply the General Plane Motion Equation.

Plane motion = Translation + Rotation about the center

$$v_B = v_A + v_{B/A} \text{ (Vectorial addition)}$$

To Draw velocity diagram [Refer Fig. (a)]

1. Take any point as o .
2. Draw $oa = 6$ cm to represent $v_A = 6$ m/sec. (Scale 1 m/sec = 1 cm) horizontally since v_A direction is horizontal.
3. Draw a line parallel to inclined plane (40°) through point o , (Here the length of the line is not known. So draw to any length) to represent v_B direction.
4. Draw a line perpendicular to the rod AB through point a to represent $v_{B/A}$ direction. (Here also, the length of the line is not known. So draw to any length)

The above two lines will intersect at b .

Now measure $ob = 5.6$ cm. So $v_B = 5.6$ m/sec

Now measure $ab = 4$ cm. so $v_{B/A} = 4$ m/sec

Analytically

by analysing triangle shown in Fig b.

Using Lami's theorem,

$$\frac{v_{BA}}{\sin 40} = \frac{v_A}{\sin 75} = \frac{6}{\sin 75} = 6.212$$

$$(\because v_A = 6 \text{ m/s})$$

$$v_{BA} = 6.212 \times \sin 40 = 4 \text{ m/s}$$

$$\frac{v_B}{\sin 65} = \frac{v_A}{\sin 75} = \frac{6}{\sin 75} = 6.212$$

$$v_B = 6.212 \times \sin 65 = 5.63 \text{ m/s}$$

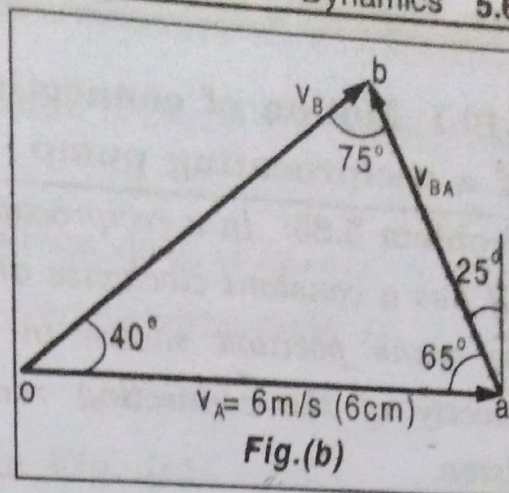
$$v_{B/A} = (AB) \times \omega$$

$$4 = 1.8 \omega$$

$$\omega = \frac{4}{1.8} = 2.222 \text{ rad/s}$$

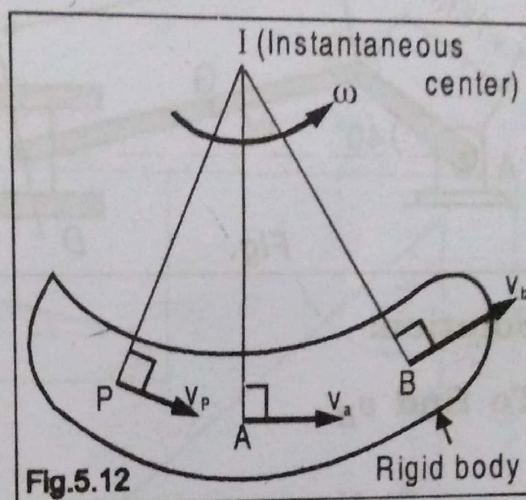
So angular velocity of the rod

$$\omega_{\text{rod}} = 2.222 \text{ rad/s.}$$



5.10 CONCEPT OF INSTANTANEOUS CENTER

A rigid body in plane motion, at any given instant of time appears as if rotating about a certain point in the plane of the body. The point which is instantaneously at rest and has zero velocity is called as the instantaneous center of rotation. The body may seem to be rotating



about one point at one instant of time and about another point at the next instant. This instantaneous center is changing every instant and is not a fixed point.

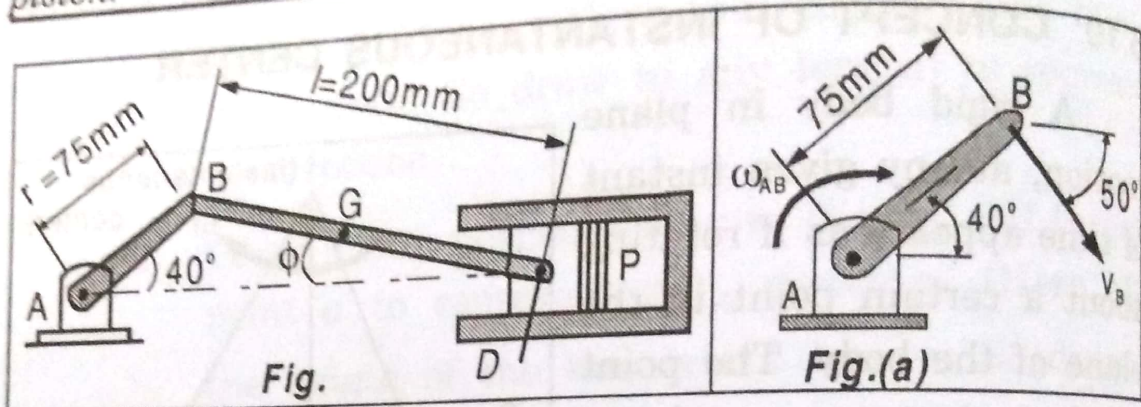
The velocity of any point in the body can be determined by assuming that point is rotating with some angular velocity ω , about the instantaneous center I at the instant.

$$\omega = \frac{v_a}{IA}; \omega = \frac{v_p}{IP}; \omega = \frac{v_b}{IB}$$

5.10.1 Motion of connecting rod of piston and crank of a reciprocating pump

Problem 5.25: In a reciprocating pump mechanism, the crank AB has a constant clockwise angular velocity of 2500 rpm. For the crank position shown in fig. determine (a) the angular velocity of the connecting rod BD. (b) The velocity of the piston.

(May/June 2010, AU)



Solution:

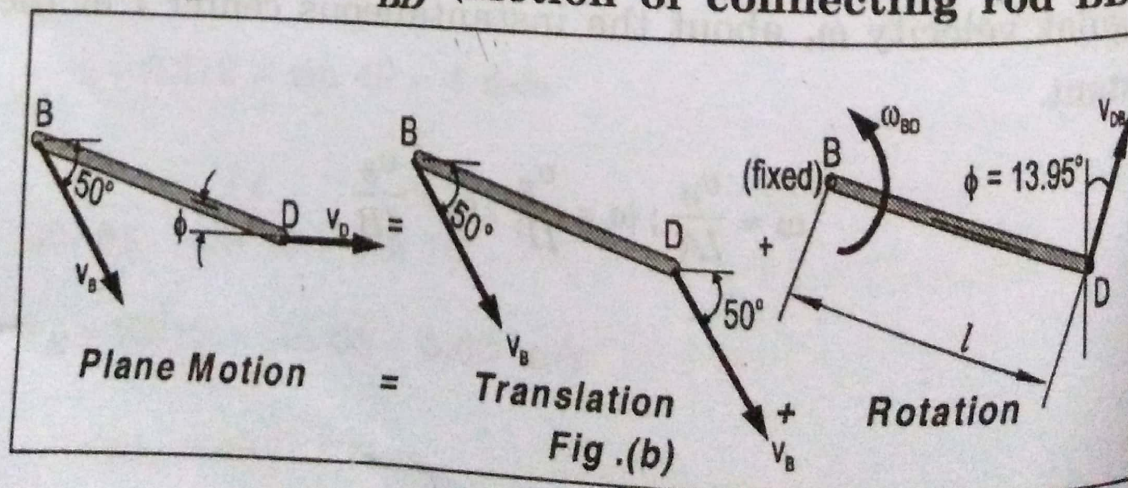
To find v_B

$$\omega_{AB} = \frac{2\pi N_{AB}}{60} = \frac{2\pi \times 2500}{60} = 261.8 \text{ rad/s.}$$

$$v_B = r\omega_{AB} = (AB) \omega_{AB} = 0.075 \times 261.8 = 19.63 \text{ m/s}$$

v_B direction is perpendicular to crank AB.

To find v_D and ω_{BD} (Motion of connecting rod BD)



First of all, find ϕ – the angle between connecting rod and horizontal.

❖ Draw ABD by using given data (**Fig.(c)**).

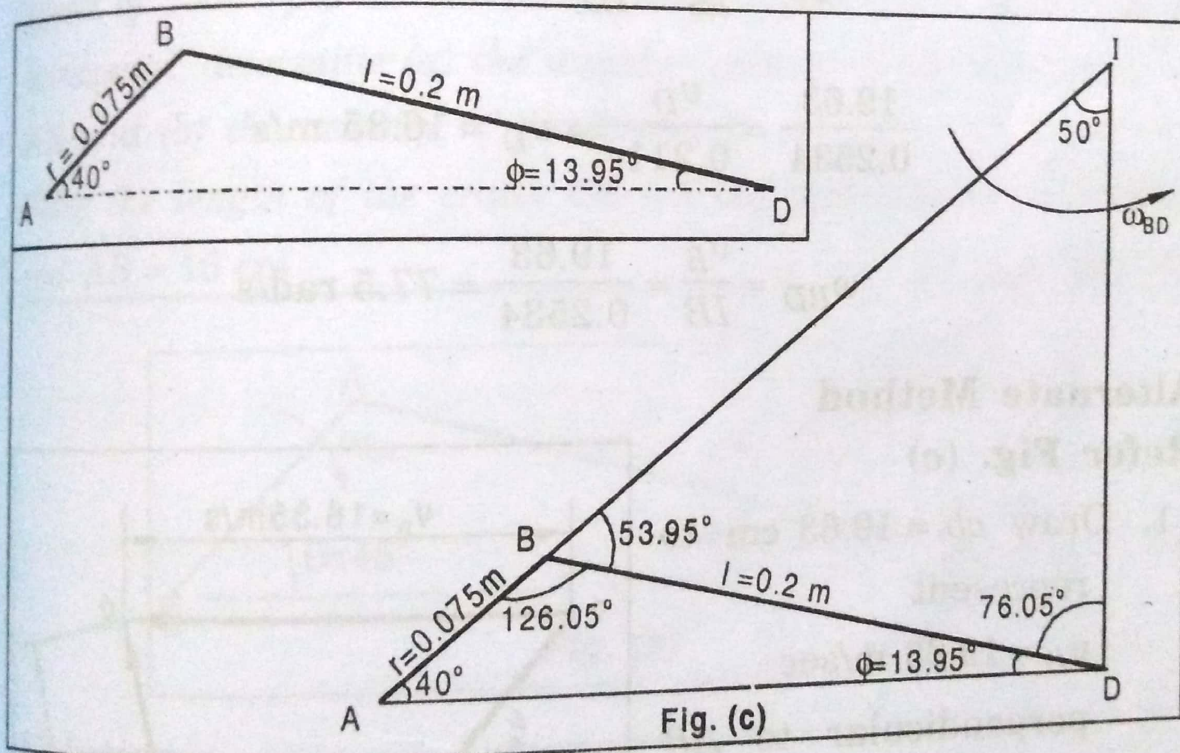
❖ Extend AB and draw vertical line from D . Both lines intersect at I .

Now by measurement from **Fig. (c)**,

We get $IB = 0.2534 \text{ m}$

$ID = 0.211 \text{ m}$

Instantaneous centre method



Using law of sines,

$$\frac{0.2}{\sin 40} = \frac{0.075}{\sin \phi}$$

$$\sin \phi = \frac{0.075 \times \sin 40}{0.2} = 0.241$$

$$\phi = 13.95^\circ$$

From $\triangle IAD$

$$\sin 40 = \frac{ID}{IB + AB}$$

From $\triangle IBD$

$$\frac{ID}{\sin 53.95} = \frac{0.2}{\sin 50} \Rightarrow I_D = 0.211 \text{ m}$$

Also,

$$\frac{IB}{\sin 76.05} = \frac{0.2}{\sin 50} \Rightarrow I_B = 0.2534 \text{ m}$$

$$\therefore \omega_{BD} = \frac{v_B}{IB} = \frac{v_D}{ID}$$

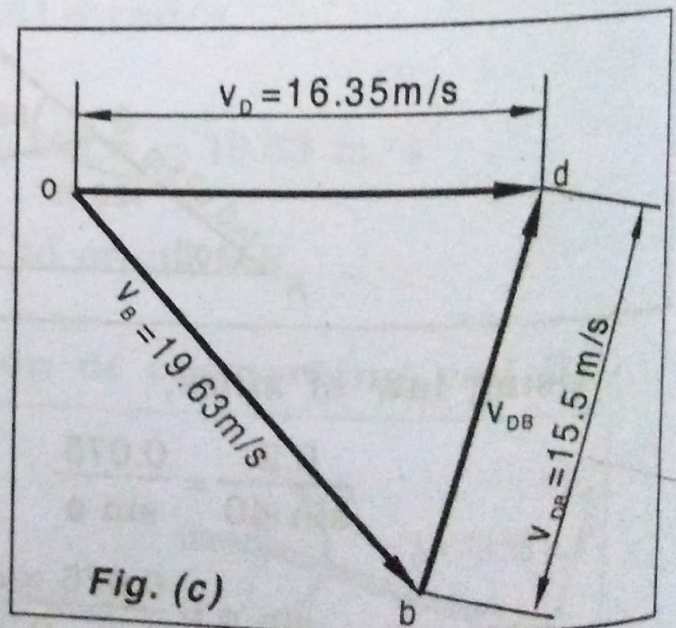
$$\frac{19.63}{0.2534} = \frac{v_D}{0.211} \Rightarrow v_D = 16.35 \text{ m/s}$$

$$\omega_{BD} = \frac{v_B}{IB} = \frac{19.63}{0.2534} = 77.5 \text{ rad/s}$$

Alternate Method

Refer Fig. (c)

1. Draw $ob = 19.63 \text{ cm}$ to represent $v_B = 19.63 \text{ m/sec}$ perpendicular to AB ie 50° to horizontal line.
2. Draw line horizontally (length not known) to represent v_D from O .



3. Draw a line perpendicular to BD from b to represent v_{DB} (length not known).
4. Both lines intersect at d .
5. Now measure $od = v_D = 16.35 \text{ m/s}$; $v_{DB} = 15.5 \text{ m/s}$

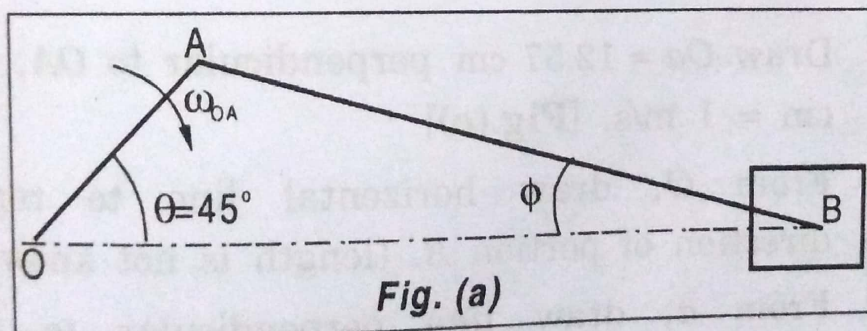
$$\text{Also } v_{D/B} = (BD) \omega_{BD}$$

$$15.5 = 0.2 \times \omega_{BD}$$

$$\omega_{BD} = 77.5 \text{ rad/s}$$

Problem 5.26: In a reciprocating pump, the piston, connecting rod and crank are shown in Fig. (a). The Crank OA has a constant speed of 1500 rpm in clockwise. When crank OA is at 45° to the horizontal, determine (a) the angular velocity of the connecting rod AB and (b) the velocity of the Piston B .

Take the length of the crank $OA = 8 \text{ cm}$ and that of connecting rod $AB = 16 \text{ cm}$ (Nov 2012 - AU)



Solution:

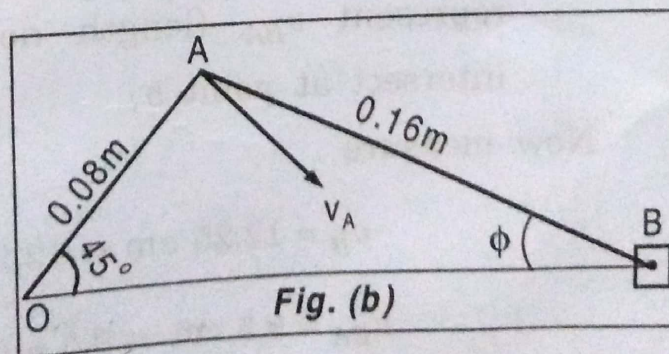
Given:

Speed of the Crank

$$= N_{OA} = 1500 \text{ rpm};$$

$$\theta = 45^\circ; OA = 0.08 \text{ m}$$

$$AB = 16 \text{ cm} = 0.16 \text{ m}$$



Angular Velocity of the Crank

$$\omega_{OA} = \frac{2\pi N_{OA}}{60} = \frac{2\pi \times 1500}{60} = 157 \text{ rad/s}$$

Linear Velocity of the crank

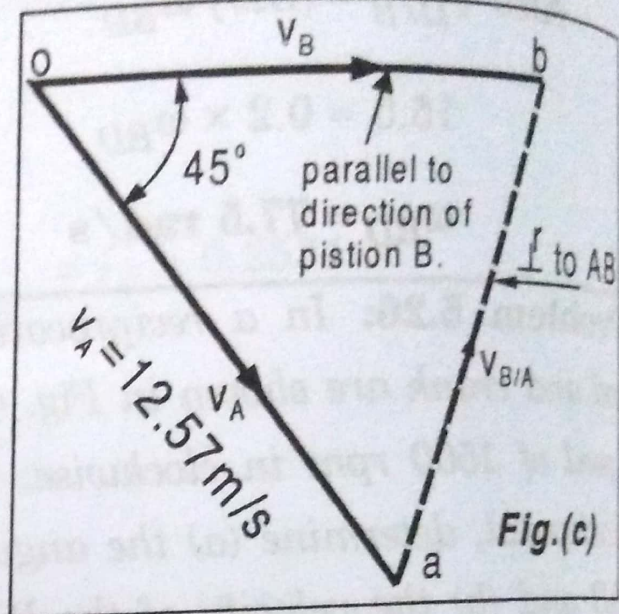
$$v_A = v_{OA} = \omega_{OA} \times OA = 157 \times 0.08 = 12.57 \text{ m/s}$$

From the Fig. (b)

$$\frac{0.08}{\sin \phi} = \frac{0.16}{\sin 45^\circ}$$

$$\sin \phi = \frac{\sin 45^\circ \times 0.08}{0.16} = 0.3535$$

$$\phi = 20.7^\circ$$



Velocity of the Point A is known in magnitude and direction, while the velocity of the Point B is known in direction only.

1. Draw $Oa = 12.57 \text{ cm}$ perpendicular to OA , scale $1 \text{ cm} = 1 \text{ m/s}$. [Fig.(c)]
2. From O , draw horizontal line to represent direction of piston B . (length is not known)
3. From a , draw line perpendicular to AB to represent v_{BA} (length not known). Both lines intersect at point b .

Now measure

$$v_B = 12.25 \text{ cm} \Rightarrow 12.25 \text{ m/s}$$

$$v_{BA} = 9.5 \text{ cm} \Rightarrow 9.5 \text{ m/s}$$

Angular velocity of the connecting rod

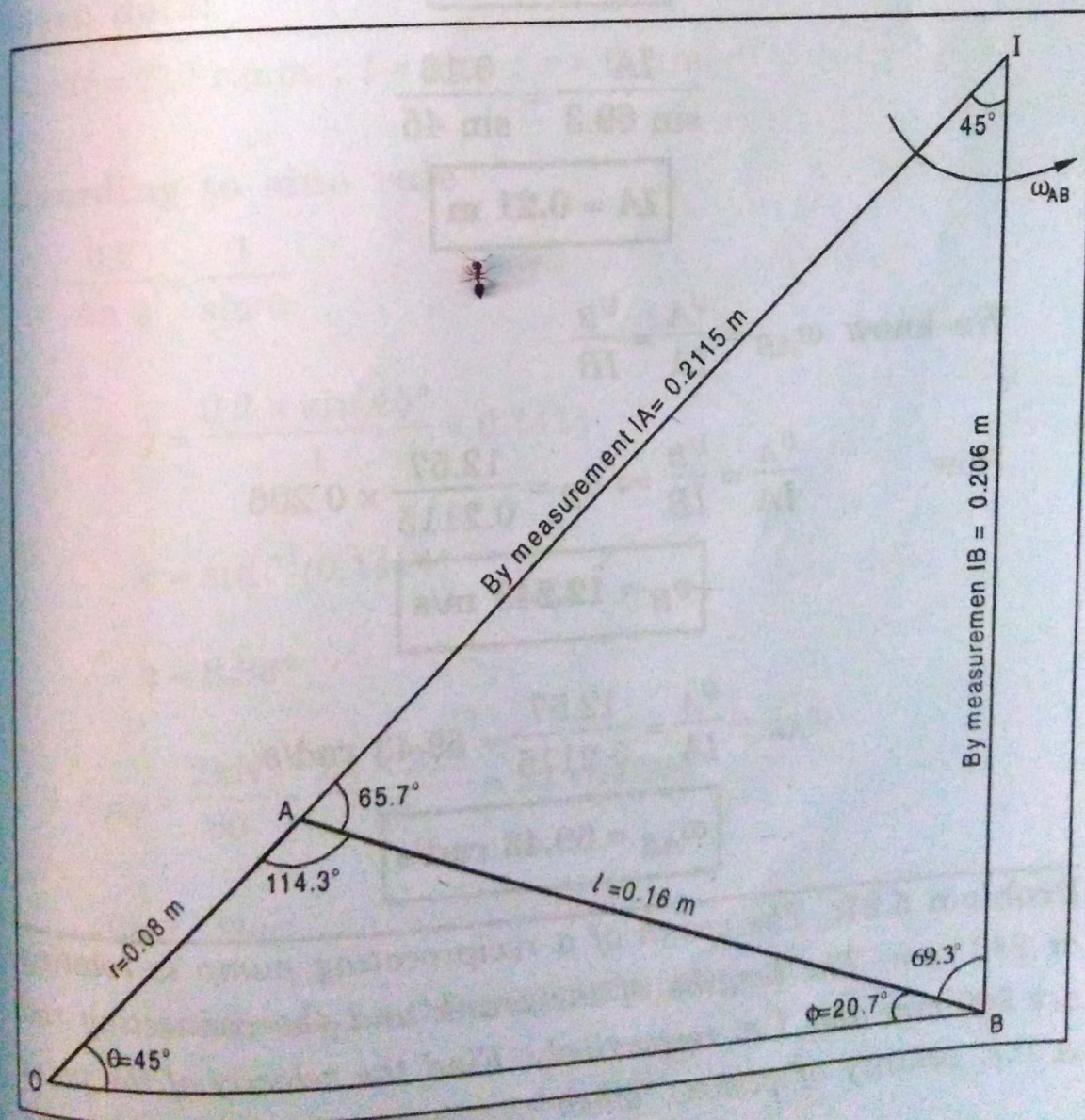
$$v_{B/A} = \omega_{AB} \times AB$$

$$\omega_{AB} = \frac{v_{B/A}}{AB} = \frac{9.5}{0.16} = 59.4 \text{ rad/s}$$

Instantaneous centre method

Draw $OA = 80 \text{ mm}$ [Take suitable scale] at an angle of 45° . Draw horizontal line from O . Take 160 mm as radius, A as centre, draw an arc on this horizontal line and get B . Now $\angle OAB = 114.3^\circ$.

Produce OA and from B , draw vertical line. Both lines intersect at I . (Instantaneous centre).



Now by measurement $IA = 0.2115 \text{ m}$ and

$$IB = 0.206 \text{ m}$$

(or)

To find out IA and IB

$$\sin 45 = \frac{IB}{IA + OA}$$

Consider $\triangle IAB$

$$\frac{0.16}{\sin 45} = \frac{IB}{\sin 65.7}$$

$$\boxed{IB = 0.21 \text{ m}}$$

$$\frac{IA}{\sin 69.3} = \frac{0.16}{\sin 45}$$

$$\boxed{IA = 0.21 \text{ m}}$$

We know $\omega_{AB} = \frac{v_A}{IA} = \frac{v_B}{IB}$

Now $\frac{v_A}{IA} = \frac{v_B}{IB} \Rightarrow v_B = \frac{12.57}{0.2115} \times 0.206$

$$\boxed{v_B = 12.243 \text{ m/s}}$$

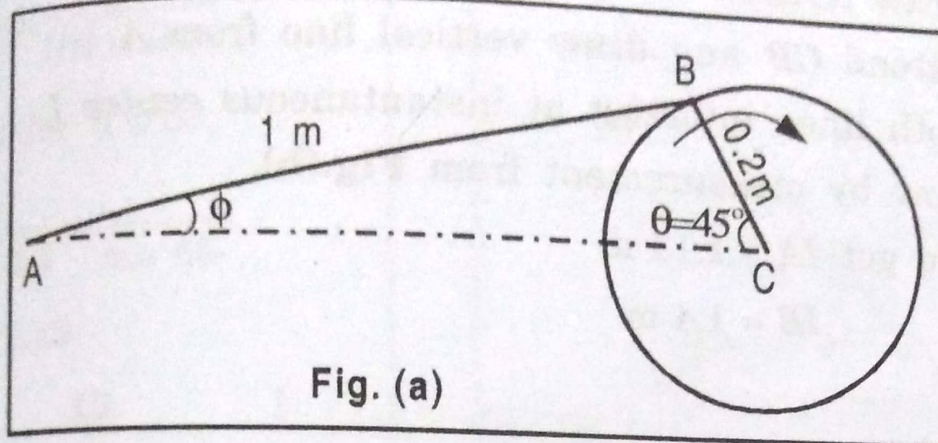
$$\omega_{AB} = \frac{v_A}{IA} = \frac{12.57}{0.2115} = 59.43 \text{ rad/s}$$

$$\boxed{\omega_{AB} = 59.43 \text{ rad/s}}$$

Problem 5.27: The crank of a reciprocating pump is rotating at 210 rpm. The lengths of the crank and the connecting rod are 200 mm and 1 m respectively. Find the velocity of the point A (i.e. velocity of piston), when the crank has turned through

an angle of 45° with the horizontal as shown in Fig. (a).

(KTU-May 2011 - Qn 2006)



Solution:

Given data:

$$N = 210 \text{ r.p.m} ; l = 1 \text{ m} ; r = 0.2 \text{ m}$$

According to sine rule

$$\frac{0.2}{\sin \phi} = \frac{1}{\sin \theta}$$

$$\sin \phi = \frac{0.2 \times \sin 45^\circ}{1} = 0.1414$$

$$\phi = \sin^{-1}(0.1414)$$

$$\phi = 8.30^\circ$$

$$\omega_{BC} = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60} = 22 \text{ rad/sec}$$

$$v_B = r \omega_{BC}$$

$$= 0.2 \times \omega_{BC} = 0.2 \times 22 = 4.4 \text{ m/s}$$

$$v_B = 4.4 \text{ m/s}$$

Refer **Fig.(b)**

To mark instantaneous centre I ,

Draw ΔABC .

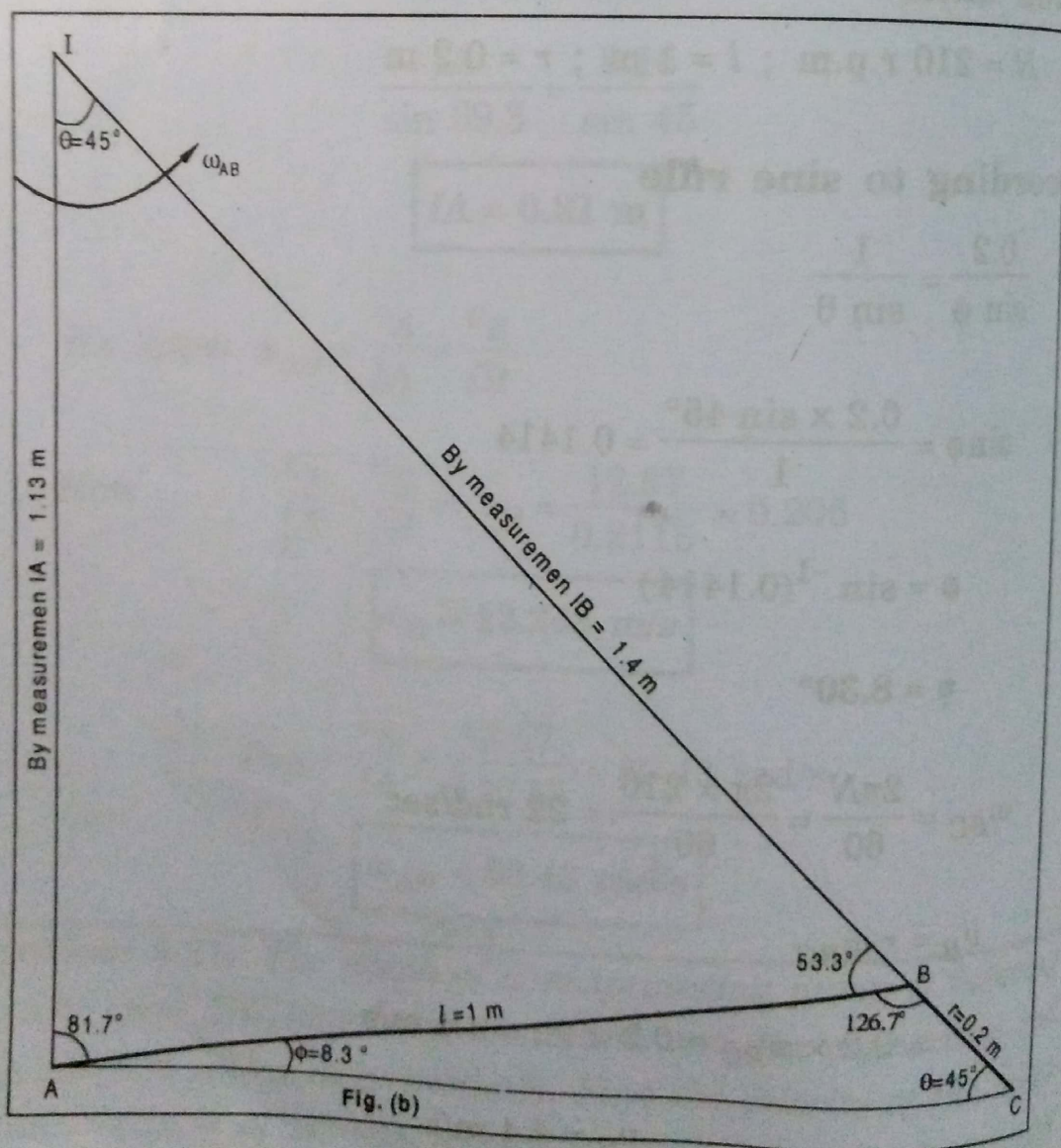
Extend CB and draw vertical line from A .

Both lines intersect at instantaneous centre I .

Now by measurement from **Fig.(b)**,

We get $IA = 1.13 \text{ m}$

$IB = 1.4 \text{ m}$



(or) Consider ΔIAC ,
from fig. (c)

$$\sin \theta = \frac{IA}{(IB + BC)}$$

Consider ΔIAB ,

$$\frac{IA}{\sin 53.3} = \frac{1}{\sin 45}$$

$$IA = 1.13$$

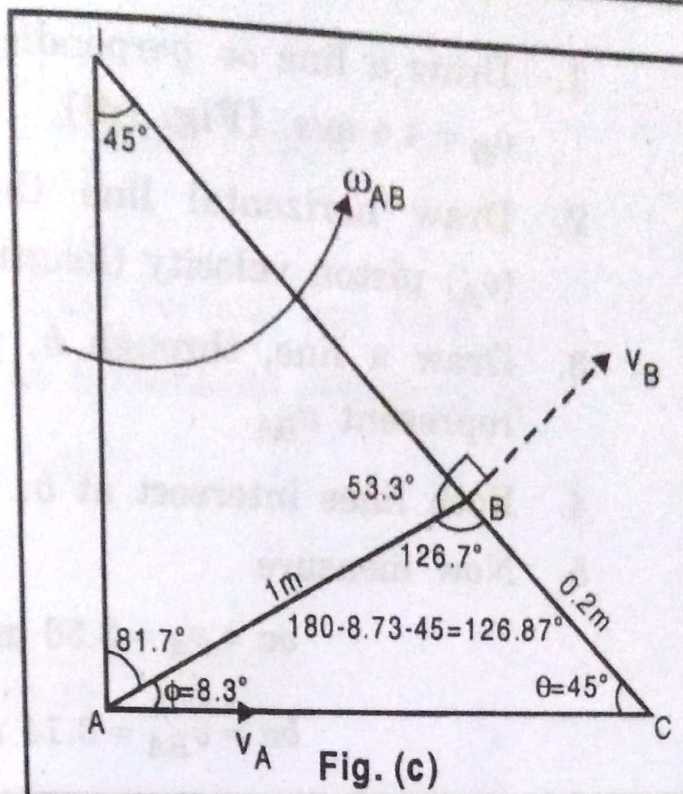
$$\text{Also, } \frac{IB}{\sin 81.7} = \frac{1}{\sin 45}$$

$$IB = 1.4 \text{ m}$$

$$\therefore \omega_{AB} = \frac{v_A}{IA} = \frac{v_B}{IB}$$

$$\omega_{AB} = \frac{v_A}{1.13} = \frac{4.4}{1.4}$$

$$= 3.143 \text{ rad/s}$$

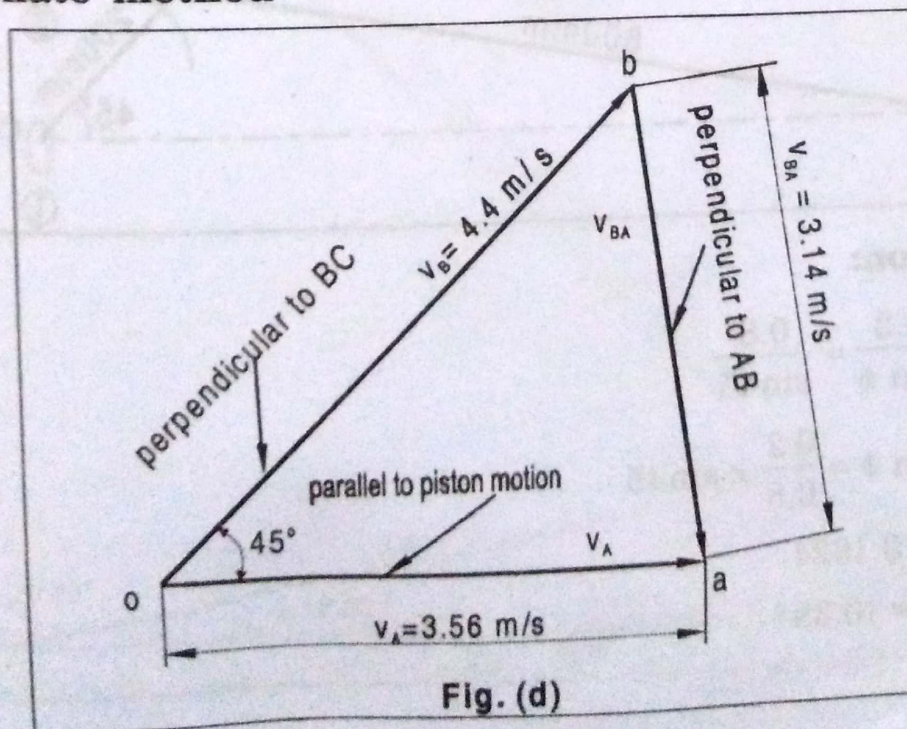


$$v_A = 1.13 \times 3.143$$

$$= 3.56 \text{ m/s}$$

$$\therefore v_A = 3.56 \text{ m/s}$$

Alternate method

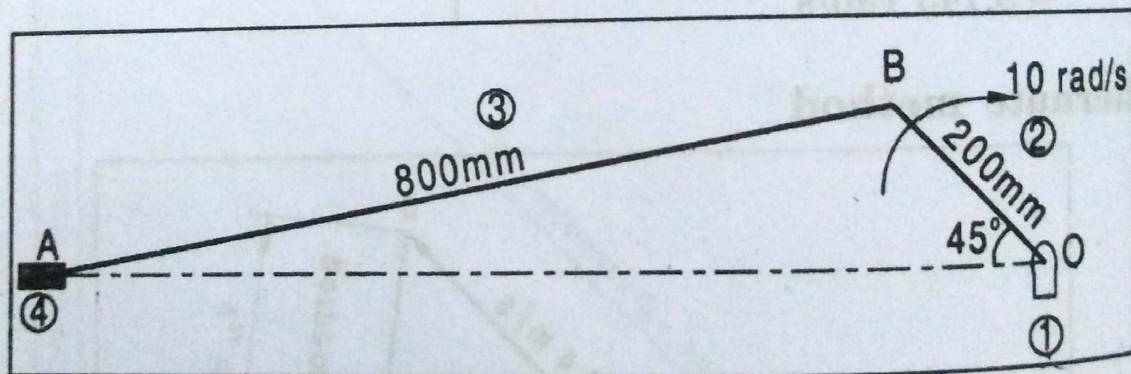


1. Draw a line ob perpendicular to BC to represent $v_B = 4.4 \text{ m/s}$. [Fig. (d)]
2. Draw horizontal line through O , to represent (v_A) piston velocity (length not known).
3. Draw a line, through b , perpendicular to AB to represent v_{BA} .
4. Both lines intersect at b .
5. Now measure

$$oa = v_A = 3.56 \text{ m/s}$$

$$ba = v_{BA} = 3.14 \text{ m/s}$$

Problem 5.28: The lengths of crank OB and connecting rod AB are 200 mm and 800 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s , find 1. velocity of the slider A and 2. Angular velocity of the connecting rod AB .



Solution:

$$\frac{0.2}{\sin \phi} = \frac{0.8}{\sin 45^\circ}$$

$$\sin \phi = \frac{0.2}{0.8} \times \sin 45^\circ$$

$$= 0.1624$$

$$\phi = 10.38^\circ$$

$$\omega_{OB} = 10 \text{ rad/s}$$

$$V_{BO} = V_B = \omega_{OB} \times OB = 10 \times 0.2 = 2 \text{ m/s}$$

Draw ABO

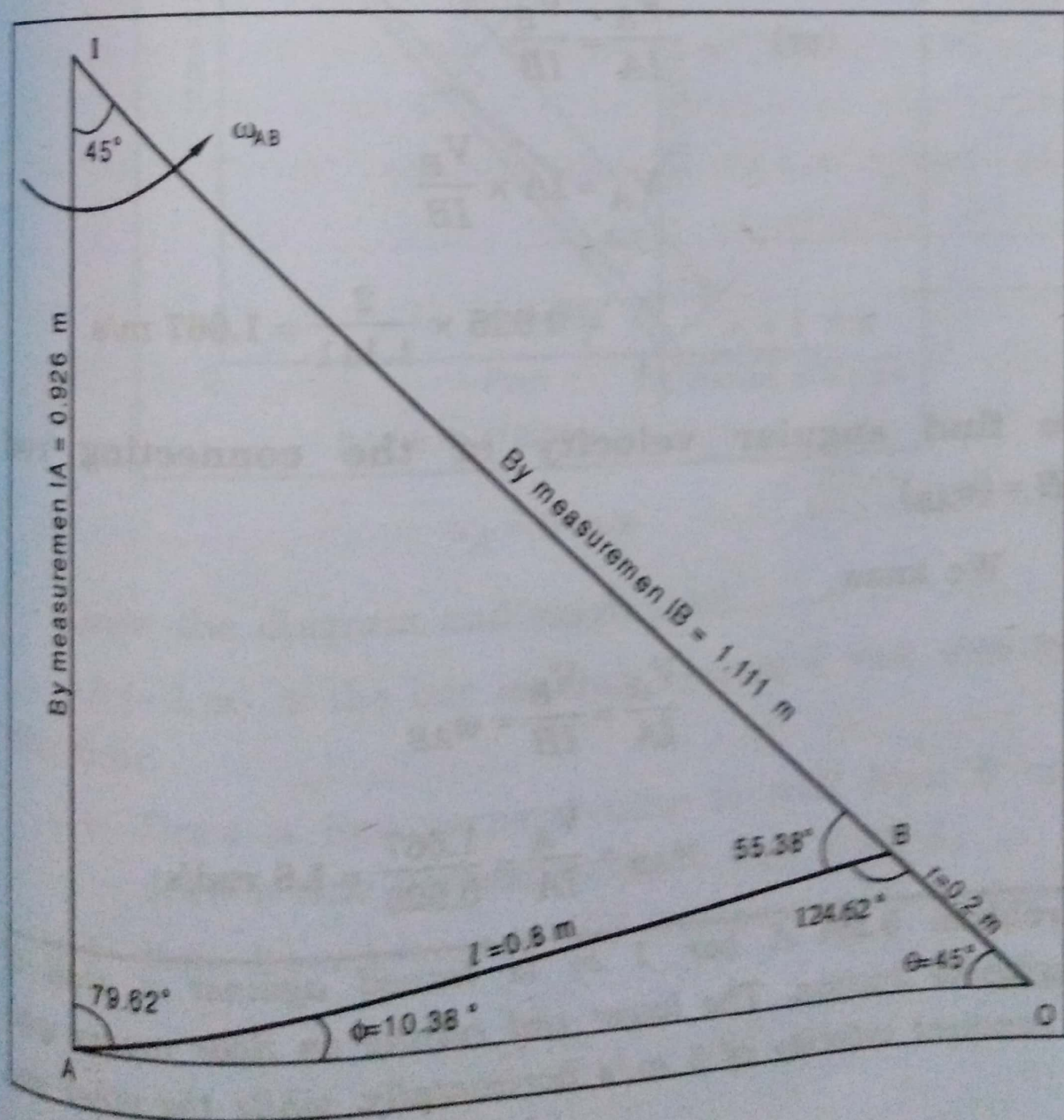
❖ Extend OB and draw vertical line from A.

Both lines intersect at Instantaneous centre at A.

(or)

Consider $\triangle IAB$,

$$\frac{IA}{\sin 55.38} = \frac{0.8}{\sin 45}$$



$$IA = 0.93 \text{ m}$$

$$\text{Similarly, } \frac{IB}{\sin 79.62} = \frac{0.8}{\sin 45}$$

$$IB = 1.1 \text{ m}$$

Now measure,

$$IA = 0.926 \text{ m}$$

$$IB = 1.111 \text{ m}$$

To find velocity of slider A

we know,

$$(\text{or}) \quad \frac{V_A}{IA} = \frac{V_B}{IB}$$

$$V_A = IA \times \frac{V_B}{IB}$$

$$= 0.926 \times \frac{2}{1.111} = 1.667 \text{ m/s}$$

To find angular velocity of the connecting rod
 $AB = (\omega_{AB})$

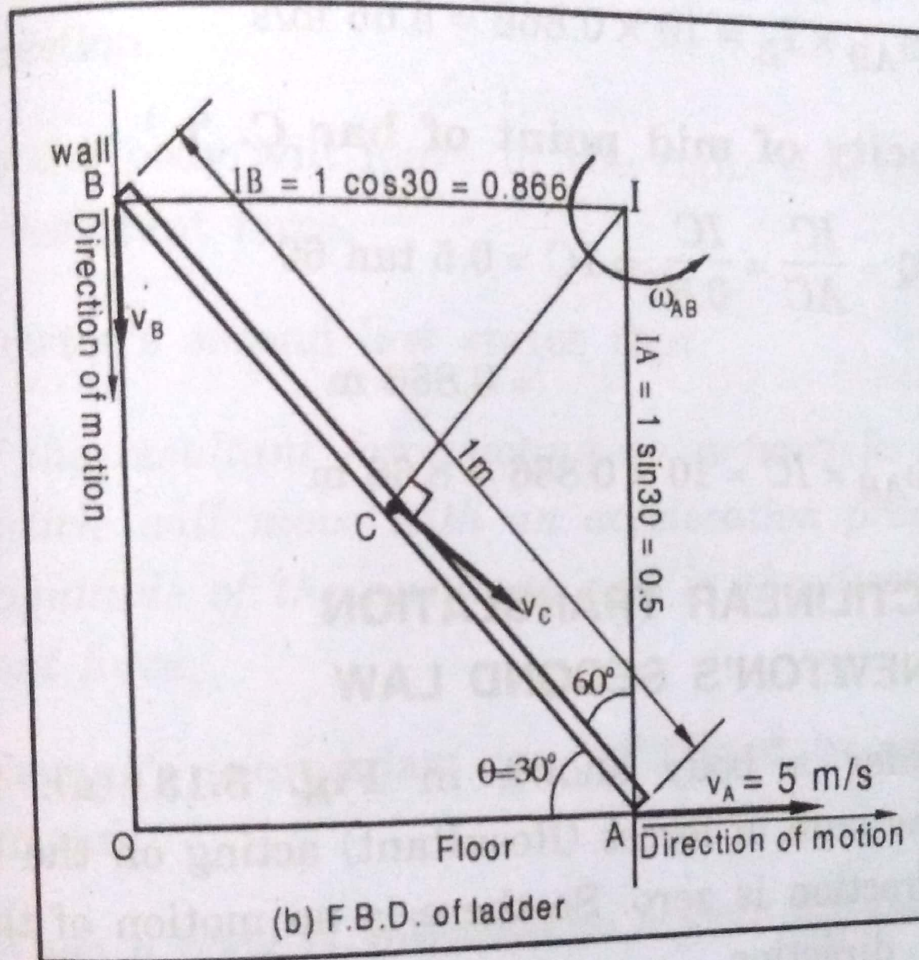
We know,

$$\frac{V_A}{IA} = \frac{V_B}{IB} = \omega_{AB}$$

$$\omega_{AB} = \frac{V_A}{IA} = \frac{1.667}{0.926} = 1.8 \text{ rad/s}$$

Problem 5.29: A bar 1 m is leaned against a wall of negligible friction. The lower end resting on floor moves with a constant velocity of 5 m/s horizontally, while the other end

slides over the wall, without leaving it. There is no friction between the floor and the bar. Find (i) the angular velocity of the bar; (ii) The velocity of the end touching the wall; and (iii) the velocity of the mid-point of the bar at the instant when the axis of the bar makes an angle of 30° with the horizontal. Solve the problem by instantaneous centre method. Neglect the weight of the bar.



$$v_A = 5 \text{ m/s}$$

Draw the diagram and mark OAB .

$AB (= 1 \text{ m})$ is the bar leaning against a wall with 30° with floor.

- ❖ Draw a line perpendicular to wall from B and draw a line perpendicular to floor from A .
- ❖ Both lines intersect at I (Instantaneous centre).

5.76 Engineering Mechanics

$$\frac{v_A}{I_A} = \frac{v_B}{I_B} = \omega_{AB}$$

(i) Angular velocity of the bar

$$\omega_{AB} = \frac{5}{0.5} = 10 \text{ rad/s}$$

(ii) Velocity of the end touching the wall [v_B]

$$v_B = \omega_{AB} \times I_B = 10 \times 0.866 = 8.66 \text{ m/s}$$

(iii) Velocity of mid point of bar C. ' v_c '

$$\tan 60 = \frac{IC}{AC} = \frac{IC}{0.5} \Rightarrow IC = 0.5 \tan 60$$

$$= 0.866 \text{ m}$$

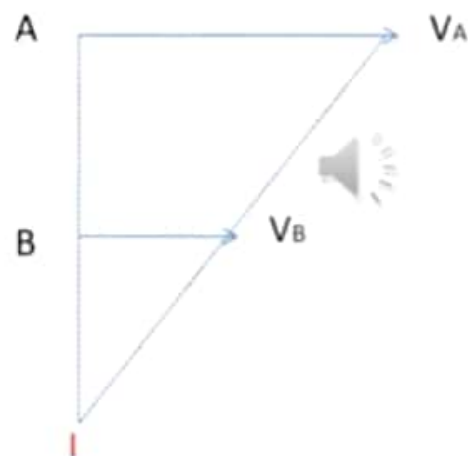
$$v_c = \omega_{AB} \times IC = 10 \times 0.866 = 8.66 \text{ m}$$

- Instantaneous Centre

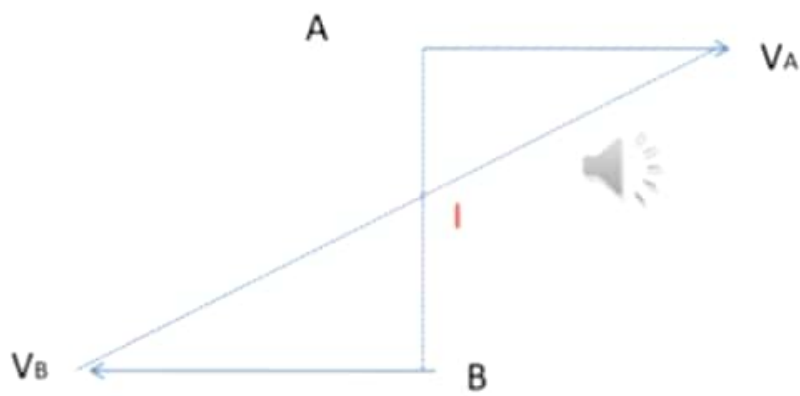
- **Instantaneous center** is a point on a member about which a body can be assumed to be rotating at the given instant
- Properties of **Instantaneous center**
- Velocity of instantaneous centre is zero
- The magnitude of velocity of any point on a body is equal to angular velocity times distance $V = r\omega$
- Direction of velocity of any point is perpendicular to line joining that point and instantaneous center

- How to find instantaneous centre (I)?

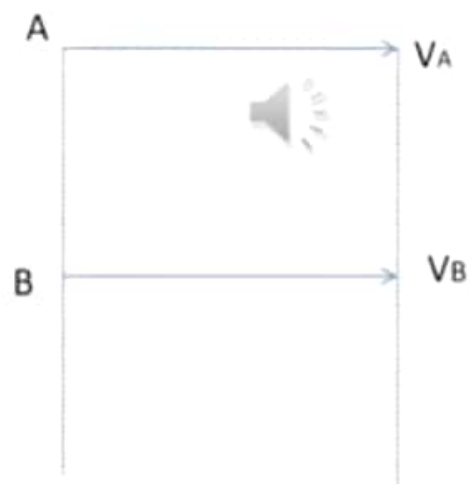
Case 1 : Velocities are parallel and not equal



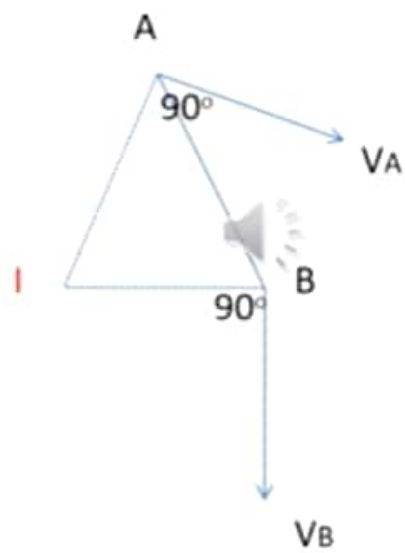
Case 2: Velocities are parallel, unequal and opposite



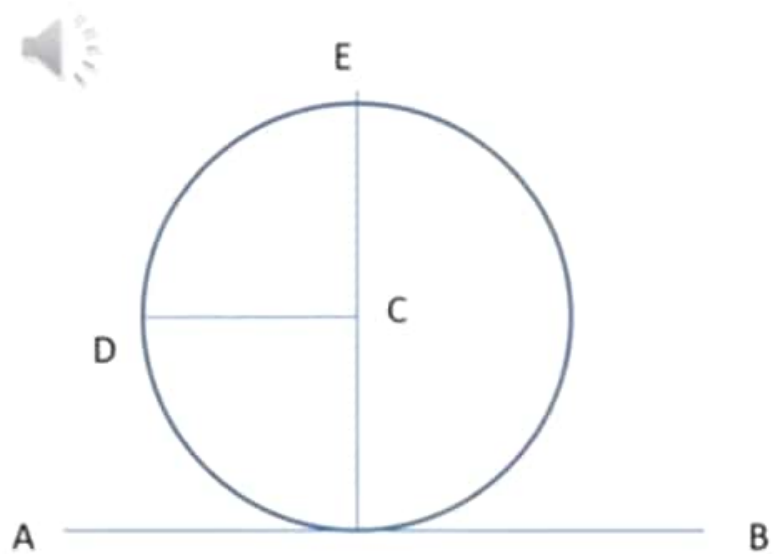
Case 3: Velocity are equal, parallel and acting in same direction



Case 4: When velocities are not parallel



1. A cylinder of radius 1m rolls without slipping along a horizontal plane AB. Its center has a uniform velocity of 20 m/s. Find velocities of the point D and E on the circumference of the cylinder as shown in figure.



- Linear velocity of I is zero. So it is the instantaneous center.

Given: $r = 1\text{m}$, $V_C = 20\text{m/s}$

$V = (\omega \times r)$ r is the distance between I and pt which we consider

$$V_C = (\omega \times IC)$$

$$20 = \omega \times 1 \quad (IC = r = 1)$$

$$\rightarrow \omega = 20 \text{ rad/s}$$



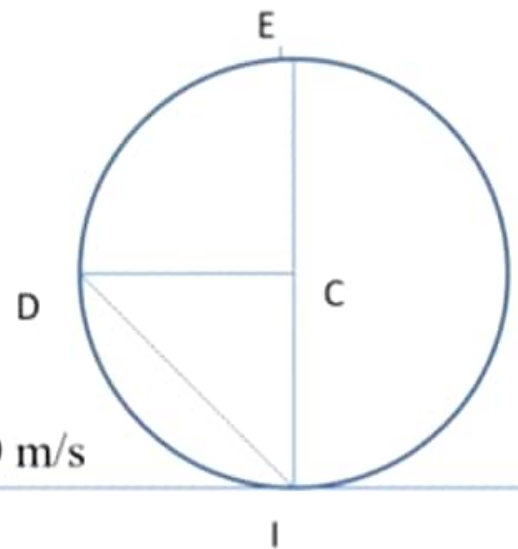
$$1. \text{Velocity at point E} = V_E = (\omega \times IE) = 20 \times 2 = 40 \text{ m/s}$$

$$2. \text{Velocity at point D} = V_D = (\omega \times ID)$$

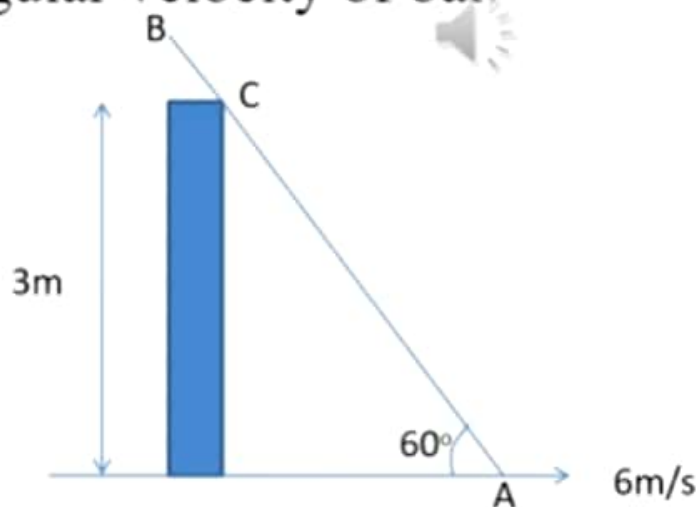
where $ID^2 = IC^2 + CD^2$

$$ID = \sqrt{2}$$

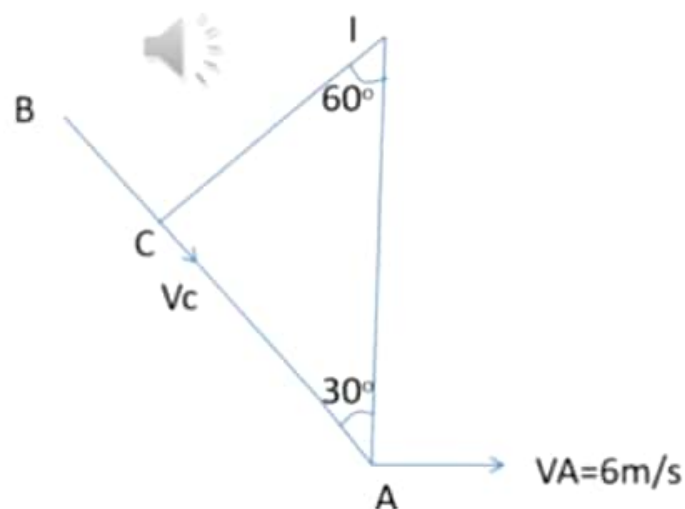
$$\rightarrow V_D = 20 \times \sqrt{2} = 28.28 \text{ m/s}$$



2. A bar AB rests at the edge of a wall of height 3m at some point C on the bar as shown in figure. The bar makes an angle of 60 degree with horizontal. If the end A moves with a constant velocity of 6 m/s, find the angular velocity of bar.

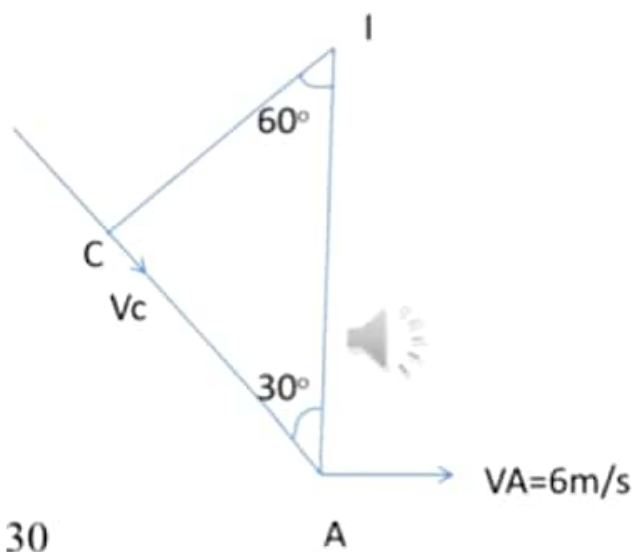


The direction of velocity of A is horizontal and direction of velocity of C is along the bar BA. The instantaneous center of bar AB is the point of intersection of the lines drawn perpendicular to the direction of velocities V_A and V_C .



$$AC \sin 60 = 3 \text{ m}$$

$$\Rightarrow AC = \frac{3}{\sin 60} = 3.46 \text{ m}$$



$$AI = AC \cos 30$$

$$AI = (3.46 \times \cos 30) = 4 \text{ m}$$

$$V_A = (\omega_{AB} \times AI)$$

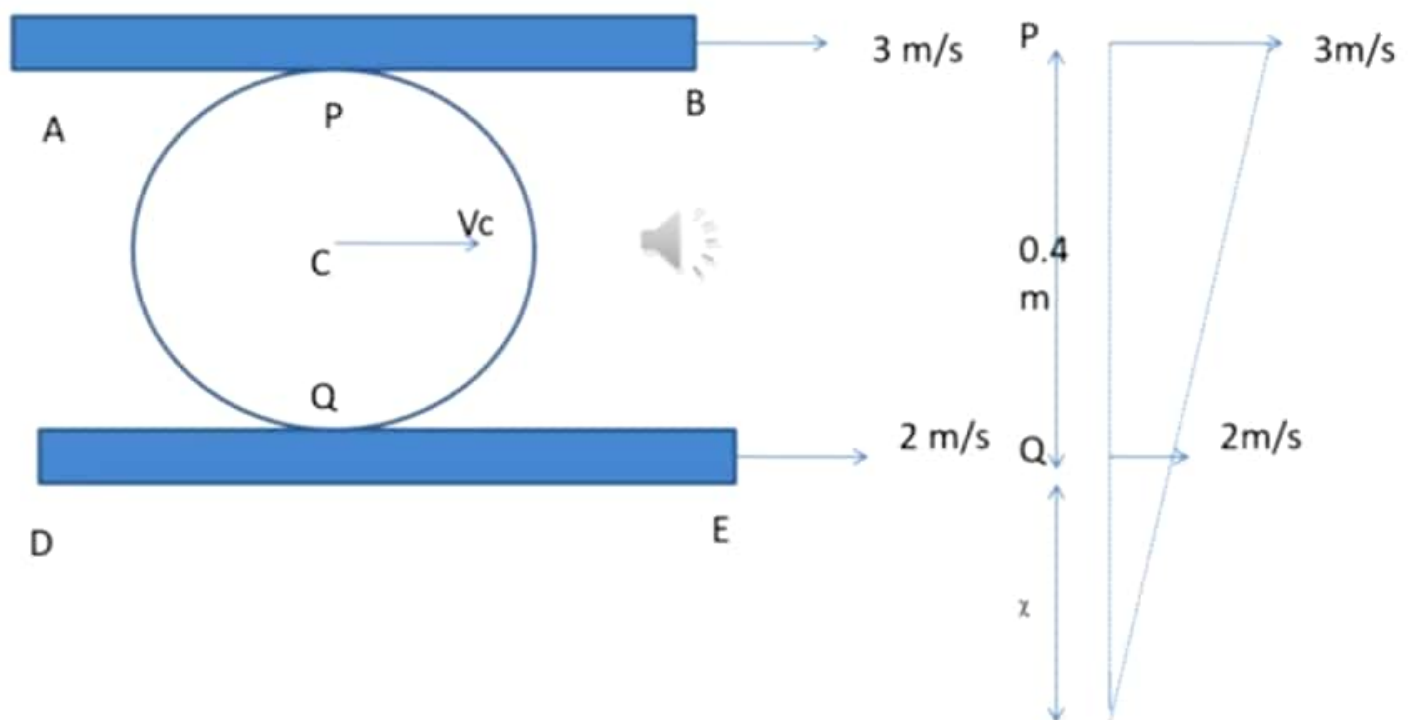
$$\Rightarrow \omega_{AB} = \frac{V_A}{AI}$$

$$= \frac{6}{4} = 1.5 \text{ rad/s}$$

3. A cylindrical roller is in contact at its top and bottom, with two conveyor belts AB and DE as shown in figure. If the belts run at uniform speed of 3 m/s and 2 m/s respectively, find the angular velocity of the roller when

- The velocities are in the same direction
- The velocities are in the opposite direction if diameter of roller is 40 cm

- Case 1



- When the velocities are in same direction. let the point of instantaneous centre be at a distance x below Q.

Velocity of P,

$$3 = \omega \times (0.4 + x) \dots \dots \dots (i)$$

Velocity of Q,

$$2 = \omega \times x \dots \dots \dots (ii)$$

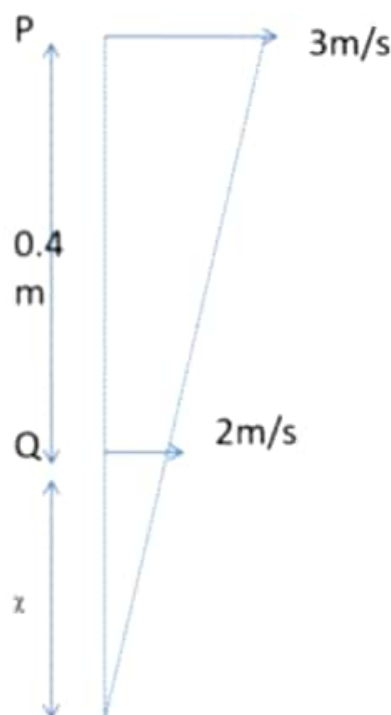
Dividing (i)/(ii) we get

$$\frac{3}{2} = \frac{\omega \times (0.4 + x)}{\omega \times x}$$

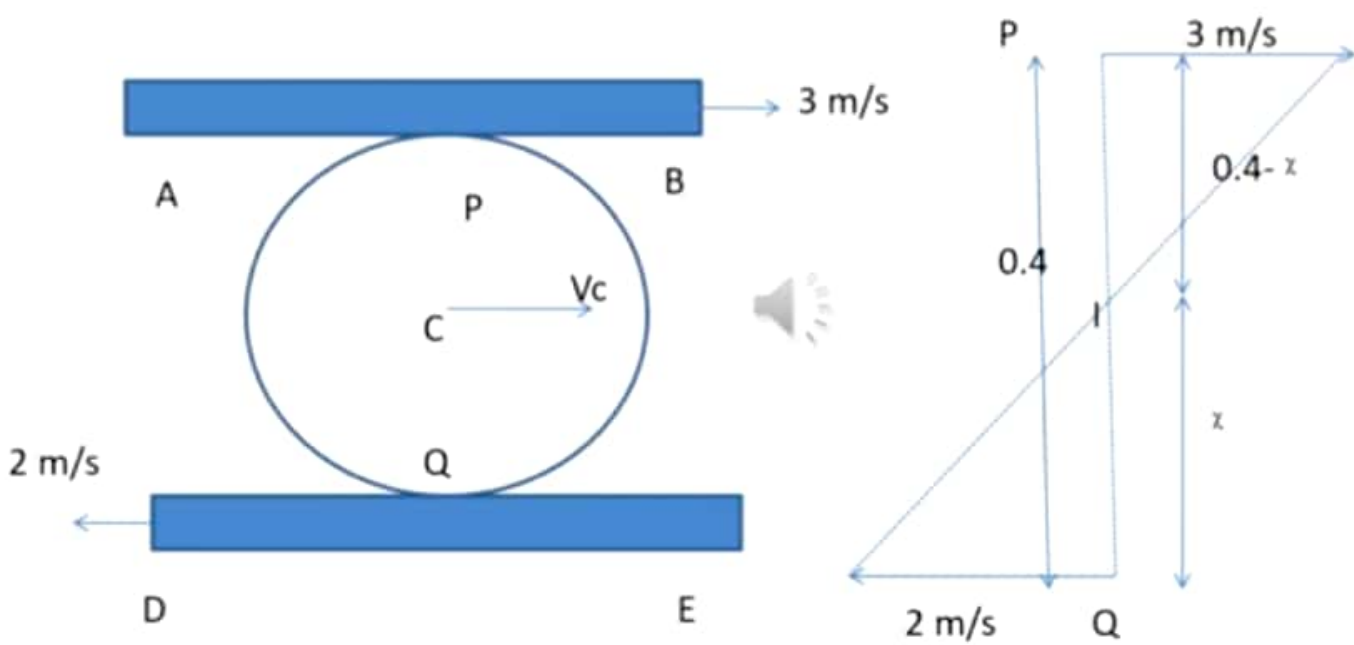
$$3x = 0.8 + 2x$$

$$x = 0.8$$

Substitute in (ii) we get $\omega = 2.5 \text{ rad/s}$



- Case 2



- When velocities are in opposite direction. Let instantaneous center be at a distance x above point Q.

Velocity of P,

$$3 = \omega \times IP = \omega \times (0.4 - x) \dots \dots \dots (i)$$

$$2 = \omega \times IQ = \omega \times x \dots \dots \dots (ii)$$

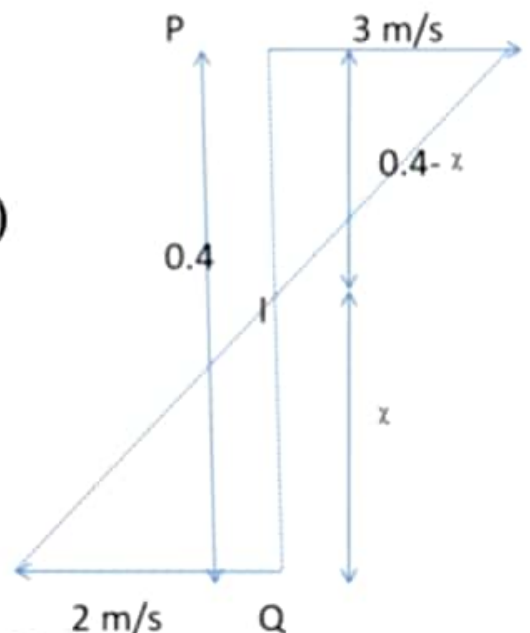
Dividing (i)/(ii) we get

$$3x = 0.8 - 2x$$

$$5x = 0.8$$

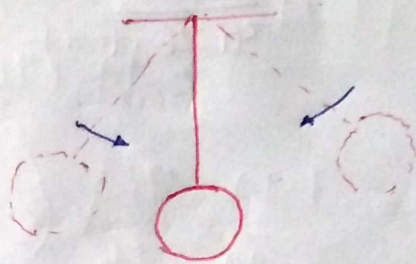
$$x = 0.16\text{m}$$

Substitute in (ii) we get $\omega = 12.5 \text{ rad/s}$



SIMPLE HARMONIC MOTION

- Types of vibration.
- Classification of vibration based on direction of motion.
- Definitions : Amplitude, Frequency, time period, cycle, stiffness of springs.
- Degrees of Freedom.
- Simple Harmonic motion.



Problems

- Problems related to spring
- Problems related to SHM.

Module 6

MECHANICAL VIBRATIONS

Mechanical Vibrations - Free and forced vibration - Degree of freedom - Simple Harmonic motion - Spring - Mass model - Period - Stiffness - Frequency - Simple numerical problems of single degree of freedom.

6.1 INTRODUCTION

Springs, beams, shafts and other elastic bodies when displaced from their equilibrium position due to the application of external forces, and then released, they execute a 'to and fro' motion called **vibratory motion**.

Vibrations are due to elastic forces. Whenever a body is displaced from its equilibrium position, work is done on the elastic constraints of body and is stored as strain energy. Now if the body is released, the internal forces cause the body to move towards equilibrium position. If the motion is frictionless, the strain energy stored in the body is converted into kinetic energy. During the period, the body reaches the equilibrium position at which it has maximum kinetic energy. The body passes through the mean position, the kinetic energy is utilised to overcome the elastic forces and is stored in the form of strain energy. Again the body begins to return to the equilibrium position and vibration is repeated indefinitely.

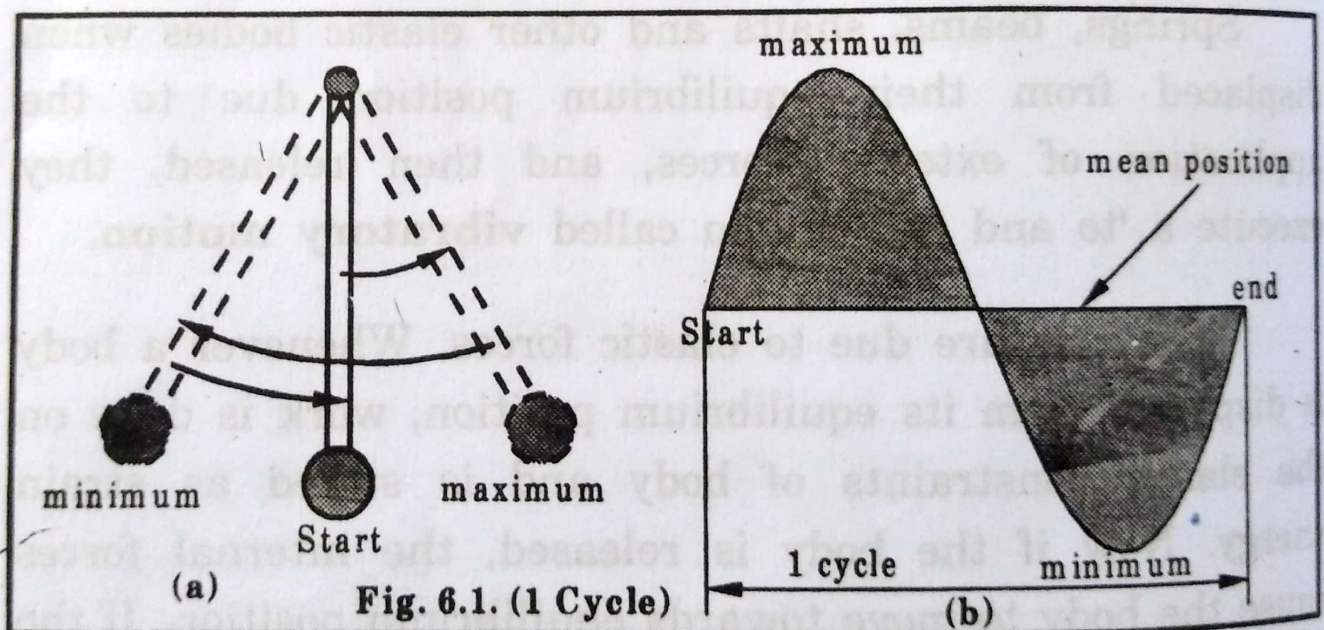
6.2 COMMONLY USED DEFINITIONS IN VIBRATORY MOTIONS (OR) BASIC FEATURES OF VIBRATING SYSTEM

1. Period of Vibration

It is the time interval after which the motion is repeated itself. It is time period to execute one complete cycle of vibration. It is expressed in second 's'.

2. Cycle

It is the motion completed in one time period. If a particle starts from mean position, goes to two extreme positions and then again comes to mean position, then it completes one cycle. (**Fig. 6.1**)



3. Frequency

Number of cycles completed in one second.

Unit: cycles/sec = Hertz = Hz.

4. Amplitude

The maximum displacement of a vibrating body from the equilibrium position.

5. Natural frequency

It is the frequency of free vibrations of body vibrating of its own without the help of an external agency.

6. Fundamental (or principal) mode of vibration

It is the mode of vibration having the lowest natural frequency.

7. Degree of freedom

The minimum number of independent coordinates required to specify the motion of system.

8. Damping

It is the resistance to the motion of vibrating body.

9. Phase difference

It is the angle by which one vibrating system is ahead or behind the other vibrating system.

10. Resonance

When the frequency of external excitation is equal to the natural frequency of a vibrating body, resonance occurs.

11. Mechanical system

A system consisting of mass, spring and damper is called mechanical system.

12. Discrete (or lumped) system

A system with finite number of degrees of freedom.

6.3 TYPES OF VIBRATORY MOTION

1. Free (or) Natural Vibrations

A vibration, in which after the initial displacement, no external forces act on the body and the motion is maintained by the internal elastic forces, is called Natural vibration.

First of all, a body is given an initial displacement. Then no external force acts on a body; Then the body is said to be under Free Vibrations or Natural Vibrations. The frequency of this vibration is called as natural frequency.

Ex: Vibration created in Guitar string, Vibration created in Diving boat, Vibration created in Twining fork, Suspension spring at two wheelers (when falling on to a pit).

2. Forced Vibrations

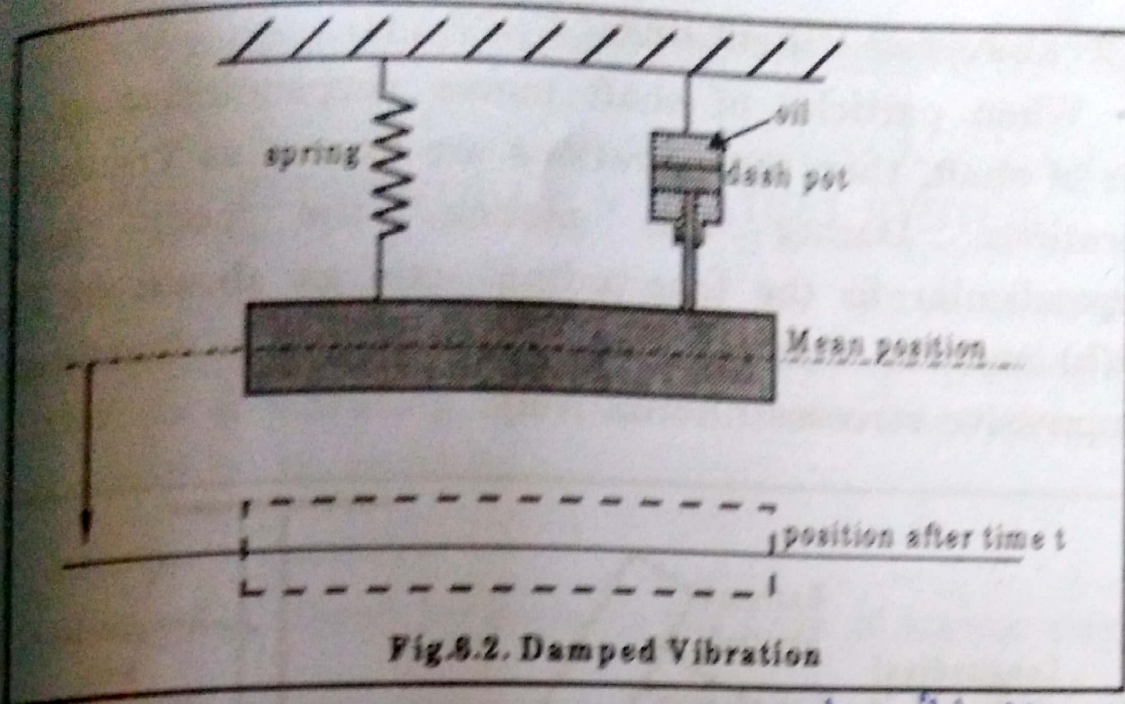
When a body vibrates under the influence of continuous periodic disturbing internal (or) external force, then the body is said to be under forced vibrations.

Ex: Vibration created by rotating and reproccating masses of IC engine.

3. Damped Vibrations

When there is reduction in amplitude over every cycle of vibration, energy possessed by a system is gradually dissipated in overcoming the internal or external resistances to motion and body comes to rest in its equilibrium position, the motion is said to be damped vibration.

Ex: Shock Absorber with Dashpot. (Fig. 6.2)



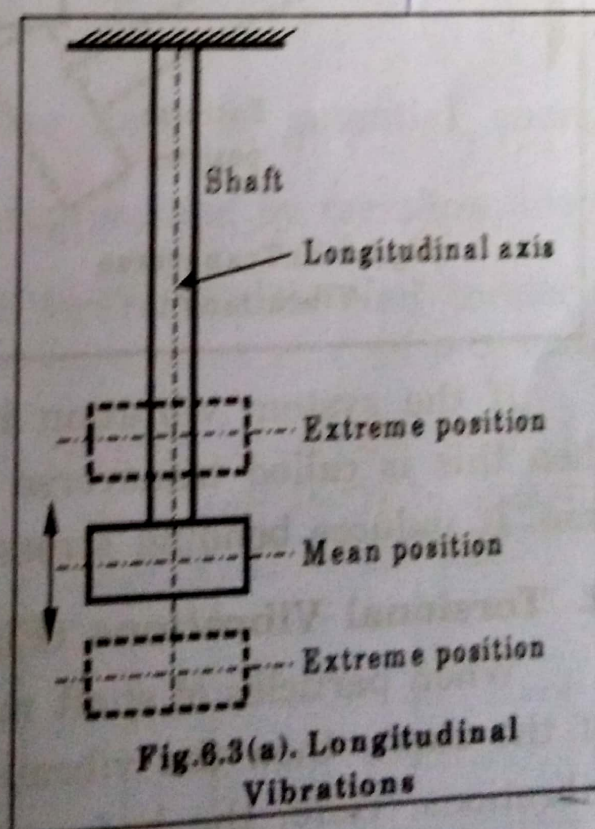
6.3.1 Types of Free Vibrations

or Classification of vibration based on direction of motion.

1. Longitudinal Vibrations

When particles of shaft move parallel to axis of the shaft, then the vibrations are known as **Longitudinal Vibrations**. During this vibration, the body moves parallel to longitudinal axis as shown in Fig. 6.3(a).

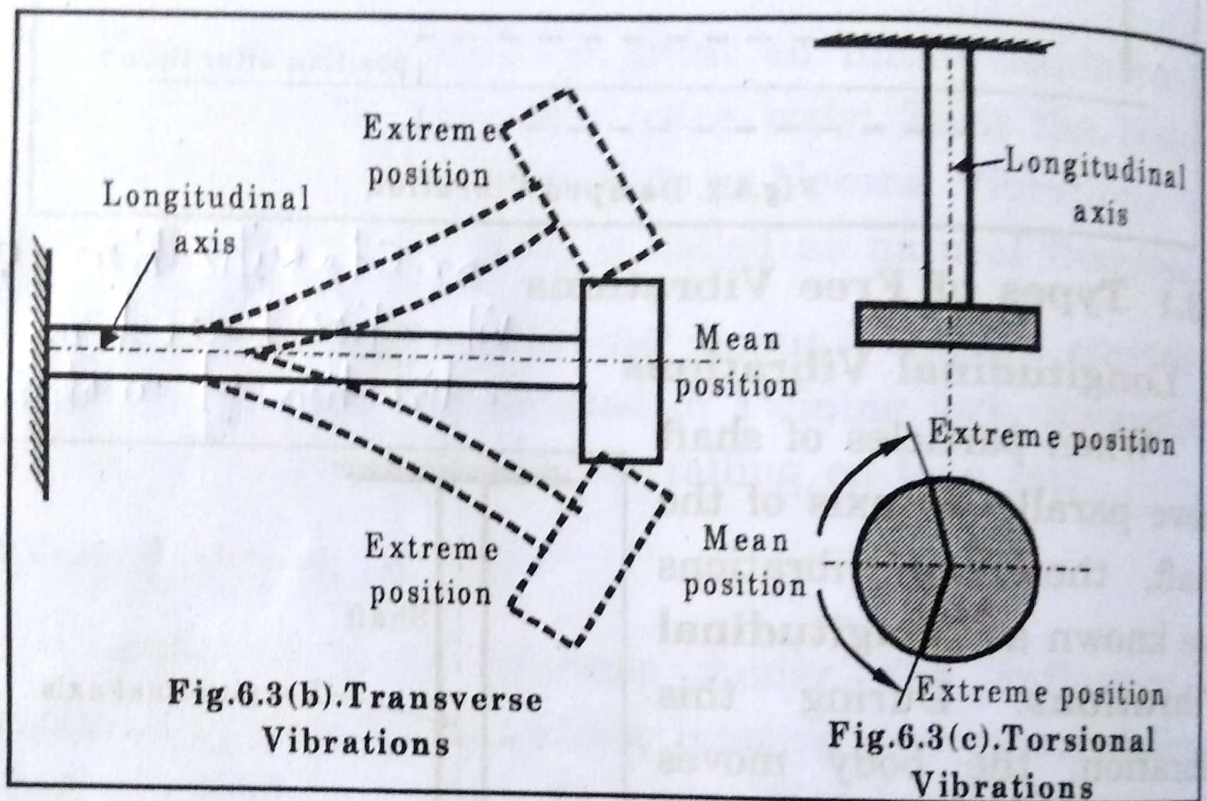
If the movement of system is parallel to the axis, then the vibration is called as longitudinal vibration.



The amplitude of longitudinal vibration is expressed in mm. Longitudinal vibration induces tensile & compressive stresses.

2. Transverse Vibrations:

When particles of shaft moves perpendicular to the axis of shaft, then the vibrations are known as Transverse vibrations. During this motion, the body moves perpendicular to the longitudinal axis as shown in Fig. 6.3(b) and the shaft is bent and subjected to tensile and compressive stresses alternatively.



If the system vibration is perpendicular to the axis, then this is called transverse vibration. It is expressed in mm. It induces bending stress.

3. Torsional Vibrations (Fig. 6.3 (c))

When particles of shaft move in a circle about the axis of the shaft, then the vibrations are known as Torsional vibrations. Here, the body will be twisted and untwisted about the longitudinal axis of the shaft.

Under the influence of twisting moment (Torque), the system is twisted and untwisted about the longitudinal

axis. This induces shear stress. This vibration is measured in radians.

6.4 BASIC ELEMENTS OF VIBRATING SYSTEM

Idealised mathematical modes of vibrating system consists of three elements, namely the mass, the spring and damper. In a vibrating body, there is exchange of energy from one form to another.

(i) Inertia elements

Energy is stored by mass in the form of kinetic energy ($\frac{1}{2} m \dot{x}^2$). These are represented by lumped mass (m) for rectilinear motion and lumped moment of inertia (I or J) for angular motion.

(ii) Restoring elements

Energy is stored in the form of potential energy ($\frac{1}{2} k x^2$). These elements are represented by massless linear (k) or torsional (k_t) springs for rectilinear and torsional motion respectively.

(iii) Damping elements

Energy is dissipated in the damper in the form of heat energy which opposes the motion of the system. These are represented by massless dampers for energy dissipation. The vibration damping elements are shown in Fig. 6.4. These are represented by c for

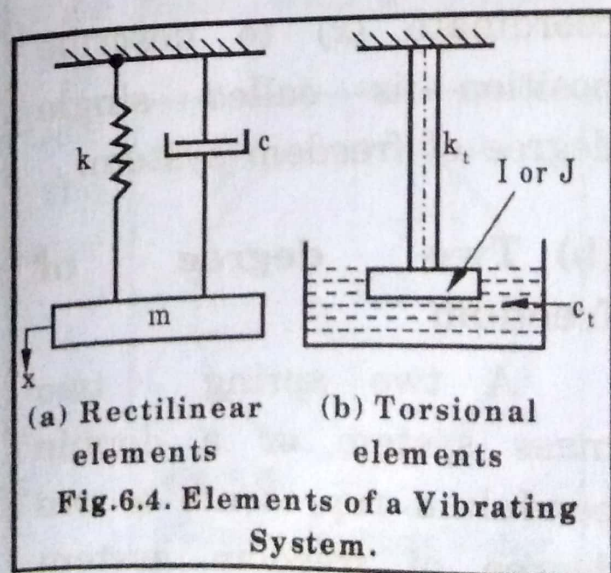


Fig.6.4. Elements of a Vibrating System.

rectilinear motion and c_t for torsional motion.

The equation of motion of such a vibrating system can be written as

$$m \ddot{x} + c \dot{x} + kx = 0$$

Where $c\dot{x}$ = Damping force, kx = spring force;

$m\ddot{x}$ = Inertia force ; $\dot{x} = \frac{dx}{dt}$ and $\ddot{x} = \frac{d^2x}{dt^2}$

6.5 DEGREES OF FREEDOM

Degrees of freedom is defined as the number of independent coordinates required to describe the motion of vibratory system.

(a) Single degree of freedom

A single spring mass system or a simple pendulum as shown in **Fig. 6.5 (a,b)** requiring only one coordinate (x) to describe position is called single degree of freedom system.

(b) Two degree of freedom

A two spring - two mass system or a double pendulum represents a two degree of freedom system

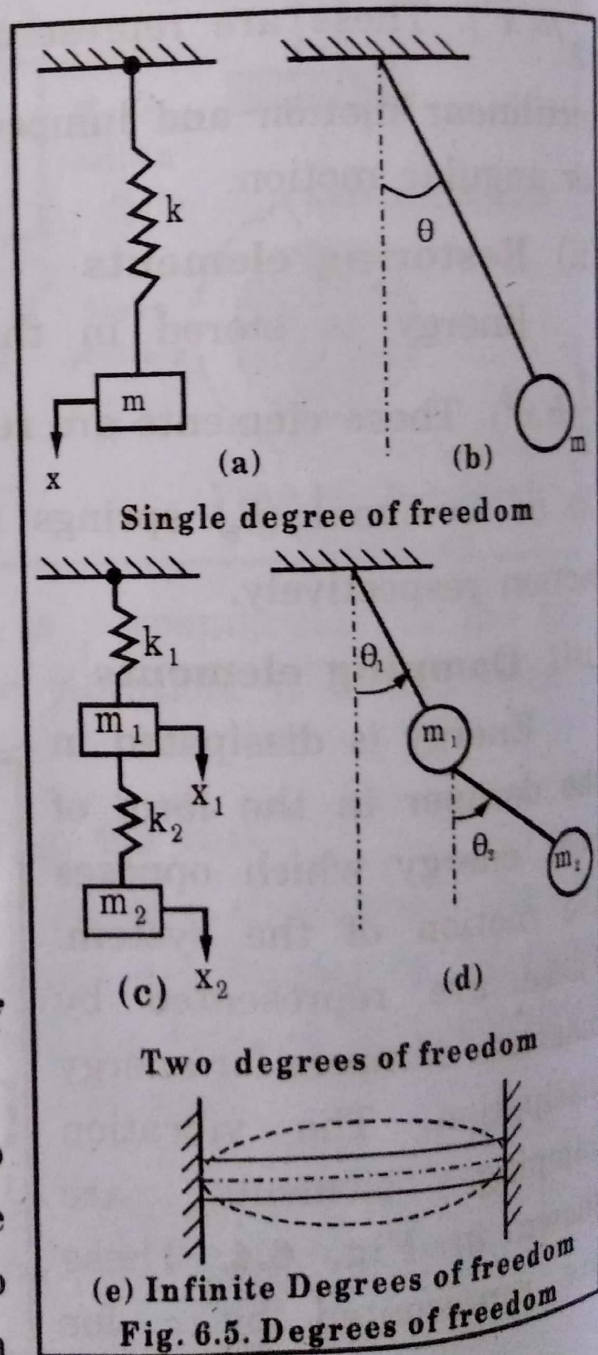


Fig. 6.5 (c,d)]. There are two coordinates required (x_1, x_2) to describe the position. Similarly for a three degree of freedom system, the coordinates required are (x_1, x_2, x_3) .

(c) Discrete or lumped system (Finite degree of freedom)

A system with finite number of degrees of freedom are called discrete or lumped system.

(d) Infinite degree of freedom

A continuous system in the form of a vibrating beam held between two supports represents an infinite degrees of freedom Fig. 6.5 (e).

6.6 SIMPLE HARMONIC MOTION

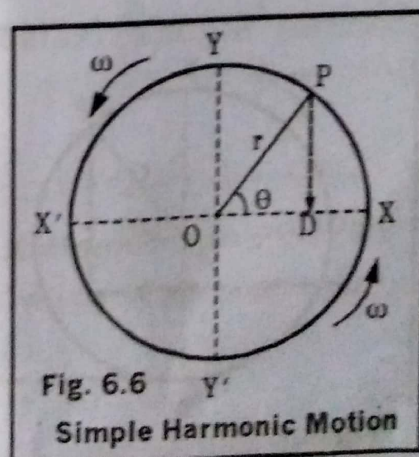
When the acceleration of a body is directly proportional to the displacement from the mean position and is always directed towards the mean position, then motion is said to be Simple Harmonic Motion (SHM).

6.6.1 Introduction

A particle is moving around the circumference of a circle of radius r with anticlockwise sense, with a constant angular velocity ω as shown in Fig.

Let P be the position of the particle at any instant and D be the projection of P on the diameter XX' of the circle.

When the point P moves along the circumference of the circle from X to Y , D moves from X to O ; When



P move from Y to X' , D moves from O to X' . Similarly when P moves from X' to Y' , D moves from X' to O . Finally when P moves from Y' to X , D moves from O to X . Hence when P completes one revolution, the point D completes one vibration about the point O . This to and fro motion of D is known as Simple Harmonic Motion (S.H.M.).

6.6.4 Terms Used in Simple Harmonic Motion

1. Amplitude

It is the maximum displacement of a body from its mean position. In Fig.6.7 OX or OX' is the amplitude of the particle P . The amplitude is always equal to the radius of the circle.

2. Periodic time

It is the time taken for one complete revolution of the particle. X to X . (or) 0° to 2π .

$$\therefore \text{Periodic time, } t_p = \frac{2\pi}{\omega} \text{ seconds}$$

We know that the acceleration,

$$a = \omega^2 x \text{ or } \omega^2 = \frac{a}{x} \text{ or } \omega = \sqrt{\frac{a}{x}}$$

$$\therefore t_p = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{x}{a}} = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} \text{ seconds}$$

It is thus obvious, that the periodic time is independent of amplitude.

3. Frequency

It is the number of cycles per second and is the reciprocal of time period, t_p .

$$\therefore \text{Frequency, } f = \frac{\omega}{2\pi} = \frac{1}{t_p} = \frac{1}{2\pi} \sqrt{\frac{a}{x}} \text{ Hz}$$

$$[\text{Hz} = \text{Hertz}]; 1 \text{ Hz} = 1 \frac{\text{cycle}}{\text{sec}}$$

When the particle moves with angular simple harmonic motion, then the periodic time,

$$t_p = 2\pi \sqrt{\frac{\text{Angular displacement}}{\text{Angular acceleration}}} = 2\pi \sqrt{\frac{\theta}{\alpha}} \text{ seconds}$$

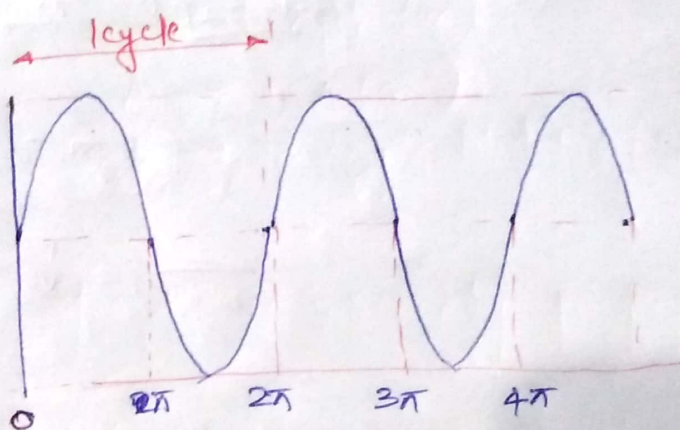
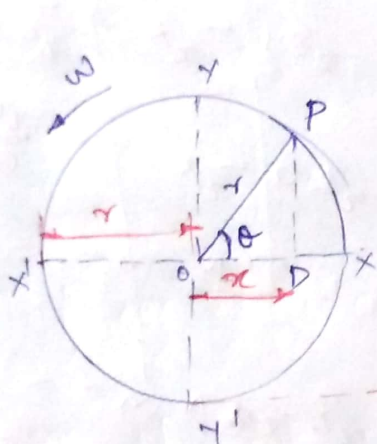
$$\text{Frequency, } f = \frac{1}{2\pi} \sqrt{\frac{\alpha}{\theta}} \text{ Hz}$$

Velocity and acceleration of a particle moving with SHM

P - Position of particle after t seconds.

θ - Angle turned by particle in t seconds

$$\omega = \theta/t \Rightarrow \boxed{\theta = \omega t}$$



Since D is projection of P on the diameter XX' , displacement of D from its mean position O is.

$$x = r \cos \theta = r \cos \omega t$$

∴ velocity of D

$$v_D = \frac{dx}{dt} = \frac{d}{dt} (r \cos \omega t)$$

$$\left[\begin{array}{l} v_D = -\omega r \sin \omega t \\ v_D = -\omega r \sin \theta \\ \sin \theta = PD/OP = \frac{\sqrt{r^2 - x^2}}{r} \\ \therefore v_D = -\omega \sqrt{r^2 - x^2} \end{array} \right]$$

Max velocity $\Rightarrow$ when $x=0$.

$$v_{D_{\max}} = -\omega r$$

Acceleration, $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt} (-\omega r \sin \theta)$

$$a = \frac{d}{dt} (-\omega r \sin \omega t)$$

$$\left[a = -\omega^2 r \cos \omega t = -\omega^2 r \cos \theta \right] \text{ or } \left[a = -\omega^2 x \right] \quad \begin{array}{c} (1) \\ \uparrow \end{array}$$

Max acceleration when $\theta=0$, $\cos \theta=1$ or $x=r$

$$\therefore a_{\max} = -\omega^2 r$$

From (1) it is clear that acceleration of D is proportional to the displacement x . $a=0$, when $x=0$ & $a=\max$, when $x=r$

Also -ve sign indicates that direction of acceleration is opposite to the direction in which x increases.

SHM is a periodic motion. All SHM are periodic motion

but all periodic motions are not SHM.

A generic motion can become SHM if it satisfies 2

Conditions

- If vibrating body's acceleration is towards the centre or mean position
- If its acceleration is proportional to the distance from mean position.

Differential Equation of SHM

From (1)

$$a = -\omega^2 x$$

$$a = \frac{d^2 x}{dt^2} \Rightarrow \boxed{\frac{d^2 x}{dt^2} + \omega^2 x = 0.}$$

Solution is given as $\boxed{x = A \cos \omega t + B \sin \omega t}$

Formulae Req'd for problems.

$$\omega = \sqrt{k/m} = \sqrt{g/l}$$

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T}$$

$$k = W/l$$

$$f = \frac{1}{2\pi} \sqrt{k/m}$$

$$= \frac{1}{2\pi} \sqrt{g/l}$$

$$v_{D, \max} = -\omega r$$

$$a_{D, \max} = -\omega^2 r$$

$$v_D = -\omega r \sin \theta$$
$$= -\omega r \sqrt{r^2 - x^2}$$

$$a_D = -\omega^2 r \cos \theta$$
$$= -\omega^2 x$$

For longitudinal vibration, $\delta = WL/AE$

$W \rightarrow$ load (N)

$L \rightarrow$ Length (m)

$A \rightarrow$ CSA (m²)

$E \rightarrow$ Young's Modulus (N/m²)

Spring Stiffness of a coiled spring is inversely proportional to the no. of coils in spring

ie $k \propto 1/n$ n : no. of coils.

$\therefore$ if n doubled, k becomes half

" n halved, " " double.

Problem 6.1: The piston of a steam engine moves with simple harmonic motion. The crank rotates at 200 r.p.m. with a stroke

of 2 metres. Find the velocity and acceleration of the piston, when it is at a distance of 0.8 metre from the centre.

Solution

Given

$$N = 200 \text{ r.p.m or } \omega = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s; } 2r = 2 \text{ m or}$$

$$r = 1 \text{ m; } x = 0.8 \text{ m}$$

Velocity of the piston

$$V = \omega \sqrt{r^2 - x^2} = 20.94 \sqrt{1 - (0.8)^2} = 12.564 \text{ m/s}$$

Acceleration of the piston

$$a = \omega^2 x = 20.94^2 \times 0.8 = 350.79 \text{ m/s}^2$$

Problem 6.2: A point moves with simple harmonic motion. When this point is 0.7 metre from the mid path, its velocity is 10 m/s and when 1.8 metres from the centre of its path its velocity is 3.2 m/s. Find its angular velocity, periodic time and its maximum acceleration.

Solution

Given

$$\text{When } x = 0.7 \text{ m, } V_{0.7} = 10 \text{ m/s; when } x = 1.8 \text{ m, } V_{1.8} = 3.2 \text{ m/s}$$

Angular velocity

ω = Angular velocity of the particle, and

r = Amplitude of the particle

Velocity of the point when it is 0.7 m from the mid path $V_{0.7}$

$$V_{0.7} = 10 = \omega \sqrt{r^2 - x^2} = \omega \sqrt{r^2 - (0.7)^2} \quad \dots(1)$$

Similarly, velocity of the point when it is 1.8 m from the centre $V_{1.8}$,

$$V_{1.8} = 3.2 = \omega \sqrt{r^2 - 1.8^2} \quad \dots(2)$$

Dividing equation (1) by equation (2)

$$\frac{10}{3.2} = \frac{\omega \sqrt{r^2 - (0.7)^2}}{\omega \sqrt{r^2 - 1.8^2}} = \frac{\sqrt{r^2 - (0.7)^2}}{\sqrt{r^2 - 1.8^2}}$$

Squaring on both sides,

$$\frac{100}{10.24} = \frac{r^2 - 0.49}{r^2 - 3.24}$$

$$100r^2 - 324 = 10.24r^2 - 5.02 \quad (\text{or}) \quad 89.76r^2 = 318.98$$

$$\therefore r^2 = 3.55 \quad (\text{or}) \quad r = 1.89 \text{ m}$$

Substituting the value of r in equation (1)

$$10 = \omega \sqrt{1.89^2 - 0.7^2} = 1.75 \omega$$

$$\therefore \omega = \frac{10}{1.75} = 5.713 \text{ rad/s}$$

Periodic time

We know that periodic time,

$$t_p = \frac{2\pi}{\omega} = \frac{2\pi}{5.713} = 1.099 \text{ s}$$

Maximum acceleration

$$a_{\max} = \omega^2 \cdot r = 5.713^2 \times 1.89 = 61.69 \text{ m/s}^2$$

Problem 6.3: A body moving with simple harmonic motion has velocities 10 m/s and 4 m/s at 2 and 4 m distance from mean position. Find amplitude and time period of the body.
(KTU 2045 - May 2014)

Given

Velocity of body $V_1 = 10 \text{ m/s}$

Velocity of body $V_2 = 4 \text{ m/s}$

Distance of point 1 from centre = 2 m

Distance of point 2 from centre = 4 m

To Find

r = Amplitude of the vibration

T = Time period of the vibration

Solution

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T} \quad \dots(1)$$

When $V_1 = 10 \text{ m/s}$, the distance $x_1 = 2 \text{ m}$

$V_2 = 4 \text{ m/s}$, the distance $x_2 = 4 \text{ m}$

$$V = \omega \sqrt{r^2 - x^2}$$

$$V_1 = \omega \sqrt{r^2 - x_1^2}$$

$$10 = \omega \sqrt{r^2 - 2^2}$$

$$\boxed{10 = \omega \sqrt{r^2 - 4}}$$

...(2)

Similarly,

$$V_2 = \omega \sqrt{r^2 - x_2^2}$$

$$4 = \omega \sqrt{r^2 - 4^2}$$

$$4 = \omega \sqrt{r^2 - 16}$$

...(3)

Dividing equation (2) by equation (3), we get

$$\frac{10}{4} = \frac{\omega \sqrt{r^2 - 4}}{\omega \sqrt{r^2 - 16}} = \frac{\sqrt{r^2 - 4}}{\sqrt{r^2 - 16}}$$

Squaring on both sides, we get

$$\left(\frac{10}{4} \right)^2 = \frac{r^2 - 4}{r^2 - 16}$$

$$6.25 = \frac{r^2 - 4}{r^2 - 16}$$

$$6.25 (r^2 - 16) = r^2 - 4$$

$$6.25r^2 - 100 = r^2 - 4$$

$$6.25r^2 - r^2 = 100 - 4$$

$$5.25r^2 = 96$$

$$5.25r^2 = 96$$

$$\boxed{r = 4.3 \text{ m}}$$

Amplitude $r = 4.3 \text{ m}$

Substituting the value r in equation (2), we get

$$10 = \omega \sqrt{4.3^2 - 16}$$

$$10 = 1.577 \omega$$

$$\omega = 6.34 \text{ rad/sec}$$

Substitute the value ω in equation (1) we get

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6.34} = 0.99 \approx 1.0 \text{ sec}$$

Time period

$$T \approx 1.0 \text{ sec}$$

Problem 6.4: A body is moving with simple harmonic motion and has an amplitude of 5 m and period of complete oscillation as 4 sec. Find the time required by the body in passing between two points which are at a distance of 3 m and 2 m from the centre and are on the same side. (KTU 1799 - April 2014)

Given Data

Amplitude $r = 5 \text{ m}$

Period $T = 4 \text{ sec}$

Distance of 1st point from centre $x_1 = 3 \text{ m}$

Distance of 2nd point from centre $x_2 = 2 \text{ m}$

Solution

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T} = \frac{2 \times \pi}{4} = 1.57 \text{ rad/sec}$$

Let

$t_1 \rightarrow$ time taken by the body from centre to the 1st point

$t_2 \rightarrow$ time taken by the body from centre to the 2nd point

Then

$t_2 - t_1$ - Time required by the body in passing between two points.

The distances of the 1st and 2nd point from centre is given by equation as,

$$x = r \cos \omega t$$

For the 1st point

$$x_1 = r \cos \omega t_1$$

$$3 = 5 \cos (1.57 \times t_1) \quad (\because \omega = 1.57)$$

$$\frac{3}{5} = \cos (1.57 \times t_1)$$

$$0.6 = \cos (1.57 \times t_1)$$

$$1.57 t_1 = (53.13)$$

$$\left| \cos^{-1} (0.6) = 53.13 \right|$$

$$1.57 \times t_1 = \left(53.13 \times \frac{\pi}{180} \text{ rad} \right)$$

$$1.57 \times t_1 = 0.9272 \text{ rad}$$

$$t_1 = 0.591 \text{ s}$$

$$\boxed{t_1 = 0.591 \text{ sec}}$$

For the 2nd point

$$x_2 = r \cos (\omega t_2)$$

$$2 = 5 \cos (1.57 \times t_2)$$

$$\frac{2}{5} = \cos (1.57 \times t_2)$$

$$0.4 = \cos (1.57 \times t_2)$$

$$1.57 \times t_2 = (66.42^\circ)$$

$$1.57 t_2 = \left(66.42 \times \frac{\pi}{180} \text{ rad} \right)$$

$$1.57 \times t_2 = 1.159$$

$$t_2 = \frac{1.159}{1.57} = 0.738 \text{ sec}$$

$$t_2 = 0.738 \text{ sec}$$

$\therefore$ Time required by the body in passing between the two points $t = t_2 - t_1 = 0.738 - 0.591 = 0.1474 \text{ sec}$

Formulae:

For finding static deflection for Longitudinal Vibrations.

$$1. \delta = \frac{Wl}{EA}$$

where $W \rightarrow$ Load attached to the shaft (or) beam in N

$l \rightarrow$ Length of shaft (or) beam in m

$E \rightarrow$ Young's Modulus (or) Modulus of elasticity (N/m^2)

$A \rightarrow$ Cross sectional area of the shaft (or) beam (m^2)

$$= \frac{\pi}{4} d^2 \text{ for Solid shaft}$$

$$= \frac{\pi}{4} (d_o^2 - d_i^2) \text{ for Hollow shaft}$$

where d = dia of shaft.

$$\omega = \text{Angular velocity} = \sqrt{\frac{k}{m}} \text{ rad/sec or } \sqrt{\frac{g}{\delta}} \quad \dots(1)$$

k - stiffness (Force needed for unit deflection)

m = mass

$$k = \frac{W}{\delta} \text{ in N/m}$$

Time Period (t_p)

It is defined as time period to complete one cycle.

$$t_p = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \text{ 'sec'} \quad \dots(2)$$

$$t_p = 2\pi \sqrt{\frac{\delta}{g}} \text{ 'sec'}$$

Frequency of natural vibration

$$f_n = \frac{1}{t_p} = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \text{ 'Hz'}. \quad \dots(3)$$

Natural Frequency of Longitudinal Vibration

$$f_n = \frac{0.4985}{\sqrt{\delta}} \text{ cycles/s (Hz)}$$

$\delta \rightarrow$ static deflection ; $f_n \rightarrow$ Natural Frequency

Problems in Longitudinal Vibration

Problem 6.5: A shaft of 100 mm diameter and 1 metre long is fixed at one end and other end carries a flywheel of mass 1000 kg. Take young's modulus as 200 GN/m²; find Natural frequency of Longitudinal Vibrations.

Given: $d = 100 \text{ mm} = 0.1 \text{ m}$; $l = 1 \text{ m}$; $m = 1000 \text{ kg}$

$$W = mg = (1000 \times 9.81) \text{ N}; E = 200 \text{ GN/m}^2 = 200 \times 10^9 \text{ N/m}^2$$

$$[\because 1 \text{ Giga N} = 1 \times 10^9 \text{ N}]$$

Solution**(i) Find Area**

$$\text{Area } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.1)^2 = 7.8539 \times 10^{-3} \text{ m}^2$$

(ii) To find deflection of shaft

$$\delta = \frac{W.l}{A.E} = \frac{(1000 \times 9.81) \times 1}{7.8539 \times 10^{-3} \times 200 \times 10^9}$$

$$\delta = 6.2453 \times 10^{-6} \text{ m}$$

(iii) Natural Frequency of Longitudinal Vibration

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$

$$f_n = \frac{0.4985}{\sqrt{6.2453 \times 10^{-6}}} = 199.47 \approx 200 \text{ Hz}$$

$$f_n = 200 \text{ Hz}$$

Problem 6.6: A cantilever shaft of 50 mm dia, 300 mm long has a disc of 100 kg attached to the free end. The Young's modulus is 200 GN/m^2 . Determine
(a) Frequency of longitudinal vibration.

Solution:

Given: $d = 50 \text{ mm}$; $l = 300 \text{ mm}$, $m = 100 \text{ kg}$,

$$E = 200 \text{ GN/m}^2$$

$$= 200 \times 10^9 \text{ N/m}^2$$

(i) To find f_n

longitudinal vibration

$$W = mg = 100 \times 9.81 = 981 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2$$

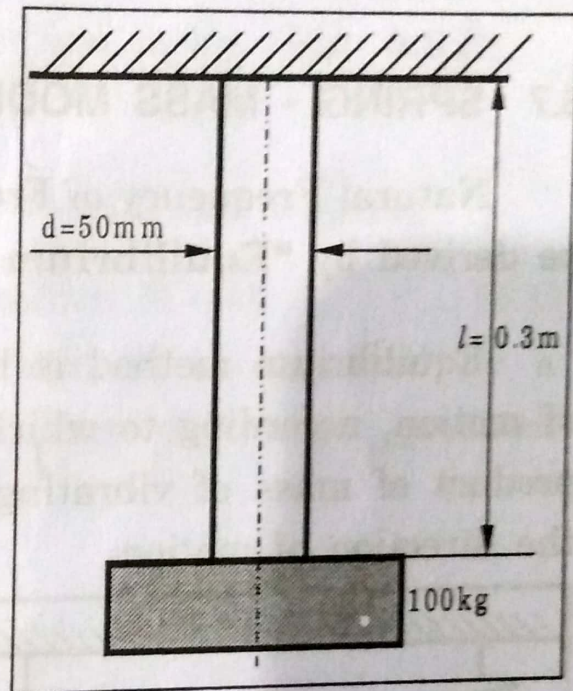
$$= 1.96 \times 10^{-3}$$

$$E = 200 \frac{\text{GN}}{\text{m}^2} = 200 \times 10^9 \text{ N/m}^2$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{W}{A}}{\frac{\delta}{l}} = \frac{Wl}{A\delta}$$

$$(\text{or}) \quad \delta = \frac{Wl}{AE} = \frac{981 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9}$$

$$\delta = 7.49 \times 10^{-7}$$



Natural frequency (f)

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} = \frac{1}{2\pi} \sqrt{\frac{9.81}{7.49 \times 10^{-7}}}$$

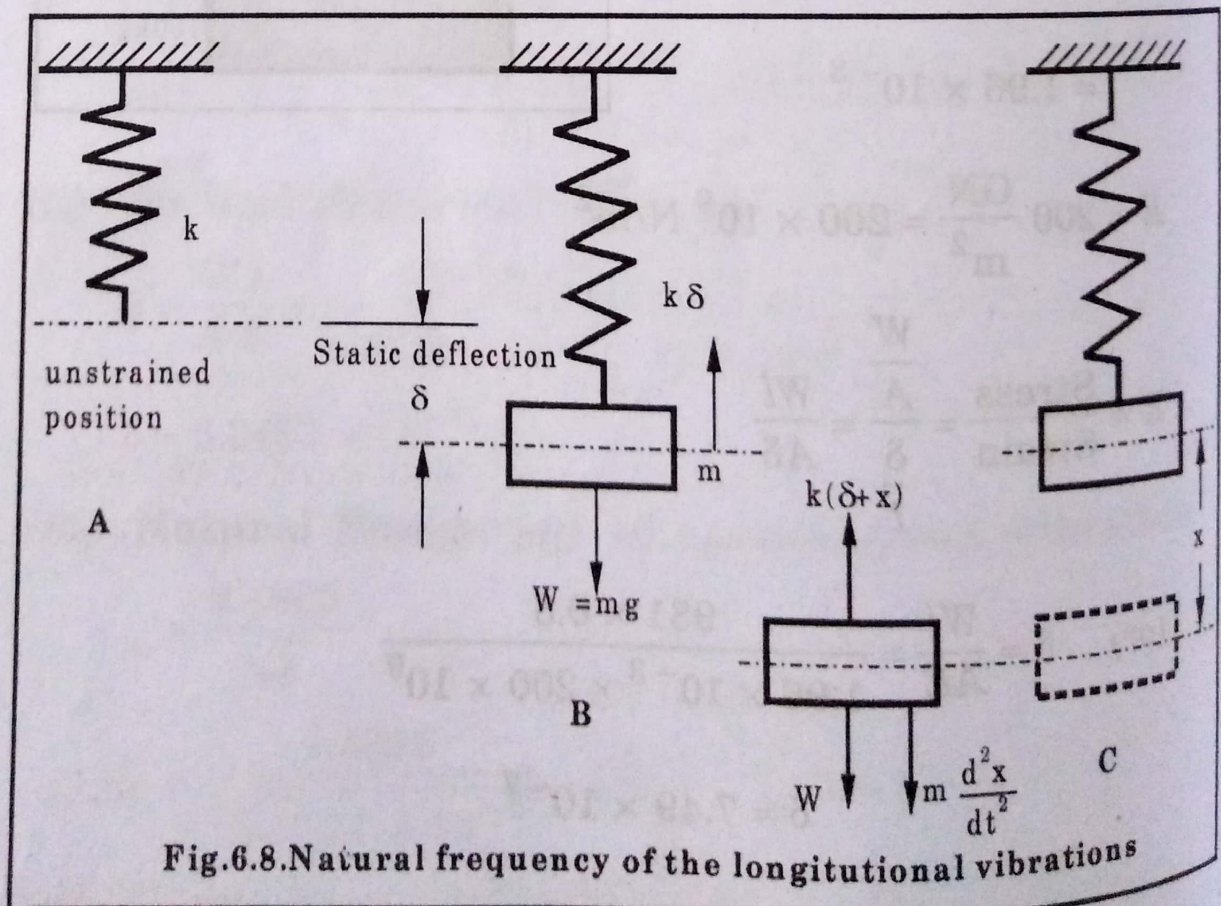
$$f_n = 575.82 \text{ Hz}$$

This is the frequency of longitudinal vibration.

6.7 SPRING - MASS MODEL

Natural Frequency of Free Longitudinal Vibrations can be derived by **"Equilibrium Method"**.

Equilibrium method is based on Newton's second law of motion, according to which inertia force is equal to the product of mass of vibrating body and its acceleration in the direction of motion.



6.25

Consider a spring - mass system as shown in **Fig. 6.8** for equilibrium

$$\begin{aligned}\text{Inertia (or disturbing) force on mass} \\ = \text{Restoring force due to spring}\end{aligned}$$

$$\text{Disturbing force} = \text{mass} \times \text{acceleration}$$

When a weight is attached at the end of the spring, the spring will be deflected (extended) by δ . The spring has attained a new equilibrium position. Now the system is in equilibrium position, by two vertical forces (**Fig. 6.8 B**).

1. W = Weight of the body $\downarrow$
2. Force of spring $k \delta \uparrow$ where k = stiffness of spring

$$\text{Under the equilibrium condition } W = k\delta \quad \dots (6.1)$$

Now pull the weight so that it is displaced by a distance of x and then release. Then the system executes a vibratory motion.

Refer the **Fig. 6.8 C**

After time t , the body is subjected to two forces

1. Weight of the body acting downward $\downarrow$
2. Upward force $= k(\delta + x) \uparrow$

So, Net force (or Restoring force)

$$= W - k(\delta + x)$$

[For our convenience, downward direction is taken as positive and upward as negative.]

$$= W - k\delta - kx$$

$$= k\delta - k\delta - kx$$

$$= -kx \quad \dots (6.2)$$

$$[\because W = k\delta \text{ from equation (6.1)}]$$

We know, by equation of motion of body,

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$-kx = m \times \frac{d^2x}{dt^2} \quad (\text{or})$$

$$m \cdot \frac{d^2x}{dt^2} + kx = 0$$

$$(\text{or}) \quad m\ddot{x} + kx = 0 \quad \dots (6.3)$$

Equation of motion

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \dots (6.3a)$$

From the equation of simple harmonic motion;

We know

$$\frac{d^2x}{dt^2} + \omega^2x = 0 \quad \dots (6.4)$$

Compare the equation (6.3) and (6.4),

We get,

$$\omega^2 = \frac{k}{m}$$

$$(\text{or}) \quad \omega = \sqrt{\frac{k}{m}} \quad \dots (6.5)$$

$$\text{Time period } 't_p' = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} \quad \dots (6.6)$$

Frequency $f = \frac{1}{t_p} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \dots (6.7)$$

We know, $W = k\delta$ from equation

$$mg = k\delta$$

$$\frac{k}{m} = \frac{g}{\delta} \quad \dots (6.7(a))$$

Substitute $\frac{k}{m} = \frac{g}{\delta}$ in equation (6.7), we get,

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} \quad \dots (6.8)$$

Substitute $g = 9.81 \text{ m/s}^2$, we get $f = \frac{0.4985}{\sqrt{\delta}} \text{ Hz} \quad \dots (6.8(a))$

(Depends on the condition of the problem, the δ value will vary.)

If the mass of the spring (m_1) is also taken into consideration, then the periodic time,

$$t_p = 2\pi \sqrt{\frac{m + \frac{m_1}{3}}{k}} \text{ seconds,}$$

$$\text{Frequency, } f = \frac{1}{2\pi} \sqrt{\frac{k}{m + \frac{m_1}{3}}} \text{ Hz}$$

6.7.1 Equivalent Stiffness of Spring

i) Spring in series

For two springs in series, equivalent stiffness is given as [Fig. 6.9(a)].

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

For n springs in series

$$\frac{1}{k_e} = \sum_{i=1}^n \frac{1}{k_i}$$

ii) Springs in parallel

For two springs in parallel, equivalent stiffness is given as [Fig. 6.9 (b)]

$$k_e = k_1 + k_2$$

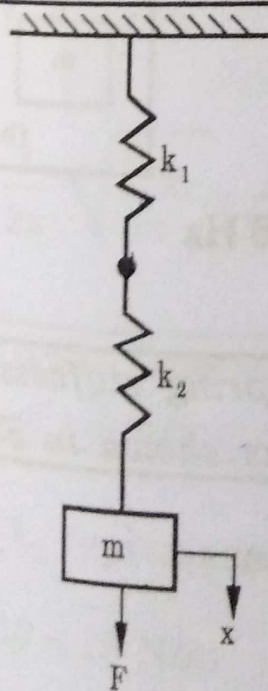


Fig.6.9(a). Springs in series.

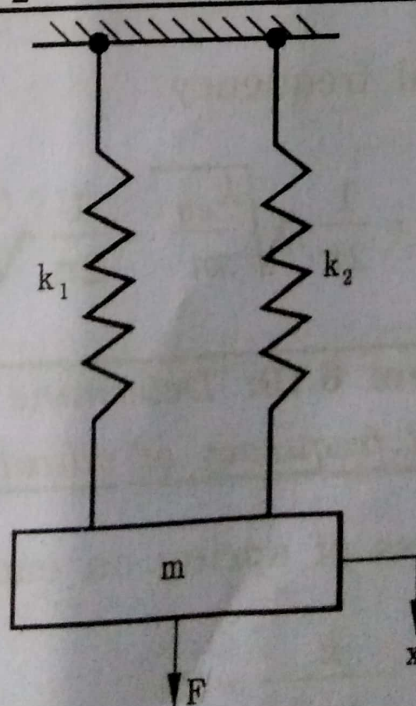
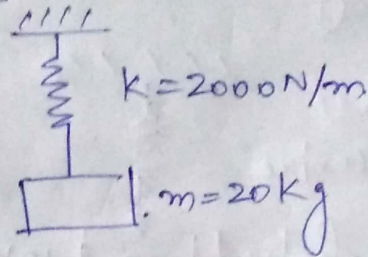


Fig.6.9(b). Springs in parallel.

Q1. Determine equivalent spring stiffness and natural frequency of vibrating system shown.

(a) $k_{eq} = 2000 \text{ N/m}$

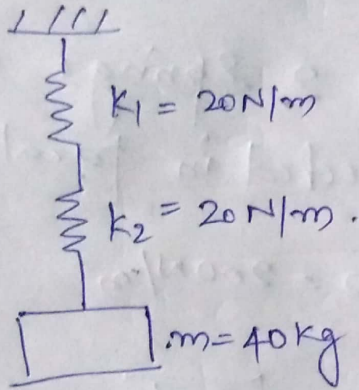


$$k_{eq} = 2000 \text{ N/m}$$

$$\omega = \sqrt{k/m} = \sqrt{\frac{2000}{20}} = 10$$

$$f = \frac{\omega}{2\pi} = \frac{10}{2\pi} = 1.5915 \text{ Hz}$$

(b)



Spring in Series.

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$$

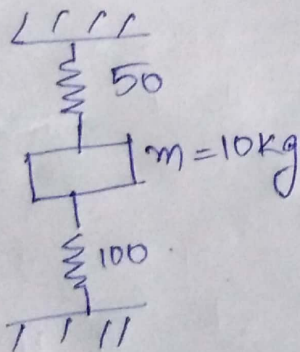
$$k_{eq} = 10 \text{ N/m}$$

$$f = \frac{1}{2\pi} \sqrt{k/m}$$

$$= \frac{1}{2\pi} \sqrt{\frac{10}{40}} = \frac{1}{4\pi} = 0.0796 \text{ Hz}$$

$$= 0.0796 \text{ Hz}$$

(c)



Parallel.

$$k_{eq} = k_1 + k_2$$

$$= 50 + 100$$

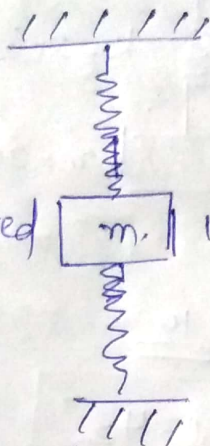
$$= 150 \text{ N/m}$$

$$f = \frac{1}{2\pi} \sqrt{k/m} = \frac{1}{2\pi} \sqrt{\frac{150}{10}}$$

$$= 0.616 \text{ Hz}$$

(d) A spring of stiffness 50 N/m was halved to keep a mass of 10 kg as shown. $\frac{1}{2}$

(A) Since the mass is kept at the centre the spring is halved $k = 50 \text{ N/m}$.



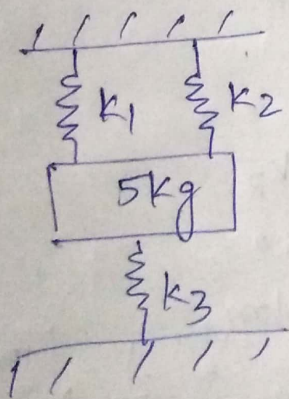
If a spring is halved, the no. of coils/spring reduced to $\frac{1}{2}$ $\therefore$ k becomes double:

$\therefore$ The system is converted to 2 springs of stiffness $k_1 = k_2 = 50 \times 2$ connected in parallel.

$$\therefore k_{eq} = k_1 + k_2 = 100 + 100 = 200 \text{ N/m}$$

$$\therefore f = \frac{1}{2\pi} \sqrt{k/m} = \frac{1}{2\pi} \sqrt{\frac{200}{10}} = \underline{\underline{0.711 \text{ Hz}}}$$

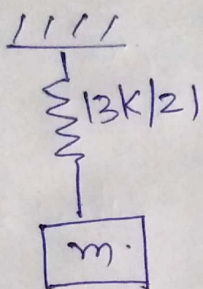
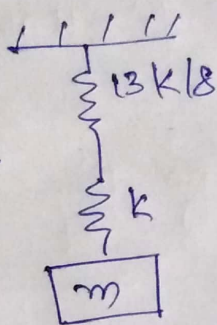
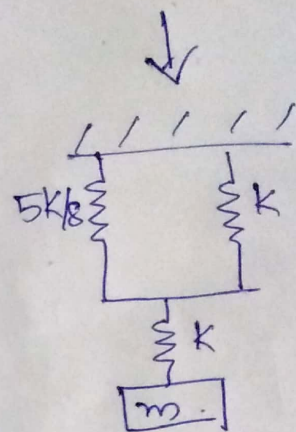
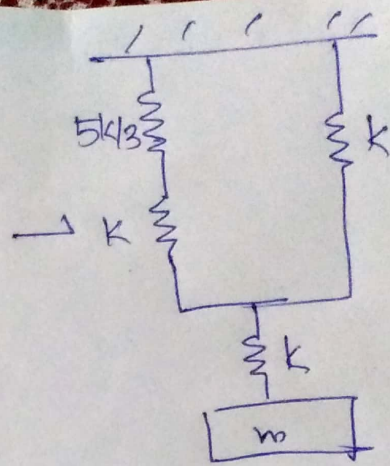
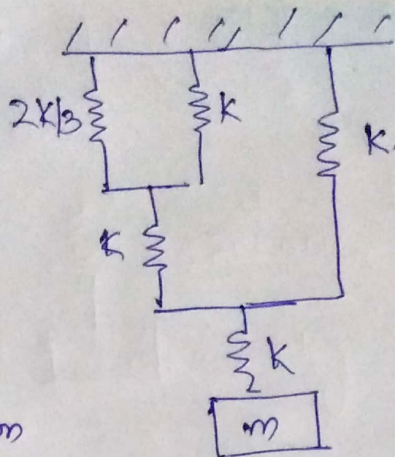
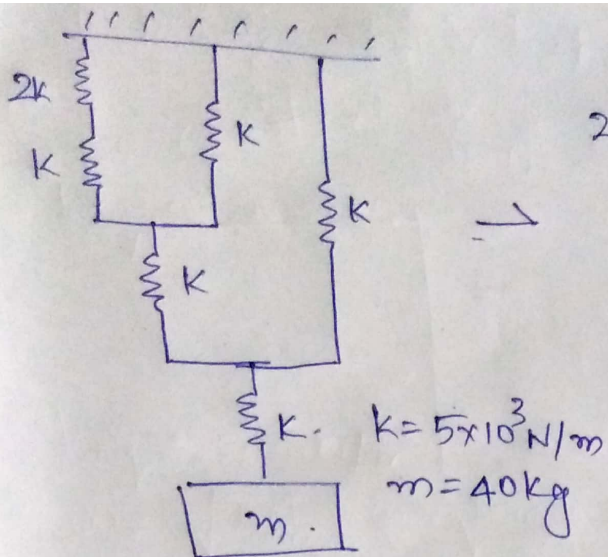
(c)



$$\begin{aligned} k_1 &= 2000 \text{ N/m} \\ k_2 &= 2500 \text{ N/m} \\ k_3 &= 3000 \text{ N/m} \end{aligned}$$

All springs in parallel $\therefore k_{eq} = k_1 + k_2 + k_3 = \underline{\underline{7500 \text{ N/m}}}$

$$\begin{aligned} f &= \frac{1}{2\pi} \sqrt{k/m} = \frac{1}{2\pi} \sqrt{\frac{7500}{5}} \\ &= \underline{\underline{6.16 \text{ Hz}}} \end{aligned}$$



$$K_{eq} = \underline{\underline{13K/24}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{13K/24}{40}}$$

MODULE VI

FREE VIBRATION

When any elastic system is displaced from its equilibrium position, the system is having a periodic motion. This periodic motion is known as vibration. Hence a vibration is a periodic motion which repeats itself after a definite interval of time. If the periodic motion takes place without any external force, the vibration is called as free vibration. The free vibrations are of three types.

1. Longitudinal vibrations.
2. Transverse vibrations.
3. Torsional vibrations.

LONGITUDINAL VIBRATION

If a body attached to one end of a weightless spindle, whose other end is said to be having longitudinal vibrations. As the weight moves up and down the spindle is subjected under compression & tension resp.

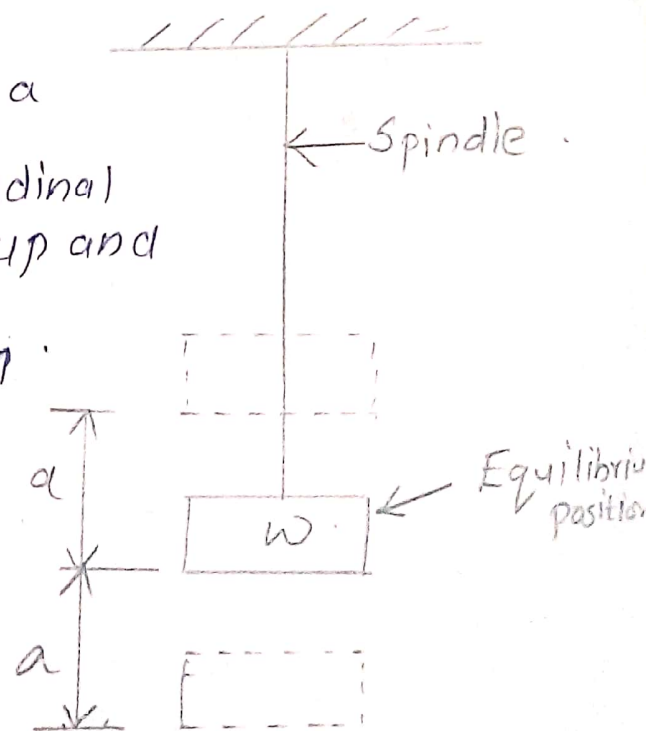
Static deflection

$$\delta = \frac{\text{Stress} \times L}{E}$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

E = Young's modulus.

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$$



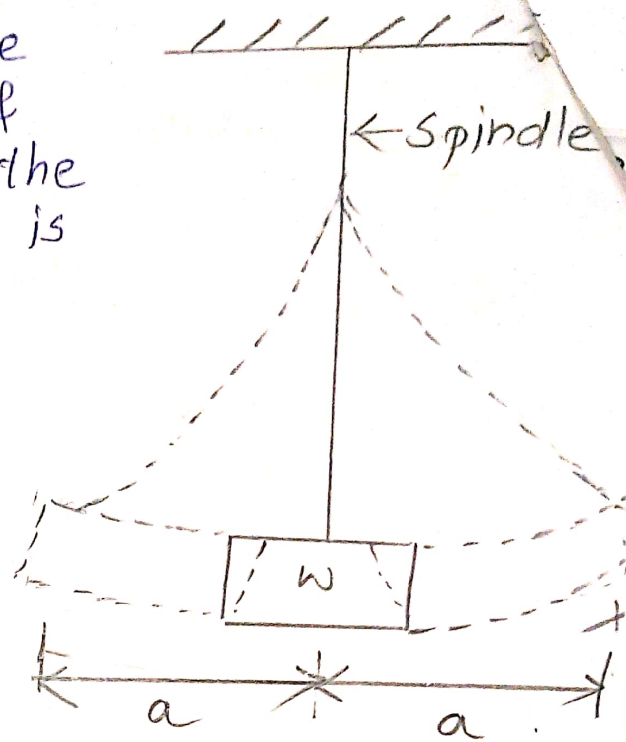
TRANSVERSE VIBRATION

If the weight w attached to the spindle is displaced to one side & released, it starts vibrating in the supporting medium. The spindle is subjected to bending.

Static deflection

$$\delta = \frac{wl^3}{3EI}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$$



TORSIONAL VIBRATION

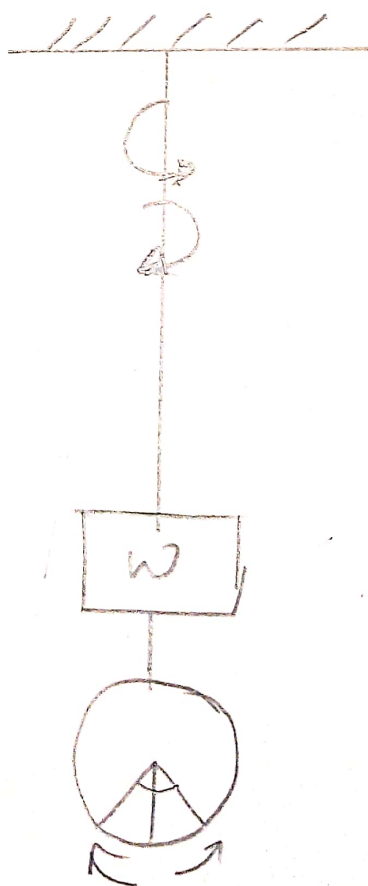
If the weight w is twisted to one side and is released, then it starts moving as in fig. This type of motion is said as Torsional vibration.

$$f = \frac{1}{2\pi} \sqrt{\frac{q}{I}}$$

$$q = \frac{T}{\theta} = \frac{GJ}{L}$$

q - Torsional stiffness.

I - Mass moment of Inertia = Mk^2 .



A vertical shaft 100mm in diameter and 1m in length has its upper end fixed to the ceiling. At the other end it carries a disc of weight 5000N having a radius of gyration of 450mm. The modulus of rigidity for the material of the shaft is $0.8 \times 10^5 \text{ N/mm}^2$. Determine the frequency of torsional vibrations & transverse vibrations if $E = 2 \times 10^5 \text{ N/mm}^2$.

$$d = 100 \text{ mm}$$

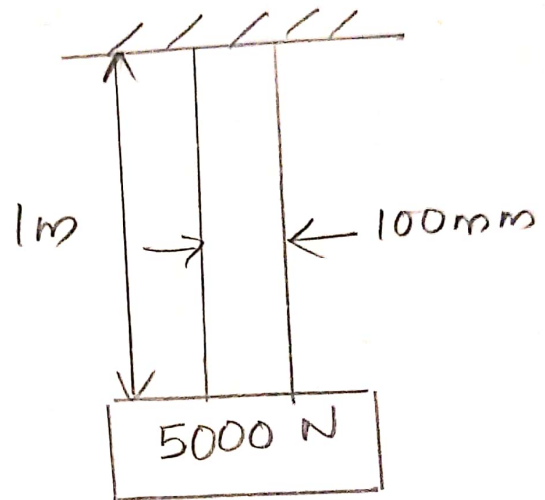
$$L = 1 \text{ m}$$

$$W = 5000 \text{ N}$$

$$k = 450 \text{ mm}$$

$$G = 0.8 \times 10^5 \text{ N/mm}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$



Transverse vibration.

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$$

$$\delta = \frac{WL^3}{3EI}$$

$$I = \frac{\pi \times d^4}{64} = \frac{\pi \times 100^4}{64} = 4908738.5 \text{ mm}^4$$

$$\delta = \frac{5000 \times 1000^3}{3 \times 2 \times 10^5 \times 4908738.5}$$

$$= 1.69 \text{ mm}$$

$$= 1.69 \times 10^{-3} \text{ m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{9.8}{1.69 \times 10^{-3}}} = \underline{\underline{12.1 \text{ cycles/s}}}$$

Torsional vibration.

$$f = \frac{1}{2\pi} \sqrt{\frac{q}{I}}$$

$$q = \frac{GJ}{L} = \frac{0.8 \times 10^5 \times 9817477.04 \frac{\text{Nmm}^2}{\text{mm}}}{1000 \text{ mm}} = 7.85 \times 10^8 \text{ Nmm}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 100^4}{32} = 9817477.04 \text{ mm}^4$$

$$I = Mk^2, \frac{MR^2}{2}$$

$$= \frac{5000 \text{ N} \times 450^2 \text{ mm}^2}{9.8 \text{ m/s}^2}$$

$$1 \text{ N} = 1 \text{ kg m/s}^2$$

$$1 \text{ N} = 1 \text{ kg} \times 1000 \text{ mm/s}^2$$

$$= 108316326.5 \text{ Nmm}^2$$

$$f = \frac{1}{2\pi} \sqrt{\frac{7.85 \times 10^8 \times 1000 \text{ Nmm}}{1.03 \times 10^8 \text{ Nmm s}^2}}$$

$$= \underline{\underline{13.89 \text{ cycles/s}}}$$

$$= \frac{5000 \text{ N}}{9.8 \times 1000 \text{ mm/s}^2}$$

$$= 5000 \text{ Nmm s}^2$$

2. A vertical shaft 5mm in diameter and 1m in length has its upper end fixed to the ceiling. At the lower end it carries a rotor of diameter 200mm and weight 20 N. The modulus of rigidity for the material of the rotor is $0.85 \times 10^5 \text{ N/mm}^2$. Calculate the frequency of torsional vibrations for the system.

$$d = 5 \text{ mm}$$

$$L = 1 \text{ m}$$

$$W = 20 \text{ N}$$

$$\text{Dia of rotor} = 200 \text{ mm}$$

$$\text{radius} = \frac{200}{2} = 100 \text{ mm}$$

$$G = 0.85 \times 10^5 \text{ N/mm}^2$$

$$f = \frac{1}{2\pi} \sqrt{\frac{q}{I}}$$

$$q = \frac{GJ}{L}$$

$$J = 0.85 \times 10^5 \times \frac{\pi \times 5^4}{32}$$

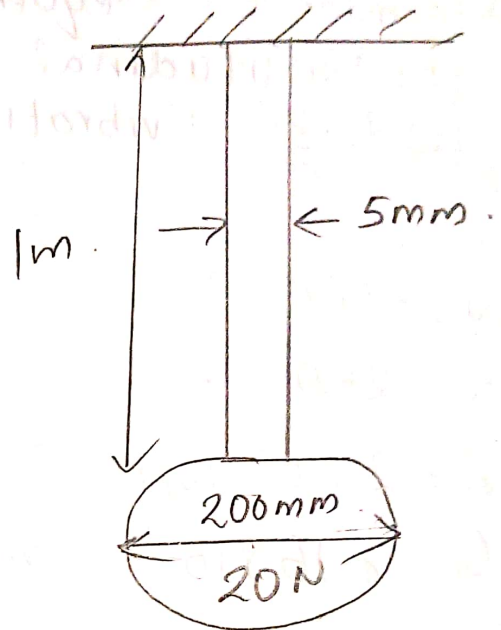
$$= 5215.53 \text{ Nmm}$$

$$\frac{1000}{1000}$$

$$I = \frac{MR^2}{2} = \frac{20 \times 100^2}{9.8 \times 1000 \times 2} = 10.2 \text{ Nmm/s}^2$$

$$f = \frac{1}{2\pi} \sqrt{\frac{5215.53 \text{ Nmm}}{10.2 \text{ Nmm/s}^2}}$$

$$= 3.6 \text{ cycles/s}$$



3. The vertical shaft is fixed at the top and carries a flywheel of weight 5000 N welded at its bottom. The radius of gyration of flywheel is 250 mm.

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$G = 8.16 \times 10^5 \text{ N/mm}^2$$

Determine the frequencies for free.

(1) longitudinal.

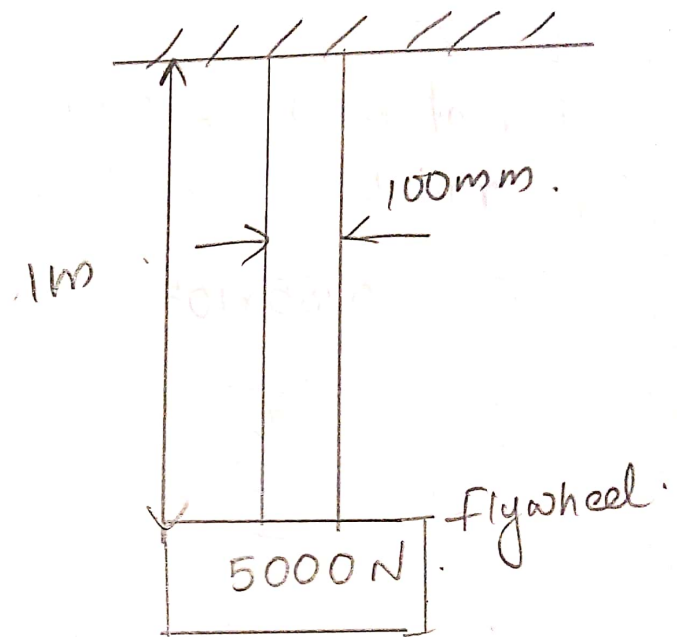
(2) torsional vibrations.

$$W = 5000 \text{ N}$$

$$K = 250 \text{ mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$G = 8.16 \times 10^5 \text{ N/mm}^2$$



i) frequency of longitudinal vibration

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}$$

$$\delta = \frac{\text{stress} \times L}{E}$$

$$= \frac{\text{Load}}{\text{Area} \times E} \times L = \frac{\pi r^2}{\pi \left(\frac{d}{2}\right)^2} \times L = \frac{\pi d^2}{4}$$

$$= \frac{1}{2\pi} \sqrt{\frac{9.8}{3.18 \times 10^{-6}}}$$

$$= \underline{\underline{279.39 \text{ cycles/s}}}$$

$$= \frac{5000 \times 1000}{\frac{\pi \times 100^2}{4} \times 2 \times 10^5}$$

$$= 3.18 \times 10^{-3} \text{ mm}$$

$$= 3.18 \times 10^{-6} \text{ m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{q}{I}}$$

$$q = \frac{G J}{L} = \frac{8.16 \times 10^5 \times \frac{\pi \times 100^4}{32}}{1000} = 8.01 \times 10^9 \text{ Nmm}$$

$$I = M K^2 = \frac{5000}{9.8 \times 1000} \times 250^2 = 31887.75 \text{ Nmm/s}^2$$

$$f = \frac{1}{2\pi} \sqrt{\frac{8.01 \times 10^9}{31887.75}}$$

$$= 79.6 \text{ cycles/s}$$

Simple Harmonic Motion

A body is said to ~~be~~ have SHM, if it moves in a straight line such that its acceleration is always proportional to its distance from a fixed point and is directed towards the fixed point.

Consider a body moving round the circumference of a circle in the anticlockwise direction as shown

Let ω = constant angular velocity of the body.

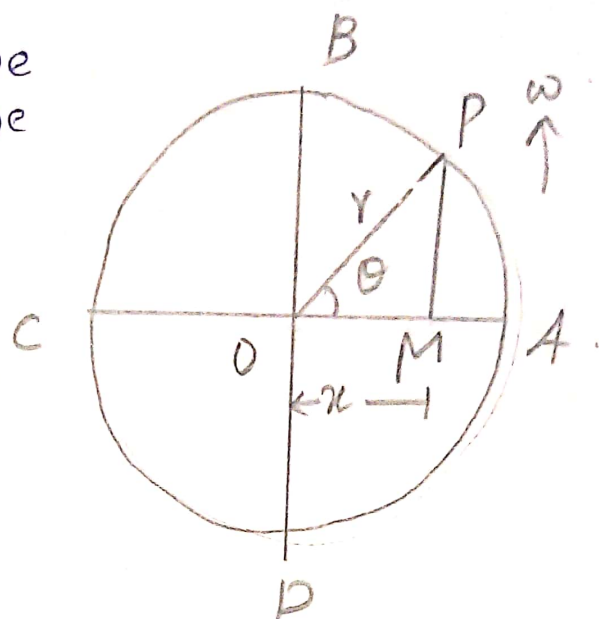
O = centre of the circle.

AC = Fixed diameter of the circle.

P = position of the body.

r = radius of the circle.

M = projection of P on diameter AC .



When the body moves round the circumference of the circle from A to B , the projection of the body on the diameter moves from A to O .

When the body moves round the circumference from B to C , the projection $O - C$.

Body moves from C to D . — projection from $C - O$.

Body moves from D to A . —→ projection from $O - A$.

TERMS IN SHM

1. Amplitude.

The maximum displacement of the point M (which is having SHM) from its mean position is called the Amplitude. O is the mean position of the point M. and maximum displacement of M from the mean position is OA or OC.

$$\text{Amplitude} = OA = \text{Radius of the circle.}$$

2. Oscillation.

The motion of M from A to C and back from C to A is called an oscillation.

3. Period of SHM.

The period of SHM is the time taken by the point M for one complete oscillation i.e. time taken by M is moving from A to C and then from C to A.

The time taken by the point M for one complete oscillation is the same as time taken by the body P moving along the circumference for one complete revolution.

$$T = \text{Period of SHM}$$

$$\omega = \text{constant angular velocity of the body P.}$$

$$T = \frac{\text{Angular displacement for one revolution}}{\text{constant angular velocity}}$$

$$T = \frac{2\pi}{\omega}$$

4. Frequency.

It is defined as the number of cycles per second.

$$\frac{1}{T} = \frac{\omega}{2\pi}$$

Cycles per second is also known as Hertz

$$x = r \cos \omega t$$

$$\theta = \omega t$$

$$V = -r\omega \sin \omega t \quad (\text{Velocity at } M)$$

$$\text{Max velocity} = \omega r$$

$$a = -\omega^2 r \cos \omega t \quad (\text{Acceleration of } M)$$

$$\text{Max acceleration} = \omega^2 r$$

$$V = -\omega \sqrt{r^2 - x^2}$$

$$a = -\omega^2 r \sin \theta$$

It is a type of periodic motion, in which restoring force $\propto$ displacement and acts in the direction opposite to that of displacement

1. Find the velocity and acceleration after 0.3s from the extreme position of a body, moving with SHM with an amplitude of 0.8m and period of complete oscillation of 1.6s.

$$\text{Amplitude, } r = 0.8 \text{ m}$$

Time taken by the body from the extreme position.

$$t = 0.3 \text{ s}$$

$$T = 1.6 \text{ s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega} = 1.6$$

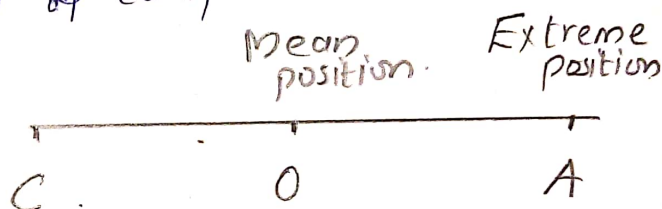
$$\omega = 3.927 \text{ rad/s}$$

$$V = -\omega^2 r \sin \omega t$$

$$= -3.927 \times 0.8 \sin(3.927 \times 0.3)$$

$$= -3.927 \times 0.8 \sin\left(3.927 \times 0.3 \times \frac{180^\circ}{\pi}\right)$$

$$= -2.9 \text{ m/s}$$



$$a = -\omega^2 r \cos \omega t$$

$$= -(3.927)^2 \times 0.8 \cos\left(3.927 \times 0.3 \times \frac{180^\circ}{\pi}\right)$$

$$= -\underline{\underline{4.72 \text{ m/s}^2}}$$

2. The piston of an engine moves with SHM. The crank rotates at 100 rpm and the stroke is 180 cm. Find the velocity and acceleration of the piston, when it is at a distance of 60 cm from the centre.

$$N = 100 \text{ rpm}$$

$$\text{Angular velocity, } \omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 100}{60}$$

$$= \underline{\underline{10.47 \text{ rad/s}}}$$

$$\text{Stroke of piston} = 180 \text{ cm}$$

$$\therefore \text{Amplitude, } r = \frac{180}{2} = 0.9 \text{ m}$$

Displacement of piston from centre.

$$x = 60 \text{ cm} = 0.6 \text{ m}$$

$$x = r \cos \omega t$$

$$0.6 = 0.9 \cos \omega t$$

$$\omega t = \cos^{-1} 0.666$$

$$\omega t = \underline{\underline{48.164}}$$

$$\text{Velocity} = -r\omega \sin \omega t$$

$$= -0.9 \times 10.47 \sin 48.164$$

$$= -\underline{\underline{7.02 \text{ m/s}}}$$

$$a = -\omega^2 r \cos \omega t = -(10.47)^2 0.9 \cos 48.164$$

$$= -\cancel{(10.47)^2} = -65.8 \text{ m/s}^2$$

3. A body is moving with SHM and has velocities of 8 m/s and 3 m/s at a distance of 1.5 m and 2.5 m resp. from the centre. Find the amplitude and time period of the body.

$$V_1 = 8 \text{ m/s} \quad x_1 = 1.5 \text{ m} \quad , \quad r = \text{amplitude of the body}$$

$$V_2 = 3 \text{ m/s} \quad x_2 = 2.5 \text{ m} \quad T = \text{time period of the body}$$

$$V = -\omega \sqrt{r^2 - x^2}$$

$$V_1 = -\omega \sqrt{r^2 - x_1^2}$$

$$8 = -\omega \sqrt{r^2 - 1.5^2} \quad \text{--- (1)}$$

$$V_2 = -\omega \sqrt{r^2 - x_2^2}$$

$$3 = -\omega \sqrt{r^2 - 2.5^2} \quad \text{--- (2)}$$

$$\textcircled{1} \div \textcircled{2}$$

$$\frac{8}{3} = \frac{-\omega \sqrt{r^2 - 1.5^2}}{-\omega \sqrt{r^2 - 2.5^2}}$$

$$2.67 = \frac{\sqrt{r^2 - 1.5^2}}{\sqrt{r^2 - 2.5^2}}$$

$$2.67^2 = \frac{r^2 - 1.5^2}{r^2 - 2.5^2}$$

$$2.67^2 (r^2 - 2.5^2) = r^2 - 1.5^2$$

$$2.67^2 r^2 - 2.67^2 \times 2.5^2 = r^2 - 1.5^2$$

$$(2.67^2 - 1) r^2 = 2.67^2 \times 2.5^2 - 1.5^2$$

$$r = \underline{\underline{2.627m}}$$

Sub r in (i)

$$8 = -\omega \sqrt{2.627^2 - 1.5^2}$$

$$\omega = 3.71 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3.71} = \underline{\underline{1.693s}}$$